bias and mean square error (MSE) for the proposed class of estimators derived up to 1st order approximation. The confidence intervals of population totals are also provided for proposed estimators.

2. SYSTEMATIC SAMPLING DESIGN WITH VARYING PROBABILITY

Most of the researcher choosed systematic sampling for its simplicity and its periodic quality. The unequal probability of systematic sample are sometimes called systematic PPS sampling. In the very first step all the units in the population are arranged at random in a list; can be sited in a specific order or may be presented in naturally happen. In the present study the sample selection procedure is then used by systematic method, when the order of the population units is random (Madow, 1949).

Then in Second, select a random start i.e. select a uniform variable d with $0 \le d < 1$. Then the n selected units are those whose index satisfied this $\Pi_{j-1} \le d + k < \Pi_j$ for some $k = 0$ and $n-1$. Since $np_i \le 1$ every one of the n integers $k = 0,1,\ldots,n-1$ will select a different sampling unit j. The joint inclusion probabilities for unit i and j only depend on the first order inclusion probabilities.

2.1 Regression Cum Ratio Estimator

A scientist who first time given the concept of combining regression and ratio methods and named it a regression-cum-ratio estimator when we have at least two auxiliary variable (Mohanty, 1967). This approach is appealing because if we have multiple auxiliary variables then it is not necessary that all have same relation with the outcome variable i.e. either the regression line passes through the origin or has some intercept. Consider N units $u_1, u_2, \ldots, u_N$ from finite population. Let Y be a quantitative character, taking the values of y_k on $u_k, (1 \le k \le N)$. Let n be a sample size of units selected from N using systematic PPS sampling without replacement when population is in random order. Furthermore, let $X_1, X_2, \ldots, X_q$ are q auxiliary variables and X_i denote the known population total of i^{th} auxiliary variable and Y is a population total for study variable.

The research also provided the variance formula for population total derived $O(N^{-4})$ an approximation of pairwise inclusion probabilities $\pi_{kk'}$ under randomized systematic PPS sampling (Hartley and Rao, 1962). The variance expression for HT estimator of population total say $t'_y = \sum_{k=1}^{n} \frac{y_k}{\pi_k}$ and retaining only $O(N)$ and they obtained

$$V(t'_y) = \sum_{k=1}^{N} \pi_k \left[1 - \frac{(n-1)}{n}\pi_k\right]\left(\frac{y_k}{\pi_k} - \frac{Y}{n}\right)^2.$$

where $\mathbf{e}_{x_r}$, $\mathbf{e}_{x_s}$ and $\mathbf{e}_y$ are the sampling errors that are expected to be very small and assumed that $E(\mathbf{e}_{x_r}) = E(\mathbf{e}_{x_s}) = E(\mathbf{e}_y) = 0$ and also

$$E(\mathbf{e}_x^2) = \left[\sum_{k=1}^{N} \Delta_k d_{x_i}^2\right]_{q\times 1} = \mathbf{\Delta}_x$$

where

$$\Delta_k = \pi_k - \frac{n-1}{n}\pi_k \pi_{k'},$$

$$d_y^2 = \left(\frac{y_k}{\pi_k} - \frac{Y}{n}\right)^2, \quad d_x^2 = \left(\frac{x_k}{\pi_k} - \frac{X}{n}\right)^2 \text{ and}$$

$$d_x d_y = \left(\frac{x_k}{\pi_k} - \frac{X}{n}\right)\left(\frac{y_k}{\pi_k} - \frac{Y}{n}\right),$$

$$d_x d_x = \left(\frac{x_k}{\pi_k} - \frac{X}{n}\right)\left(\frac{x_{k'}}{\pi_{k'}} - \frac{X}{n}\right)$$

where X and Y is population total.

