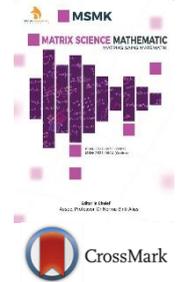


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REVIEW ARTICLE

DIFFERENTIAL TRANSFORMATION METHOD FOR SOLVING MALARIA –HYGIENE MATHEMATICAL MODEL

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ABSTRACT

In this study, we proposed a malaria-hygiene mathematical model using non-linear differential equation. The model equations are divided into seven compartments consisting of five human compartments (Hygienic Susceptible, Unhygienic Susceptible, Hygienic Infected, Unhygienic Infected, and Recovered) and two vector compartments (Non-Disease Carrier vector and Disease carrier vector). Differential Transformation Method (DTM) is applied to solve the mathematical model. The solutions obtained by DTM are compared with Runge-Kutta order 4th method (RK4). The graphical solutions illustrate similarity between DTM and RK4. It therefore imply that DTM can be consider a reliable alternative solution method.

KEYWORDS

Malaria, Hygiene, Mathematical model, Runge-Kutta order 4th, DTM.

1. INTRODUCTION

Malaria is a parasitic disease which spread by the female *Anopheles* mosquitoes. The prevalence of malaria has continued to be a global health challenge with spontaneous infection rate in the last five decades, and resulting into huge medical restive economic hardship (Jasminka et al., 2019; Yibeltal et al., 2018). It is transmitted from person to person through an infected female *Anopheles* mosquito bite (Azeb et al., 2018). Worldwide, 229 million cases of malaria were recorded with 409,000 deaths in 2019. The sub-Sahara African regions are the endemic ancient home of malaria with 94% reported cases and death (World Health Organization, 2021). According to a study, the number of mosquitoes continues to increase due to the following environmental and human population related factors (Singh et al., 2003):

- (i) Discharge of household wastes such as garbage and trash in residential areas,
- (ii) Open drainage of sewage in residential areas,
- (iii) Plantations of vegetation and hedges in residential areas and in parks,
- (iv) Industries and transport systems producing wastes in residential areas,
- (v) Open water storage tanks and ponds.

Also the presence of overgrown vegetation and stagnant water with “bola” activities such as unhealthy hygienic practices of sanitary in and around residential environment increases exponential growth of *Anopheles* mosquitoes (Mauti et al., 2015; Enebeli et al., 2019; Musoke et al., 2018).

Thus, unhygienic environmental conditions in the habitat caused by human populations become responsible for the fast-growing numbers of mosquitoes. DTM is one of the methods used to solve different kinds of linear and nonlinear differential equations. It was first introduced by Zhou in a study about electrical circuits (Zhou, 1986). DTM constructs a semi-analytic numerical technique that uses Taylor series for the solution of differential equations in the form of polynomials.

It is possible to solve integro-differential equations, linear Fredholm integro-differential equation, differential equations, difference equations, differential difference equations, fractional differential equations, pantograph equations, fourth-order parabolic partial differential equations, volterra integral equations and quadratic riccati differential equations by this method (Azuaba and Akinwande, 2018; Arikoglo and Ozkol, 2008; Maleknejad and Kajani, 2004; Maleknejad et al., 2004; Ozdemir and Kaya, 2006; Arikoglo and Ozkol, 2006a; Arikoglo and Ozkol, 2006b; Arikoglo and Ozkol, 2007; Keskin et al., 2007; Ibis, 2014; Jothika and Savitha, 2018; Abiodun et al., 2015). The main advantage of this method is that it can be applied directly to linear and nonlinear ordinary differential equations without linearization, discretization or perturbation. Some works has been done to solve model equation with differential transformation method (DTM); applied DTM to solve a deterministic model of infectious disease, the result shows a positive correlation between DTM and Runge-Kutta solution (Adebisi et al., 2019). In a study, DTM was applied to determine the approximate solution to a sterile insect technology model for controlling zika virus vector (Atokolo et al., 2021).

