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REVIEW ARTICLE

COMPARATIVE STUDY OF MATHEMATICAL MODEL OF EBOLA VIRUS DISEASE VIA USING DIFFERENTIAL TRANSFORM METHOD AND VARIATION OF ITERATION METHOD

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ABSTRACT

This study investigates the application of differential transformation method and variational iteration method in finding the approximate solution of Ebola model. Variational iteration method uses the general Lagrange multiplier to construct the correction functional for the problem while differential transformation method uses the transformed function of the original nonlinear system. The result revealed that both methods are in complete agreement, accurate and efficient for solving systems of ODEs.

KEYWORDS

Ebola virus disease, mathematical modeling, vaccination, differential transformation method, Lagrange multipliers, variational iteration method, transformed function.

1. INTRODUCTION

Mathematical modeling of infectious diseases is very useful in investigating human infectious diseases, such as Ebola virus. This helps in the understanding of the disease dynamics and give useful predictions about the potential transmission of the disease and the effective-ness of possible control measures, which can provide helpful information for public health policy makers [1]. Ebola virus disease (EVD) also known as Ebola hemorrhagic fever that is a viral hemorrhagic fever of humans and other primates caused by Ebola viruses. Ebola is a rare and deadly disease caused by infection with one of the Ebola virus species. Signs and symptoms typically start between two days and three weeks after contracting the virus with a fever, sore throat, muscular pain, and headaches. Then vomiting, diarrhea and rashes usually follow along with decreased function of the liver and kidneys. Currently some people begin to bleed both internally and externally. The most commonly implemented models in epidemiology are the SIR and SEIR models. The SIR model consists of three compartments: Susceptible individuals S, Infectious individuals I , and Recovered individuals R . The SEIR model takes into consideration that, in many infectious diseases there is an exposed period after the transmission of the infection from susceptible to potentially infective members but before these potential infective can transmit infection. That is why an extra compartment is introduced, which is the exposed class E . Hence the SEIR model gives a generalization of the basic SIR model. Many researchers have employed both the SIR and the SEIR models in the modeling of the Ebola outbreak [1-3]. We describe the transmission of Ebola virus by a SEIR model, where the population is divided into four groups such that the susceptible individuals at time t, denoted by $S(t)$, enter the exposed class $E(t)$ before they become infectious. The infectious class at time t denoted by $I(t)$ represents the individuals that are infected with the disease and are suffering the symptoms of Ebola, then the recovered class which at time t is denoted by $R(t)$. The total population assumed constant during the short period of time under study is given by

$$N(t) = S(t) + I(t) + E(t) + R(t) \tag{1}$$

at any instant of time t . The SEIR model of Ebola virus transmission with vaccination is then described by the following set of nonlinear ordinary differential equations (ODEs):

$$\left.\begin{aligned}
\frac{dS(t)}{dt} &= -\beta S(t)I(t) - \gamma S(t) \\
\frac{dE(t)}{dt} &= \beta S(t)I(t) - \gamma E(t) \\
\frac{dI(t)}{dt} &= \gamma E(t) - \mu I(t) \\
\frac{dR(t)}{dt} &= \mu I(t) + \nu E(t)
\end{aligned}\right\} \tag{2}$$

Transitions between different states are described by the following parameters: β is the rate of transmission, γ is the rate of infection and μ is the rate of recovery, ν is the rate of vaccination. This model is very simple and assumes that the population is constant in the period of time under study so that the sum of the right-hand side of the equations of system (2) is zero. Many researchers have worked on the mathematical modelling of Ebola Virus Disease. Chowell G and Nishiura H in their work showed that mathematical modeling offers useful insights into the risk of a major epidemic of EVD and the assessment of the impact of basic public health measures on disease spread [3]. Rachah and Torres in 2015 gave a comparison between two different mathematical models used in the description of the Ebola virus propagation in West Africa [4]. They investigated the two models to improve the prediction and the control of the propagation of the virus and particularly studied the case when the two models generate similar results.

In this paper, Equation (2) is solved by using variational iteration method and the differential transformation method for numerical comparison. Differential transformation method is one of the well-known techniques to solve linear and nonlinear equations. It was first introduced by Zhou for solving linear and nonlinear initial value problems in electrical circuit analysis [5]. This method has been used to solve differential algebraic equation, Schr ö dinger equations, fractional differential equation, Lane-Emden type equation, free vibration analysis of rotating beams, unsteady rolling motion of spheres equation in inclined tubes [6-8]. Important advantage is that this method is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate. Also, the basic idea of the variational iteration method is to construct an iteration method based on correction functional that include a generalized Lagrange multiplier [9-11]. The VIM was proposed where the value of the multiplier was chosen using variational theory so that each improves the accuracy of the solution [9]. The initial approximation i.e. trial function usually includes unknown coefficient which can be determined to satisfy any boundary and initial conditions. VIM does not require specific transformation for nonlinear terms as required by other techniques and is now widely used by many researchers to study autonomous ordinary differential equation, Integro-differential systems, Linear Helmholtz partial differential equation and other fields [12-19]. In this method the solution is given in an infinite series usually convergent to an accurate solution.