The general class of estimators by combining the regression approach with ratio as follows

$$t_{gm_{rcr}} = \left[t'_y + a\sum_{i=1}^{r}\alpha_i\left(t'_{x_i} - X_i\right)\right]\left[\prod_{i=r+1}^{s}\left(\frac{X_i}{t'_{x_i}}\right)^{c\gamma_i}\right]$$

(2.1.1)

where a and c are appropriate constants to be chosen for making the special cases. Let t'_{x_i} denote the sample total of i^{th} auxiliary variable and t'_y be the sample total of study variable, where t'_{x_i} and t'_y are HT estimators of total. The value of $\alpha's$ and $\gamma's$ are determined by minimizing the MSE. As in suggested class, we divided q auxiliary variables in two groups $r + s = q$; where the first r variables are considered for regression estimator, second s variables are considered for ratio estimator . The unknown constants $\alpha's$ and $\gamma's$ is getting by minimizing the MSE of $t_{gm_{rcr}}$ given in (2.1.1).

Now let $t'_y = Y + e_y$ and $t'_{x_i} = X_i + e_{x_i}$ are sampling errors of study and auxiliary variables respectively. The expression of regression cum ratio estimator becomes

$$t_{gm_{rcr}} = \left[\left(Y + e_y\right) + a\sum_{i=1}^{r}\alpha_i\left(X_i + e_{x_i} - X_i\right)\right]$$
$$\left[\prod_{i=r+1}^{s}\left(\frac{X_i}{X_i + e_{x_i}}\right)^{c\gamma_i}\right]$$

$$t_{gm_{rcr}} = \left[Y + e_y + a\sum_{i=1}^{r}\alpha_i\left(e_{x_i}\right)\right]\left[\left(1 - \sum_{i=r+1}^{s}\frac{c\gamma_i}{X_i}e_{x_i}\right)\right]$$

Ignoring 2nd and higher order terms and we will get

$$\left(t_{gm_{rcr}} - Y\right) = e_y + a\sum_{i=1}^{r}\alpha_i\left(e_{x_i}\right) - Yc\sum_{i=r+1}^{s}\frac{\gamma_i}{X_i}e_{x_i}$$
$$-c\sum_{i=r+1}^{s}\frac{\gamma_i}{X_i}e_{x_i}e_y - ac\sum_{i=1}^{r}\alpha_i\left(e_{x_i}\right)\sum_{i=r+1}^{s}\frac{\gamma_i}{X_i}e_{x_i}$$

In matrix notation can be written as

$$t_{gm_{rcr}} - Y = e_y + a\boldsymbol{\alpha}'\mathbf{e}_r - cY\boldsymbol{\gamma}'\mathbf{X}_s\mathbf{e}_s$$
$$-c\boldsymbol{\gamma}'\mathbf{X}_s\mathbf{e}_s e_y - ac\boldsymbol{\alpha}'\mathbf{e}_r\mathbf{e}_s\mathbf{X}_s\boldsymbol{\gamma} \qquad (2.1.2)$$

where $\boldsymbol{\alpha}'$ and $\mathbf{e}_r$ are the vectors of unknown coefficients of order $(1\times r)$ and sampling errors of order $(r\times 1)$ respectively for r auxiliary variables. Also $\boldsymbol{\gamma}'$ and $\mathbf{e}_s$ are the vectors of unknown coefficients of order $(1\times s)$ and sampling errors of order $(s\times 1)$ respectively for S auxiliary variables. Let consider diagonal matrix of S variables are $\mathbf{X}_s = \left[\frac{1}{X_i}\right]_{s\times s}$.

The expression of bias by taking expectation of eq (2.1.2)as

$$Bias(t_{gm_{rcr}}) = -c\boldsymbol{\gamma}'\mathbf{X}_s\mathbf{S}_{yx_s} - ac\boldsymbol{\alpha}'\mathbf{\Sigma}_{x_r x_s}\mathbf{X}_s\boldsymbol{\gamma}.$$

(2.1.3)

where $\mathbf{e}_r$, $\mathbf{e}_s$ and $\mathbf{e}_y$ are the sampling errors that are expected to be very small quantities and assumed that $E(\mathbf{e}_r) = E(\mathbf{e}_s) = E(\mathbf{e}_y) = 0$ and also

$$E\left(\mathbf{e}_r\mathbf{e}'_s\right) = \left[\sum_{k=1}^{N}\sum_{l=1}^{N}\Lambda_{kl}\hat{x}_{ki}\hat{x}_{lj}\right]_{r\times s} = \mathbf{\Sigma}_{x_r x_s} \text{ and}$$

$$E\left(\mathbf{e}_s\mathbf{e}_y\right)=\left[\sum_{k=1}^{N}\Lambda_{kl}\hat{y}_k\hat{x}_{ki}\right]_{s\times 1}=\mathbf{S}_{yx_s}\cdot$$