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The method as applied provides fast convergence rate and was considered a variable alternative tool for solving non-linear and linear problem in science and engineering. A group researcher carried out a study to solve a mathematical model for dengue fever using DTM (Eguda et al., 2019). The work shows that DTM is a very effective tool for solving ordinary differential equation problems. Some researchers solved a mathematical model of yellow fever dynamics incorporating secondary host applying DTM, from the work it showed that DTM was a valid alternative solution method (Somma et al., 2019). With all the study carried out none has solved to the best of the author's knowledge a malaria hygiene mathematical method using DTM. Hence this work is aimed to solve a malaria hygiene mathematical model. The paper is organized as follows: Section 2 is the model formulation, section 3 presents DTM, section 4 includes model solution, section 5 is presents graphical solution of the model, in section 6 we conclude the study.

2. MODEL FORMULATION

In this model, the total human population denoted by N_H is subdivided into Unhygienic susceptible human population S_u , Hygienic Susceptible Human population S_h , Unhygienic infected human population I_u , hygienic infected human population I_h and the Recovered Human population R_h . The mosquito population denoted by N_v is subdivided into susceptible mosquitoes S_v and infected mosquitoes I_v . Therefore, we have the following sub populations:

$$N_H = S_u + S_h + I_u + I_h + R. \tag{1}$$

$$N_v = S_v + I_v. \tag{2}$$

Let Λ_H be the recruitment rate of the human population. A fraction $(1 - \alpha)\Lambda_H$ enters unhygienic susceptible human class while the remaining fraction $\alpha\Lambda_H$ enters the hygienic susceptible human class. The unhygienic susceptible class is increased by the rate at which unhygienic human class lose immunity after recovery given as ω_u , and reduced by the rate of progression to hygienic class τ_1 , the force of infection for the unhygienic class λ_u and natural human death rate μ_H . The hygienic susceptible human compartment is increased by the τ_1 , the rate at which hygienic human loss immunity after recovery at ω_h , while the compartment is reduced by natural human death rate μ_H and the force of infection for the hygienic class $(1 - \zeta)\lambda_h$. The unhygienic infected human class I_u is increased by λ_u and reduced by natural human death rate μ_H , rate of progression from I_u to I_h given as τ_2 , malaria induced death for unhygienic human class δ_u and recovery for unhygienic human θ_u . The hygienic infected class I_h is increased by $(1 - \zeta)\lambda_h$ and τ_2 then reduced by the recovery rate for a hygienic human class given as θ_h , malaria induced death for hygienic human class δ_h and natural death rate μ_H . The Human recovery class R is increased by θ_h and θ_u , then reduced by μ_H , ω_h and ω_u . The susceptible mosquito class S_v is increased by the Mosquito recruitment rate given as Λ_v , reduced by the mosquito's death rate μ_v , and force of infection for mosquito given as λ_v . The infected mosquito class I_v is increased by λ_v and

Given the above description and definitions of variables and parameters in Table 1 and 2, the following are the model equations:

$$\frac{dS_u}{dt} = (1 - \alpha)\Lambda_H - (\tau_1 + \lambda_u + \mu_H)S_u + \omega_u R$$

$$\frac{dS_h}{dt} = \alpha\Lambda_H + \omega_h R + \tau_1 S_u - ((1 - \zeta)\lambda_h + \mu_H)S_h,$$

$$\frac{dI_u}{dt} = \lambda_u S_u - (\tau_2 + \delta_u + \theta_u + \mu_H)I_u$$

$$\frac{dI_h}{dt} = (1 - \zeta)\lambda_h S_h + \tau_2 I_u - (\delta_h + \theta_h + \mu_H)I_h, \tag{3}$$

$$\frac{dR}{dt} = \theta_u I_u + \theta_h I_h - (\omega_u + \omega_h + \mu_H)R,$$

$$\frac{dS_v}{dt} = \Lambda_v - \lambda_v S_v - \mu_v S_v,$$

$$\frac{dI_v}{dt} = \lambda_v S_v - \mu_v I_v$$

where

$$\lambda_u = \frac{b_1 \beta_{vh} I_v}{N_H}, \lambda_h = \frac{b_2 \beta_{vh} I_v}{N_H}, b_1 > b_2, \lambda_v = \frac{b_3 \beta_{hv} (I_u + \rho I_h)}{N_H}, \delta_u > \delta_h, \theta_h > \theta_u. \tag{4}$$

Table 1. Variables	
Symbols	Description
S_u	Unhygienic Susceptible Human
S_h	Hygienic Susceptible Human
I_u	Unhygienic Infected Human
I_h	Hygienic Infected Human
R	Recovered Human
S_v	Non – disease carrier Mosquitoes
I_v	Disease carrier Mosquitoes