Table 1: Fundamental operation of Differential transform method

Original Function	Transformed Function
$y(x)=g(x)\pm h(x)$	$Y(k)=G(k)\pm H(k)$
$y(x)=\alpha g(x)$	$Y(k)=\alpha G(k)$
$y(x)=\frac{dg(x)}{dx}$	$Y(k)=(k+1)G(k+1)$
$y(x)=\frac{d^2g(x)}{d^2x}$	$Y(k)=(k+1)(k+2)G(k+2)$
$y(x)=\frac{d^mg(x)}{d^mx}$	$Y(k)=(k+1)(k+2)...(k+m)G(k+m)$
$y(x)=1$	$Y(k)=\delta(k)$
$y(x)=x$	$Y(k)=\delta(k-1)$
$y(x)=x^m$	$Y(k)=\delta(k-m)=\begin{cases}1, & \text{if} \quad k=m\\ 0, & \text{if} \quad m\neq n\end{cases}$
$y(x)=g(x)h(x)$	$Y(k)=\sum_{m=0}^{k}H(m)G(k-m)$
$y(x)=e^{(\lambda x)}g(x)h(x)$	$Y(k)=\frac{\lambda^k}{k!}$
$y(x)=(1+x)^m$	$Y(k)=\frac{m(m-1)...(m-k+1)}{k!}$

2.THE DIFFERENTIAL TRANSFORMATION METHOD

An arbitrary function $f(x)$ can be expanded in Taylor series about a point $x=0$.

$$f(x)=\sum_{k=0}^{\infty}\frac{x^k}{k!}\left[\frac{d^kf}{dx^k}\right]_{x=0}. \quad (3)$$

The differential transformation of $f(x)$ is defined as arbitrary function $f(x)$ can be expanded in Taylor series about a point $x=0$.

$$f(x)=\frac{1}{k!}\left[\frac{d^kf}{dx^k}\right]_{x=0}. \quad (4)$$

Then the inverse differential transform is

$$f(x)=\sum_{k=0}^{\infty}x^kF(k). \quad (5)$$

The fundamental mathematical operations performed by differential transform method are listed in Table 1.

3.VARIATIONAL ITERATION METHOD

According to the variational iteration method we consider the following general differential equation [9,10,13]:

$$Lu+Nu=g(x), \quad (6)$$

where L is a linear operator N is a nonlinear operator and $g(x)$ is an inhomogeneous term. We can construct a correctional function as follows

$$u_{n+1}=u_n(x)\int_0^x\lambda\{Lu_n(s)+N\tilde{u}_n(s)-g(s)\}ds, \quad (7)$$

Where λ is a Lagrangian multiplier which can be identified optimally via variational theory [13]. The subscript n denotes the n^{th} approximation and $\tilde{\mu}_n$ is considered as a restricted variation i.e $\delta\tilde{\mu}_n=0$. Consider the stationary condition of the above correction functional then the Lagrange multiplier can be expressed as

$$\lambda_i(w)=\frac{(-1)^m}{(m-1)!}(w-x)^{m-1}, \quad (8)$$

where m is the highest order of the differential equation.

4.APPLICATION OF DIFFERENTIAL TRANSFORMATION METHOD

Using the transformed function of the original function in Table 1, we obtained the recurrence relation of Equation (2) as

$$\left.\begin{aligned}S(k+1)&=\frac{1}{k+1}\left[\beta\sum_{m=0}^{k}S(m)I(k-m)-\upsilon S(k)\right]\\E(k+1)&=\frac{1}{k+1}\left[\beta\sum_{m=0}^{k}S(m)I(k-m)-\gamma E(k)\right]\\I(k+1)&=\frac{1}{k+1}\left[\gamma E(k)-\mu I(k)\right]\\R(k+1)&=\frac{1}{k+1}\left[\mu I(k)+\upsilon S(k)\right]\end{aligned}\right\} \quad (9)$$

SEIR model with initial value of susceptible $S(0)=0.88$, Exposed $E(0)=0.07$, Infected $I(0)=0.05$ and recovered $R(0)=0$ transmission rate $\beta=0.02$, Infectious rate $\gamma=0.1887$ and recovered rate $\mu=0.1$ then System (9) with $\nu=0$,