The expression for mean square error to term order (1/n) from (2.1.2) fomed as:

$$E\left(t_{g_{m_{rcr}}}-Y\right)^2=E\left(e_y+\boldsymbol{\alpha}'\mathbf{e}_r-Y\boldsymbol{\gamma}'\mathbf{X}_s\mathbf{e}_s\right)^2, \qquad (2.1.4)$$

and for optimum value of $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$, differentiating the above equation of (2.1.4) w.r.t $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ simultaneously and equate to zero

$$\frac{d}{d\boldsymbol{\alpha}}E\left(e_y\boldsymbol{\alpha}'\mathbf{e}_r-Y\boldsymbol{\gamma}'\mathbf{X}_s\mathbf{e}_s\right)^2=0,$$

or

$$-E\left(\mathbf{e}_re_y\right)-E\left(\mathbf{e}_r\mathbf{e}_r'\right)\boldsymbol{\alpha}+YE\left(\mathbf{e}_r\mathbf{e}_s'\right)\mathbf{X}_s\boldsymbol{\gamma}=0,$$

$$-\mathbf{S}_{yx_r}-\boldsymbol{\Sigma}_{x_r}\boldsymbol{\alpha}+Y\boldsymbol{\Sigma}_{x_rx_s}\mathbf{X}_s\boldsymbol{\gamma}=0,$$

$$-\mathbf{S}_{yx_r}=\boldsymbol{\Sigma}_{x_r}\boldsymbol{\alpha}-Y\boldsymbol{\Sigma}_{x_rx_s}\mathbf{X}_s\boldsymbol{\gamma}\cdot \qquad (2.1.5)$$

where $E\left(\mathbf{e}_r\mathbf{e}_y\right)=\left[\sum_{k=1}^{N}\Lambda_{kl}\hat{y}_k\hat{x}_{ki}\right]_{r\times 1}=\mathbf{S}_{yx_r}$,

$$E\left(\mathbf{e}_r\mathbf{e}_r'\right)=\left[\sum_{k=1}^{N}\sum_{l=1}^{N}\Lambda_{kl}\hat{x}_{ki}\hat{x}_{lj}\right]_{r\times r}=\boldsymbol{\Sigma}_{x_r} \text{ and}$$

$$E\left(\mathbf{e}_r\mathbf{e}_s'\right)=\left[\sum_{k=1}^{N}\sum_{l=1}^{N}\Lambda_{kl}\hat{x}_{ki}\hat{x}_{lj}\right]_{r\times s}=\boldsymbol{\Sigma}_{x_rx_s}\cdot$$

Now by differentiating the equation (2.1.4) w.r.t $\boldsymbol{\gamma}$, we have

$$\frac{d}{d\boldsymbol{\gamma}}E\left(e_y+\boldsymbol{\alpha}'\mathbf{e}_r-Y\boldsymbol{\gamma}'\mathbf{X}_s\ \mathbf{e}_s\right)^2=0,$$

or

$$-E\left(\mathbf{e}_se_y\right)-E\left(\mathbf{e}_s\mathbf{e}_r'\right)\boldsymbol{\alpha}+YE\left(\mathbf{e}_s\mathbf{e}_s'\right)\mathbf{X}_s\boldsymbol{\gamma}=0,$$

$$-\mathbf{S}_{ys}-\boldsymbol{\Sigma}_{x_sx_r}\boldsymbol{\alpha}+Y\boldsymbol{\Sigma}_{x_s}\mathbf{X}_s\boldsymbol{\gamma}=0,$$

or

$$-\mathbf{S}_{ys}=\boldsymbol{\Sigma}_{x_sx_r}\boldsymbol{\alpha}+Y\boldsymbol{\Sigma}_{x_s}\mathbf{X}_s\boldsymbol{\gamma}\cdot \qquad (2.1.6)$$