Table 2. Model Parameters	
Parameters	Definitions
Λ_H	Recruitment rate of Human Population
Λ_v	Recruitment rate of mosquitoes
τ_1	progression from S_u to S_h
τ_2	progression from I_u to I_h
δ_u	disease – induced death for the unhygienic human class
δ_h	disease – induced death for hygienic human class
b_1	biting rate of mosquito for unhygienic human class
b_2	biting rate of mosquito for hygienic human class
β_{vh}	transmission probability of infection from mosquito to human
β_{hv}	transmission probability of infection from human to mosquito
λ_u	the force of infection for unhygienic human class
λ_h	the force of infection for hygienic human class
λ_v	force of infection for mosquitoes
b_3	biting rate of mosquitoes
ζ	rate of reduction of infection for hygienic class
ρ	Modification Parameter
θ_u	rate of recovery for unhygienic human class
θ_h	rate of recovery for hygienic human class
ω	rate at which recovered human become susceptible
α	hygienic rate
μ_H	Natural human death rate
μ_v	natural death rate of mosquitoes
N_H	Total Human Population

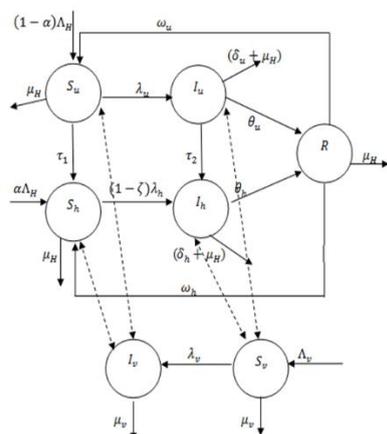


Figure 1. Model Schematic diagram

3. DIFFERENTIAL TRANSFORMATION METHOD (DTM)

In this section, the basic principle of DTM is being utilized as follows. Given

an arbitrary function $f(t)$ that can be expanded in Taylor series at the point $t = 0$ as

$$f(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k f}{dt^k} \right]_{t=0} \tag{5}$$

We define the differential transformation as

$$F(k) = \frac{1}{k!} \left[\frac{d^k f}{dt^k} \right]_{t=0} \tag{6}$$

The inverse differential transform is

$$f(t) = \sum_{k=0}^{\infty} t^k F(k) \tag{7}$$

The table below consists of some properties of the DTM. Given that $c(x)$ and $f(x)$ are arbitrary functions with $C(k)$ and $F(k)$ as the respective transformation functions.

Table 3. Basic Properties of DTM		
S/N	Original Functions	Transformed Functions
1	$y(x) = c(x) \pm f(x)$	$Y(k) = C(k) \pm F(k)$
2	$y(x) = \rho c(x)$	$Y(k) = \rho C(k)$, where ρ is a constant
3	$y(x) = \frac{dc(x)}{dx}$	$Y(k) = (k + 1)C(k + 1)$
4	$y(x) = \frac{d^2c(x)}{dx^2}$	$Y(k) = (k + 1)(k + 2)C(k + 2)$
5	$y(x) = \frac{d^nc(x)}{dx^n}$	$Y(k) = (k + 1)(k + 2) \dots (k + n)C(k + n)$
6	$y(x) = 1$	$Y(k) = \delta(k)$
7	$y(x) = x$	$Y(k) = \delta(k - 1)$. δ is the Kronecker delta
8	$y(x) = e^{\lambda x}$	$Y(k) = \frac{\lambda^k}{k!}$
9	$y(x) = c(x)f(x)$	$Y(k) = \sum_{m=0}^{\infty} F(m)C(k - m)$
10	$y(x) = (1 + x)^n$	$Y(k) = \frac{n(n-1)(n-2)\dots(n-k+1)}{k!}$

4. MODEL SOLUTION

Here, DTM is applied to solve the model equations (3):

