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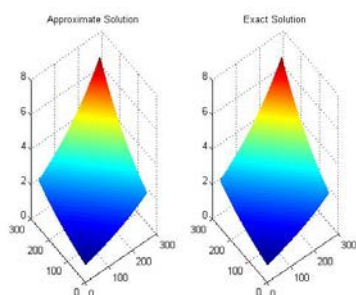


Figure 1: (Test Problem 1)

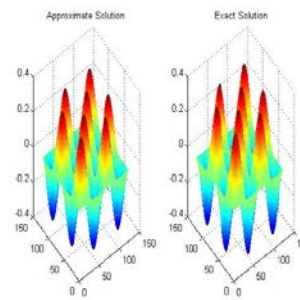


Figure 2: (Test Problem 2)

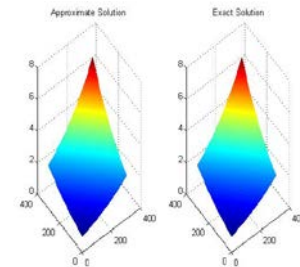


Figure 3: (Test Problem 3)

Table 1: (Test Problem 1) Error norm, the accuracy order of the scheme for different value of γ .

h	τ	$\ e\ _{l^2}$	Order
$\frac{1}{4}$	$\frac{1}{4}$	$4.3151e^{-3}$	-----
$\frac{1}{8}$	$\frac{1}{8}$	$2.0464 \cdot e^{-4}$	0.0763
$\frac{1}{16}$	$\frac{1}{16}$	$4.0356e^{-5}$	2.3422
$\frac{1}{32}$	$\frac{1}{32}$	$6.0877e^{-6}$	2.7288
$\frac{1}{64}$	$\frac{1}{64}$	$7.6237e^{-7}$	2.9973
$\frac{1}{128}$	$\frac{1}{128}$	$5.7646e^{-8}$	3.7252
$\frac{1}{256}$	$\frac{1}{256}$	$1.4849e^{-8}$	1.9569

Table 2: (Test Problem 2) Error norm, the accuracy order of the scheme for different value of γ .

h	τ	$\ e\ _{l^2}$	Order
$\frac{1}{4}$	$\frac{1}{4}$	$6.800e^{-3}$	-----
$\frac{1}{8}$	$\frac{1}{8}$	$4.5763 \cdot e^{-4}$	3.8933
$\frac{1}{16}$	$\frac{1}{16}$	$2.2629e^{-5}$	4.3379
$\frac{1}{32}$	$\frac{1}{32}$	$3.2192e^{-6}$	2.8134
$\frac{1}{64}$	$\frac{1}{64}$	$7.2640e^{-7}$	2.1478

Table 3: (Test Problem 3)Error norm, the accuracy order of the scheme for different value of γ .

h	τ	$\ e\ _{l^2}$	Order
$\frac{1}{4}$	$\frac{1}{4}$	$3.5100e^{-2}$	-----
$\frac{1}{8}$	$\frac{1}{8}$	$1.8700.e^{-2}$	0.9084
$\frac{1}{16}$	$\frac{1}{16}$	$5.500e^{-3}$	1.7655
$\frac{1}{32}$	$\frac{1}{32}$	$2.3037e^{-4}$	4.5774
$\frac{1}{150}$	$\frac{1}{150}$	$9.4135e^{-5}$	1.2911
$\frac{1}{350}$	$\frac{1}{350}$	$8.3189e^{-6}$	3.5003
$\frac{1}{1024}$	$\frac{1}{1024}$	$9.5671e^{-7}$	3.1202