where $E\left(\mathbf{e}_s\mathbf{e}_y\right)=\left[\sum_{k=1}^{N}\Lambda_{kl}\hat{y}_k\hat{x}_{ki}\right]_{s\times 1}=\mathbf{S}_{yx_s}$,

$$E\left(\mathbf{e}_s\mathbf{e}_s'\right)=\left[\sum_{k=1}^{N}\sum_{l=1}^{N}\Lambda_{kl}\hat{x}_{ki}\hat{x}_{lj}\right]_{s\times s}=\boldsymbol{\Sigma}_{x_s} \text{ and}$$

$$E\left(\mathbf{e}_r\mathbf{e}_s'\right)=\left[\sum_{k=1}^{N}\sum_{l=1}^{N}\Lambda_{kl}\hat{x}_{ki}\hat{x}_{lj}\right]_{r\times s}=\boldsymbol{\Sigma}_{x_rx_s}\cdot$$

Now (2.1.5) and (2.1.6) can be written in matrix form as

$$\begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\gamma}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\Sigma}_{x_r} & -Y\boldsymbol{\Sigma}_{x_rx_s}\mathbf{X}_s\\ \boldsymbol{\Sigma}_{x_sx_r} & -Y\boldsymbol{\Sigma}_{x_s}\mathbf{X}_s\end{bmatrix}^{-1}\begin{bmatrix}-\mathbf{S}_{yx_r}\\ -\mathbf{S}_{yx_s}\end{bmatrix}.$$

The results for computing the inverse of matrix of matrices has been provided (Ahmad et al., 2014). According to their results we have

$$\begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\gamma}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\Sigma}_{x_r}^{-1}+Y\boldsymbol{\Sigma}_{x_r}^{-1}\boldsymbol{\Sigma}_{x_rx_s}\mathbf{X}_s\mathbf{H}_s^{-1}\boldsymbol{\Sigma}_{x_sx_r}\boldsymbol{\Sigma}_{x_r}^{-1} & -Y\boldsymbol{\Sigma}_{x_r}^{-1}\boldsymbol{\Sigma}_{x_rx_s}\mathbf{X}_s\mathbf{H}_s^{-1}\\ \mathbf{H}_s^{-1}\boldsymbol{\Sigma}_{x_sx_r}\boldsymbol{\Sigma}_{x_r}^{-1} & -\mathbf{H}_s^{-1}\end{bmatrix},\ \begin{bmatrix}-\mathbf{S}_{yx_r}\\ -\mathbf{S}_{yx_s}\end{bmatrix}$$

where $\mathbf{H}_s^{-1}=Y^{-1}\mathbf{W}_s^{-1}$,

$$\mathbf{W}_s^{-1}=-Y^{-1}\left(\boldsymbol{\Sigma}_{x_s}\mathbf{X}_s-\boldsymbol{\Sigma}_{x_sx_r}\boldsymbol{\Sigma}_{x_r}^{-1}\boldsymbol{\Sigma}_{x_rx_s}\mathbf{X}_s\right)^{-1},$$

or

$$\begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\gamma}\end{bmatrix}=\begin{bmatrix}-\boldsymbol{\Sigma}_{x_r}^{-1}\mathbf{S}_{yx_r}-\boldsymbol{\beta}_{1_{rs}}\mathbf{X}_s\mathbf{W}_s^{-1}\boldsymbol{\beta}_{1_{sr}}\mathbf{S}_{yx_r}+\boldsymbol{\beta}_{1_{rs}}\mathbf{X}_s\mathbf{W}_s^{-1}\mathbf{S}_{yx_s}\\ Y^{-1}\mathbf{W}_s^{-1}\left(\mathbf{S}_{yx_s}-\boldsymbol{\beta}_{1_{sr}}\mathbf{S}_{yx_r}\right)\end{bmatrix}$$

where $\boldsymbol{\beta}_{1_{rs}}=\boldsymbol{\Sigma}_{x_r}^{-1}\boldsymbol{\Sigma}_{x_rx_s}$,

$$\boldsymbol{\alpha}=-\boldsymbol{\Sigma}_{x_r}^{-1}\mathbf{S}_{yx_r}-\boldsymbol{\beta}_{1_{rs}}\mathbf{X}_s\mathbf{W}_s^{-1}\boldsymbol{\beta}_{1_{sr}}\mathbf{S}_{yx_r}+\boldsymbol{\beta}_{1_{rs}}\mathbf{X}_s\mathbf{W}_s^{-1}\mathbf{S}_{yx_s}\cdot$$