Using the properties in Table 3, we obtain the following system of transformed equations:

$$S_u(k + 1) = \frac{1}{k+1} \left[(1 - \alpha)\Lambda_H - (\tau_1 + \mu_H)S_u(k) - \frac{b_1\beta_{vh}}{N_H} \sum_{m=0}^k S_u(m)I_v(k - m) + \omega_u R(k) \right] \tag{8}$$

$$S_h(k + 1) = \frac{1}{k+1} \left[\alpha\Lambda_H + \omega_h R(k) + \tau_1 S_u(k) - \mu_H S_h(k) - \frac{(1-\zeta)b_2\beta_{vh}}{N_H} \sum_{m=0}^k S_h(m)I_v(k - m) \right] \tag{9}$$

$$I_u(k + 1) = \frac{1}{k+1} \left[\frac{b_1\beta_{vh}}{N_H} \sum_{m=0}^k S_u(m)I_v(k - m) - (\tau_2 + \delta_u + \theta_u + \mu_H)I_u(k) \right] \tag{10}$$

$$I_h(k + 1) = \frac{1}{k+1} \left[\frac{(1-\zeta)b_2\beta_{vh}}{N_H} \sum_{m=0}^k S_h(m)I_v(k - m) + \tau_2 I_u(k) - (\delta_h + \theta_h + \mu_H)I_h(k) \right] \tag{11}$$

$$R(k + 1) = \frac{1}{k+1} [\theta_u I_u(k) + \theta_h I_h(k) - (\omega_u + \omega_h + \mu_H)R(k)] \tag{12}$$

$$S_v(k + 1) = \frac{1}{k+1} \left[\Lambda_v - \frac{b_3\beta_{hv}}{N_H} \sum_{m=0}^k S_v(m)(I_u(k - m) + \rho I_h(k - m)) - \mu_v S_v(k) \right] \tag{13}$$

$$I_v(k + 1) = \frac{1}{k+1} \left[\frac{b_3\beta_{hv}}{N_H} \sum_{m=0}^k S_v(m)(I_u(k - m) + \rho I_h(k - m)) - \mu_v I_v(k) \right] \tag{14}$$

With initial conditions given as

$S_u(0) = 40; S_h(0) = 50; I_u(0) = 60; I_h(0) = 35; R = 40; S_v(0) = 25; I_v(0) = 20$. We have the following initial values and parameters;

$S_u(1) = 75.58826667; S_u(2) = -4.184918845; S_u(3) = 29.96909653;$
 $S_u(4) = 6.952730975;$
 $S_h(1) = 87.59373333; S_h(2) = 10.71830970; S_h(3) = 26.60343544;$
 $S_h(4) = 8.700966496;$
 $I_u(1) = -40.78426667; I_u(2) = 13.88464134; I_u(3) = -3.147393685;$

$I_u(4) = 0.5393056537;$
 $I_h(1) = 22.66086667; I_h(2) = -12.56513789; I_h(3) = 3.195120115;$
 $I_h(4) = -0.559047512;$
 $R(1) = -54.96760; R(2) = 44.11645520; R(3) = -23.63798303;$
 $R(4) = 9.42007805$
 $S_v(1) = 999.9055775; S_v(2) = 498.1294006; S_v(3) = 333.1743809;$
 $S_v(4) = 249.770719,$
 $I_v(1) = 0.09186200; I_v(2) = 1.842149461; I_v(3) = 0.149469510; I_v(4) = 0.224539509$

and are transformed as follows:

$$s_u(t) = \sum_{n=0}^k S_u(k)t^k = 40 + 75.58826667t - 4.184918845t^2 + 29.96909653t^3 + 6.952730975t^4 + \dots$$

$$s_h(t) = \sum_{n=0}^k S_h(k)t^k = 50 + 87.59373333t + 10.71830970t^2 + 26.60343544t^3 + 8.700966496t^4 + \dots$$

$$i_u(t) = \sum_{n=0}^k I_u(k)t^k = 60 - 40.78426667t + 13.88464134t^2 - 3.147393685t^3 + 0.5393056537t^4 + \dots$$

$$i_h(t) = \sum_{n=0}^k I_h(k)t^k = 35 + 22.66086667t - 12.56513789t^2 + 3.195120115t^3 - 0.559047512t^4 + \dots$$

$$r(t) = \sum_{n=0}^k R(k)t^k = 40 - 54.96760t + 44.11645520t^2 - 23.63798303t^3 + 9.42007805t^4 + \dots$$

$$s_v(t) = \sum_{n=0}^k S_v(k)t^k = 25 + 999.9055775t + 498.1294006t^2 + 333.1743809t^3 + 249.770719t^4 + \dots$$

$$i_v(t) = \sum_{n=0}^k I_v(k)t^k = 20 + 0.09186200t + 1.842149461t^2 + 0.149469510t^3 + 0.224539509t^4 + \dots$$

5. GRAPHICAL PRESENTATION OF MODEL SOLUTION

In this section, the graphical illustration obtained from the analytical solution is presented using Maple software. By using the initial conditions and parameter values from table 4, figures (2) to (8) show the solutions plots by DTM and RK4.