$S(1)=-0.174856,\ S(2)=0.016644526,$
$S(3)=-0.01056266,$
$E(1)=-0.004409,\ E(2)=0.004015131,$
$E(3)=0.00345805,$
$I(1)=0.00829,\ I(2)=-0.00083813,$
$I(3)=0.000280489,$
$R(1)=0.005,\ R(2)=0.0004145,$
$R(3)=0.000041906,$

when $\nu=0.1$ then recovery rate $R(1)=0.0325144$, $R(2)=0.008736763,\ R(3)=0.000548824.$ Then closed form of the solution is given by

$$\left.\begin{aligned}s(t)&=\sum_{m=0}^{k}S(k)t^k=0.08-0.174856t+0.016644526t^2-0.01056266t^3+...\\e(t)&=\sum_{m=0}^{k}E(k)t^k=0.07-0.004409t+0.004015131t^2-0.00345805t^3+...\\i(t)&=\sum_{m=0}^{k}I(k)t^k=0.05+0.00829t-0.00083813t^2+0.000280489t^3+...\\r(t)&=\sum_{m=0}^{k}R(k)t^k=0-0.005t+0.0004145t^2-0.000041906t^3+...\end{aligned}\right\} \quad (10)$$

5.APPLICATION OF VARIATION OF ITERATION METHOD

Applying the variational iteration method in (2), we derive the correctional functional as follows:

$$\left.\begin{aligned}
s_{n+1}(t) &= s_n(t) + \int_0^t \lambda_1(w)\{s_n'(w) + \beta\tilde{s}_n(w)\tilde{i}_n(w) + \nu\tilde{s}_n(w)\}dw, \\
e_{n+1}(t) &= e_n(t) + \int_0^t \lambda_2(w)\{e_n'(w) - \beta\tilde{s}_n(w)\tilde{i}_n(w) + \gamma\tilde{e}_n(w)\}dw, \\
i_{n+1}(t) &= i_n(t) + \int_0^t \lambda_3(w)\{i_n'(w) + \gamma\tilde{e}_n(w) - \mu\tilde{i}_n(w)\}dw, \\
r_{n+1}(t) &= r_n(t) + \int_0^t \lambda_4(w)\{r_n'(w) - \mu\tilde{i}_n(w) - \nu\tilde{s}_n(w)\}dw.
\end{aligned}\right\} \quad (11)$$

Where λ_1, λ_2, λ_3, λ_4 are general Lagrange multipliers and $\tilde{s}_n(w)$, $\tilde{e}_n(w)$, $\tilde{i}_n(w)$, $\tilde{r}_n(w)$ are considered restricted variation which means that $\delta\tilde{s}_n(w) = \delta\tilde{e}_n(w) = \delta\tilde{i}_n(w) = \delta\tilde{r}_n(w) = 0$. From Equation (8) $\lambda_1(w) = \lambda_2(w) = \lambda_3(w) = \lambda_4(w) = -1$, Equation (11) becomes

$$\left.\begin{aligned}
s_{n+1}(t) &= s_n(t) - \int_0^t \{s_n'(w) + \beta\tilde{s}_n(w)\tilde{i}_n(w) + \nu\tilde{s}_n(w)\}dw, \\
e_{n+1}(t) &= e_n(t) - \int_0^t \{e_n'(w) - \beta\tilde{s}_n(w)\tilde{i}_n(w) + \gamma\tilde{e}_n(w)\}dw, \\
i_{n+1}(t) &= i_n(t) - \int_0^t \{i_n'(w) + \gamma\tilde{e}_n(w) - \mu\tilde{i}_n(w)\}dw, \\
r_{n+1}(t) &= r_n(t) - \int_0^t \{r_n'(w) - \mu\tilde{i}_n(w) - \nu\tilde{s}_n(w)\}dw.
\end{aligned}\right\} \quad (12)$$

With initial approximation $s(0) = 0.88,\ e(0) = 0.07,\ i(0) = 0.05$ and $r(0) = 0$, $\beta = 0.02,\ \gamma = 0.1887$ and $\mu = 0.1$ with $\nu = 0$ which gives

$$\left.\begin{aligned}
s_1(t) &= 0.88 - 0.174856t \\
e_1(t) &= 0.07 - 0.004409t \\
i_1(t) &= 0.05 + 0.00829t \\
r_1(t) &= 0 - 0.005t
\end{aligned}\right\} \quad (13)$$

It can be observed that the result of the epidemic system of Equation (2) is in complete agreement with the result obtained by the differential transform method.

6.CONCLUSION

In this paper, differential transformation method (DTM) and variational iteration method (VIM) has been successfully employed to obtain the approximate solution of model with initial condition. Result obtained by this method shows that both are in excellent agreement which indicates their effectiveness and reliability. These two methods can be considered as an alternative method for solving a wide class of linear and non-linear problems which arise in various fields of study.

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