(2.1.7)

$$\boldsymbol{\gamma}=Y^{-1}\mathbf{W}_s^{-1}\left(\mathbf{S}_{yx_s}-\boldsymbol{\beta}_{1_{sr}}\mathbf{S}_{yx_r}\right)\cdot \qquad (2.1.8)$$

The MSE obtained by taking square and then expectation of equation (2.1.2). Also by substituting the value of $\boldsymbol{\alpha}'s$ and $\boldsymbol{\gamma}'s$ from (2.1.7) and (2.1.8), the final expression of mean square error as

$$MSE\left(t_{gm_{rcr}}\right)=s_{y_{sys}}^2-\mathbf{S}_{yx_r}'\boldsymbol{\Sigma}_{x_r}^{-1}\mathbf{S}_{yx_r}+\left(\mathbf{S}_{yx_s}'-\mathbf{S}_{yx_r}'\boldsymbol{\beta}_{1_{rs}}\right)\cdot$$

$$\mathbf{W}_s^{-1}\mathbf{X}_s\left(\boldsymbol{\beta}_{1_{sr}}\mathbf{S}_{yx_r}-\mathbf{S}_{yx_s}\right)$$

(2.1.9)

Where $E\left(\mathbf{e}_y^2\right)=\sum_{k=1}^{N}\Lambda_{kl}\hat{y}_{ki}^2=s_{y_{sys}}^2$,

$$E\left(\mathbf{e}_r\mathbf{e}_y\right)=\left[\sum_{k=1}^{N}\Lambda_{kl}\hat{y}_k\hat{x}_{ki}\right]_{r\times 1}=\mathbf{S}_{yx_r}$$

$$E\left(\mathbf{e}_s\mathbf{e}_y\right)=\left[\sum_{k=1}^{N}\Lambda_{kl}\hat{y}_k\hat{x}_{ki}\right]_{s\times 1}=\mathbf{S}_{yx_s} \text{ and}$$

$$E\left(\mathbf{e}_r\mathbf{e}_r'\right)=\left[\sum_{k=1}^{N}\sum_{l=1}^{N}\Lambda_{kl}\hat{x}_{ki}\hat{x}_{lj}\right]_{r\times r}=\boldsymbol{\Sigma}_{x_r}\cdot$$

$$E\left(\mathbf{e}_r\mathbf{e}_r'\right)=\left[\sum_{k=1}^{N}\sum_{l=1}^{N}\Lambda_{kl}\hat{x}_{ki}\hat{x}_{lj}\right]_{r\times r}=\boldsymbol{\Sigma}_{x_r}\cdot$$

2.2 Jackknife Replication Variance

The Jackknife variance estimator for the estimator developed in (2.1.1) is considered and the Jackknife replicates for (2.1.1) after successively deleting k^{th} units are:

$$t_{gm_{rcr}}^{(k)}=\left[t_y'^{(k)}+a\sum_{i=1}^{r}\alpha_i\left(t_{x_i}'^{(k)}-X_i\right)\right]\left[\prod_{i=r+1}^{s}\left(\frac{X_i}{t_{x_i}'^{(k)}}\right)^{c\gamma_i}\right]. \qquad (2.2.1)$$

Where

$$t_y'^{(k)}=\begin{cases}\sum_{s_k}\frac{y_k}{\pi_k} & if \quad k\in s_k\\ \sum_{s}\frac{y_k}{\pi_k} & if \quad k\in s\end{cases}$$

(2.2.2)

and

$$t_{x_i}'^{(k)}=\begin{cases}\sum_{s_k}\frac{x_k}{\pi_k} & if \quad k\in s_k\\ \sum_{s}\frac{x_k}{\pi_k} & if \quad k\in s\end{cases}$$

(2.2.3)

where the sample with k^{th} units being deleted belongs to S_k and complete sample belongs to *s*.