Table 4: Parameter values of Model		
Symbols	Values	Source
Λ_H	100	(Oluwatayo, 2019)
Λ_v	1000	(Bakare and Nwozo, 2017)
τ_1	0.25	(Assumed)
τ_2	0.5	(Assumed)
δ_u	0.13	(Assumed)
δ_h	0.06	(Assumed)
b_1	0.17	(Assumed)
b_2	0.1	(Assumed)
β_{vh}	0.03	(Olaniyi et al., 2018)
β_{hv}	0.09	(Olaniyi et al., 2018)
b_3	0.12	(Olaniyi and Obabiyi, 2013)
ζ	0.08	(Assumed)
ρ	0.5	(Assumed)
θ_u	0.05	(Assumed)
θ_h	0.15	(Assumed)
ω	0.7902	(Bakare and Nwozo, 2017)
α	0.46	(Assumed)
μ_H	0.00004	(Oluwataya, 2019)
μ_v	0.0000569	(Bakare and Nwozo, 2017)

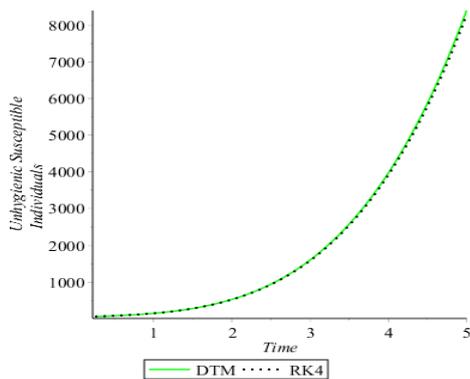


Figure 2: Solution of Unhygienic Susceptible Individuals

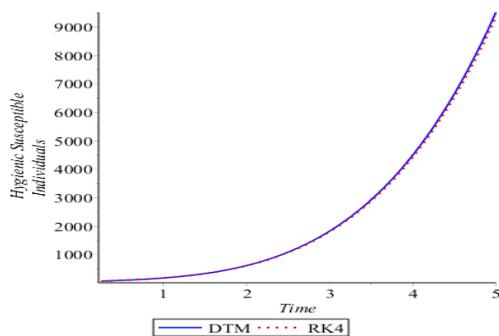


Figure 3: Solution of Hygienic Susceptible Individuals

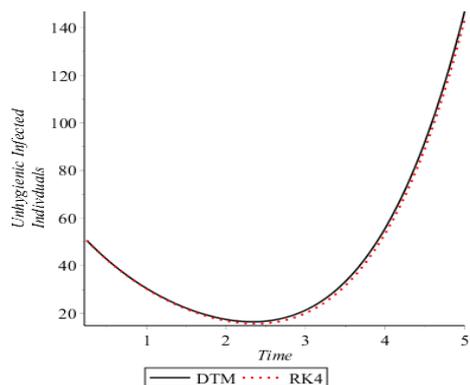


Figure 4: Solution of Unhygienic Infected Individuals

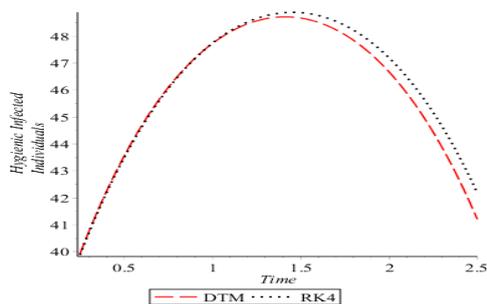


Figure 5: Solution of Hygienic Infected Individuals

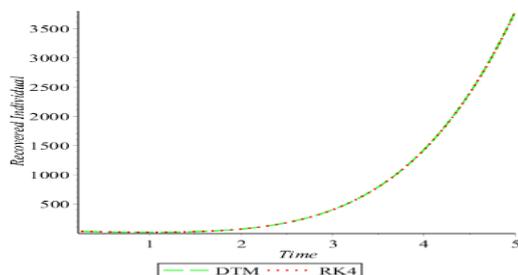


Figure 6: Solution of Recovered Individuals

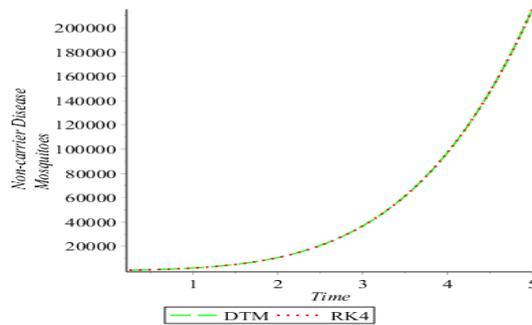


Figure 7: Solution of Non-Disease carrier Mosquitoes

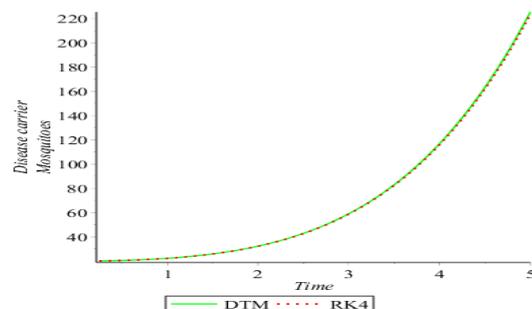


Figure 8: Solution of Disease carrier Mosquitoes

6. CONCLUSION

In this study, we present a mathematical model to assess the effect of hygiene in malaria transmission and solved the system of equations using the DTM. Graphical illustrations were presented and compared with DTM result and classical Runge-Kutta order 4th method (RK4). It is shown that DTM is a piecewise efficient convergent, cost effective method for solving nonlinear differential equations in the bounded domains. The method gives rapidly converging series solutions and the solutions can be improved by expanding the series. It is observed that the series solutions obtained with DTM can be written in exact closed form.