The pseudo-value defined as

$$t_{j_{rcr}}=nt_{gm_{rcr}}-(n-1)t_{gm_{rcr}}^{(k)},$$

where $t_{gm_{rcr}}$ is defined in (2.1.1) and $t_{gm_{rcr}}^{(k)}$ defined in (2.2.1).

The Jackknife variance estimator is

$$v(t_{j_{rcr}})=\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(t_{j_{rcr}}-t_{JK_{rcr}}\right)^2,$$

where $t_{JK_{rcr}}$ defined as

$$t_{JK_{rcr}} = \frac{1}{n}\sum_{i=1}^{n} t_{j_{rcr}} .$$

The Jackknife variance estimator form as

$$v(t_{j_{rcr}}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(t_{j_{rcr}} - t_{JK_{rcr}}\right)^2 ,$$

$$= \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\left(nt_{gm_{rcr}} - (n-1)t^{(k)}_{gm_{rcr}}\right) - \frac{1}{n}\sum_{i=1}^{n}\left(nt_{gm_{rcr}} - (n-1)t^{(k)}_{gm_{rcr}}\right)\right)^2 ,$$

$$= \frac{(n-1)^2}{n(n-1)}\sum_{i=1}^{n}\left(t^{(k)}_{gm_{rcr}} - \frac{1}{n}\sum_{i=1}^{n} t^{(k)}_{gm_{rcr}}\right)^2 ,$$

Or

$$v(t_{j_{rcr}}) = \frac{n-1}{n}\sum_{i=1}^{n}\left(\left[t'^{(k)}_{y} + a\sum_{i=1}^{r}\alpha_i\left(t'^{(k)}_{x_i} - X_i\right)\right]\left[\prod_{i=r+1}^{s}\left(X_i / t'^{(k)}_{x_i}\right)^{c\gamma_i}\right]\right.$$
$$\left. - \frac{1}{n}\sum_{i=1}^{n}\left(\left[t'^{(k)}_{y} + a\sum_{i=1}^{r}\alpha_i\left(t'^{(k)}_{x_i} - X_i\right)\right]\left[\prod_{i=r+1}^{s}\left(X_i / t'^{(k)}_{x_i}\right)^{c\gamma_i}\right]\right)\right)^2 ,$$

or

$$v(t_{j_{rcr}}) = \frac{n-1}{n}\sum_{s}\left(\left[\sum_{s_k} y_k/\pi_k + a\sum_{i=1}^{r}\alpha_i\left(\sum_{s_k} x_{ki}/\pi_k - X_i\right)\right]\left[\prod_{i=r+1}^{s}\left(X_i \Big/ \sum_{s_k} x_{ki}/\pi_k\right)^{c\gamma_i}\right]\right.$$
$$\left. - \frac{1}{n}\sum_{s}\left(\left[\sum_{s_k} y_k/\pi_k + a\sum_{i=1}^{r}\alpha_i\left(\sum_{s_k} x_{ki}/\pi_k - X_i\right)\right]\left[\prod_{i=r+1}^{s}\left(X_i \Big/ \sum_{s_k} x_{ki}/\pi_k\right)^{c\gamma_i}\right]\right)\right)^2 . \quad (2.2.4)$$

Different special cases of proposed class have also been developed under systematic sampling with varying probabilities. The expressions of Bias, mean square and their Jackknife replication variance estimators for the special class of estimators as follows;

2.3 Special Cases

The performance of regression estimator is similar to ratio estimator when relationship passing through origin. But in many practical situations when regression line not passed through the neighbourhood of origin in this case regression performs better than ratio estimator (Singh and Espejo, 2003). The number of special cases of proposed class given in (2.1.1) with their expression of bias, mean square errors and Jackknife replication variance estimators are given in the followingtable;