REFERENCES

Abiodun, A.O., Sunday, O.E., Hilary, I.O., Grace, O.A., 2015. A novel approach for solving quadratic riccati differential equations. *International Journal of Applied Engineering Research*, 10 (11), Pp. 29121-29126.

Adebisi, A.F., Peter, O.J., Ayoola, T.A., Ayoade, A.A., Faniyi, O.E., Ganiyu, A.B., 2019. Semi Analytic Method for Solving Infectious Disease Model. *Science World Journal*, 14 (1), Pp. 88-91.

Arikoglo, A., Ozkol, I., 2006. Solution of difference equations by using differential transformation method. *Applied Mathematics and Computation*, 174, Pp. 1216-1228.

Arikoglo, A., Ozkol, I., 2006. Solution of differential-difference equations by using differential transformation method. *Applied Mathematics and Computation*, 181, Pp. 153-162.

Arikoglo, A., Ozkol, I., 2007. Solution of fractional differential equations by using differential transformation method. *Chaos Soliton. Fract.*, 34, Pp. 1473-1481.

Arikoglo, A., Ozkol, I., 2008. Solutions of integral and integro-differential equation systems by using differential transformation method. *Applied Mathematics and Computation*, 56, Pp. 2411-2417. doi: 10.1016/j.camwa.2008.05.017

Atokolo, W., Aja, R.O., Omale, D., Tenuche, B.S., Mbah, G.C.E., 2021. Numerical Solution of the sterile insect technology for the control of Zika virus vector population. *J. Math. Comput. Sci.*, 11 (2), Pp. 1400-1412.