Table 2.1 Special Cases under Systematic Sampling

Special cases	Special Case-I (Regression Estimator) when $a=1$, $c=0$ substitute in 2.1.1	Special Case-II (Ratio Estimator) When $c=1$ and $a=0$ substitute in 2.1.1
Estimator	$t_{gm_r} = t'_y + \sum_{i=1}^{r}\alpha_i\left(t'_{x_i} - X_i\right)$ Where $\alpha_i = -\Sigma^{-1}_{x_r}\mathbf{S}_{yx_r}$	$t_{gm_r} = t'_y\prod_{i=r+1}^{s}\left(\frac{X_i}{t'_{x_i}}\right)^{\gamma_{1i}}$
Bias	$Bias(t_{gm_{lr}}) = 0$	$Bias(t_{gm_r}) = -\gamma'_1\mathbf{X}_s\mathbf{S}_{yx_s}$ where $\gamma_1 = Y^{-1}\mathbf{X}^{-1}_s\Sigma^{-1}_{x_s}\mathbf{S}_{yx_s}$
Mean Square Error	$MSE(t_{gm_{lr}}) = s^2_{y_{lr}} - \mathbf{S}'_{yx_r}\Sigma^{-1}_{x_r}\mathbf{S}_{yx_r}$	$MSE(t_{gm_r}) = s^2_{y_{gm}} - \mathbf{S}'_{yx_s}\Sigma^{-1}_{x_s}\mathbf{S}_{yx_s}$
Replication Variance estimator	$v(t_{j_{lr}}) = \frac{(n-1)}{n}\sum_{s}\left(t^{(k)}_{gm_{lr}} - \frac{1}{n}\sum_{i=1}^{n}t^{(k)}_{gm_{lr}}\right)^2$ Where $t^{(k)}_{gm_{lr}} = \sum_{s_k}\frac{y_k}{\pi_k} + \sum_{i=1}^{r}\alpha_{li}\left\{\sum_{s_k}\frac{x_k}{\pi_k} - X_i\right\}$	$v(t_{j_r}) = \frac{(n-1)}{n}\sum_{s}\left(t^{(k)}_{gm_r} - \frac{1}{n}\sum_{i=1}^{n}t^{(k)}_{gm_r}\right)^2$ Where $\sum_{s_k} y_k/\pi_k\prod_{i=r+1}^{s}\left(X_i \Big/ \sum_{s_k}\frac{x_{ki}}{\pi_k}\right)^{\gamma_{1i}}$

3.Simulation Study

In this section conducted a simulation study, A finite artificial population of size $N = 1000$ with six auxiliary variables $(x_1, x_2, ..., x_6)$, where the population elements are independently generated from $x_1 \sim N(2.1, 0.79)$, $x_2 \sim N(7, 1.2)$, $x_3 \sim N(11, 1.8)$, $x_4 \sim N(9, 2.3)$, $x_5 \sim N(6, 4.5)$, and $x_6 \sim N(12, 2.7)$. The model is $y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_6 x_6 + e_i$ with $(\beta_0, \beta_1, \beta_2, ..., \beta_6) = (0.23, 3, 2.3, 2.20, 1.75, 1.30, 5)$ and $e_i \sim N(0,1)$. The variable $z_i \sim N(8, 3.5)$ was used as a size measure for the unequal probability sampling.

The results generated for 4,000 samples where a sample of $n = 200$ is selected by employing the scheme of sampling, when the order of the population units is random (Madow, 1949). This systematic selection procedure is given in R language software and by default its command is s = UPrandomsystematic(pik,eps=1e-6) where pik is a vector of inclusion probability and eps control value, which is by default equal to 1e-6. Inclusion probabilities calculated by $\pi_k = \frac{nX_k}{X}$, it can also get by default command in R language, i.e. pik=inclusionprobabilities(a,n) where "a" is vector of positive integer and n size of sample.

Firstly, for replication variance estimation, we estimate parameter from the whole sample $n = 200$ then dropping one unit of sample and then recalculated the statistic of interest based on the incomplete sample $n - 1 = 199$. A pseudo-value is then computed as the difference between the whole sample estimate and the partial estimate from incomplete sample. Using these pseudo values we calculated estimated total, Jackknife replication variance estimate and also 95% confidence interval.

3.1Analysis of Proposed Estimators

For each of the $S = 4000$ samples, percent relative bias of the total estimator and the Jackknife variance estimates is calculated with respect to the parameter of interest (population total T) and true mean squared error respectively i.e.,

$$P.R.B = \frac{E(t) - T}{T} \times 100 ,$$

where $$E(t) = \frac{1}{4000}\sum_{i=1}^{4000} t_i .$$

$$P.R.B = \frac{E(V_J(t)) - MSE_{true}}{MSE_{true}} \times 100 ,$$

where $E(V_J(t)) = \frac{1}{4000}\sum_{i=1}^{4000} V_J(t_i)$ and $MSE_{true} = \frac{1}{4000}\sum_{i=1}^{4000}(t_i - T)^2 .$

The 95 percent confidence interval coverage by each of the variance estimators for the proposed estimators is:

$$mean(pseudo) \pm t_{1-\alpha/2, n-1}\sqrt{\mathrm{var}(pseudo)/n}$$

3.2Simulation Results

The results for 4,000 samples for total estimators with their bias, mean square error, replication variance, percent relative bias and confidence intervals are present in Table1, 2 and 3.

Table 1: Bias, MSE & Replication Variance Estimates for (4,000 Samples)

Estimator	Bias	Mean Square Error	Replication Variance Estimates
$t_{g_{m_{lr}}}$	-89.10782	40136582	43247768
$t_{g_{m_r}}$	-38.04006	8613089	482425.88
$t_{g_{m_{rcr}}}$	13.02923	4198198	70335115

Table-1 represents the results of bias, mean square error and replication variance for the point estimators, based on systematic sampling design with varying probabilities are regression, ratio and regression-cum-ratio estimator. According to table-1 there are regression-cum-ratio estimator which have minimum MSE with smaller but positive bias about (13.0292) than other estimators. The regression and ratio have high MSE but negative bias. From table-1 we can see that the variance estimator using Jackknife replication variance is smaller for ratio while regression-cum-ratio estimator has greater replication variance.

Table 2: Relative bias (RB) of Total for the proposed estimators (4,000 samples)

Estimator	Estimated Total	RB (%)
$t_{g_{m_{lr}}}$	1480.608	-0.9868941
$t_{g_{m_r}}$	1853.468	-0.9896118
$t_{g_{m_{rcr}}}$	1887.988	-0.9880399

Table-2 shows the percent relative biases (PRB) of the eight types of estimators for total. All the biases are less than 1% in the absolute value and according to findings of relative biases all these estimators behave reasonably in term of point estimation. Table-3 represents the percent relative bias (PRB) of Jackknife variance estimator and confidence interval (C-I) for the proposed estimators. The results show that, regression estimator has smaller positive relative bias as compared to other estimators. On the other hand ratio having negative PRB.

Table 3: Relative bias (RB) of Jackknife Variance Estimator and Confidence Interval (C-I) for the Proposed estimators (4,000 samples)

Estimator	Estimated Total	S.E	RB (%)	95% Confidence Interval
$t_{g_{m_{lr}}}$	1480.608	465.14	0.0775	563.40 – 2397.82
$t_{g_{m_r}}$	1853.468	49.11	-0.9439	1756.63 – 1950.31
$t_{g_{m_{rcr}}}$	1887.988	593.02	15.7536	718.62 - 3057.10

Table-3 represents the confidence interval for population total using standard error which generated through Jackknife variance estimator. For this purpose generate 4000 samples of size 200 and for each sample, we compute the Jackknife 95% confidence interval. According to those results, we are 95% confident to say that this interval will contain the true value of population total. It is important to note that the narrow intervals of estimators leads to a higher precision than wider intervals. The ratio estimator have narrow limits to each other than others.

4.Conclusion

In this study, we have developed a general class of estimators for estimating the population total using multi-auxiliary variables under systematic PPS sampling design with varying probabilities. The overall result showed that the variances are calculated by Jackknife variance estimators with smaller bias and confidence intervals using these replication estimates provides precise interval. We may also conclude from simulation regarding performance of proposed estimators, the class of regression-cum-ratio has smaller MSE with least bias than all suggested estimators so we preferred this estimators.

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