Azeb, E.Y., Habtamu, D.E., Yitayal, A.G., 2018. Utilization and Associated Factors of Insecticide Treated Bed Net among pregnant women attending Antenatal Clinic of Addis Zemen Hospital, North-Western Ethiopia: An institutional Based Study. *Malaria Research and Treatment*, Article ID 3647184. <https://doi.org/10.1155/2018/3647184>

- Azuaba, E., Akinwande, N.I., 2018. Analytical Solution of the Mathematical Model of Ebola Disease Dynamics Incorporating Infection-age Structure in the Quarantined Compartment with Treatment. *Journal of the Nigerian Association of Mathematical Physics*, 45 (3), Pp. 369-378.
- Bakare, E.A., Nwozo, C.R., 2017. Bifurcation and sensitivity analysis of Malaria – Schistosomiasis Coinfection Model. *International Journal of Applied Computational Mathematics*, doi:10.1007/s40819-017-0394-5.
- Eguda, F.Y., Andrawus, J., Babuba, S., 2019. The Solution of a mathematical Model for Dengue Fever Transmission using Differential Transformation Method. *J. Nig. Soc. Phys. Sci.*, 1, Pp. 82-87.
- Enebeli, U.U., Amadi, A.N., Iro, O.K., Oparaocha, E.T., Nwoke, E.A., Ibe, S.N.O., Oti, N.N., Chukwuocha, U.M., Nwufu, C.R., Amadi, C.O., Esomonu, I., 2019. Sanitation and Hygiene Practices and the Occurrence of Childhood Malaria in Abia State, Nigeria. *Research journal's Journal of Public Health*, 5 (6), Pp. 1-15.
- Ibis, B., 2014. Application of reduced differential transformation method for solving fourth-order parabolic partial differential equations. *Journal of Mathematics and Computer Science*, 12, Pp. 124-131.
- Jasminka, T., Ivana, S., Tamara, A., Melita, J., Aleksandar, V., 2019. Malaria: The Past and the Present. *Microorganism*, 7 (179).
- Jothika, K., Savitha, S., 2018. Solving volterra integral equations by using differential transformation method. *International Journal of Engineering Development and Research*, 6 (3), Pp. 592-595.
- Keskin, Y., Kurnaz, A., Kiris, M.E., Oturanc, G., 2007. Approximate solutions of generalized pantograph equations by the differential transformation method. *International Journal of Nonlinear Sciences*, 8, Pp. 159-164.
- Maleknejad, K., Kajani, M.T., 2004. Solving linear integro-differential equation system by Galerkin method with hybrid functions. *Applied Mathematics and Computation*, 159, Pp. 603-612.
- Maleknejad, K., Mirzaee, F., Abbasbandy, S., 2004. Solving linear integro-differential equations system by using rationalize haar functions method. *Applied Mathematics and Computation*, 155, Pp. 317-328.
- Mauti, G.O., Mauti, E.M., Kowanga, K.D., 2015. Evaluation of Malaria spread in relation to poor environmental conditions at Kibaha District (Tanzania). *Journal of Scientific & Innovative Research*, 4 (5), Pp. 203-206.
- Musoke, D., Miiro, G., Ndejjo, R., Karani, G., Morris, K., Kasassa, S., 2018. Malaria prevention practices and associated environmental risk factors in a rural community in Wakiso district, Uganda. *PLoS ONE*, 13 (10), Pp. e0205210. <https://doi.org/10.1371/journal.pone.0205210>
- Olaniyi, S., Obabiyi, O.S., 2013. Mathematical Model for Malaria transmission dynamics in human and mosquito population with non-linear forces of infection. *Int. J. Pure Appl. Math.*, 88 (1), Pp. 125 -156.
- Olaniyi, S., Okosun, K.O., Adesanya, S.O., Emmanuel, A.A., 2018. Global Stability and Optimal Control Analysis of Malaria Dynamics in the Presence of Human Travelers. *The Open Infectious Diseases Journal*, 10, Pp. 166 – 186.
- Oluwatayo, M.O., 2019. Mathematical Modelling of the Co-infection Dynamics of Malaria-Toxoplasmosis in the Tropics. *Biometrical Letters*, 56 (2), Pp. 139-163.
- Ozdemir, O., Kaya, M.O., 2006. Flapwise bending vibration analysis of a rotating tapered cantilever Bernoulli-Euler beam by differential transformation method. *Journal of sound vibration*, 289, Pp. 413-420.
- Singh, S., Chandra, P., Shukla, J.B., 2003. Modelling and analysis of the spread of carrier dependent infectious diseases with environmental effects. *Journal of Biological Sciences*, 11 (3), Pp. 325-335.
- Somma, S.A., Akinwande, N.I., Abah, R.A., Oguntolu, F.A., Ayebusi F.D., 2019. Semi-analytical solution for the Mathematical Modelling of yellow fever dynamics incorporating secondary host. *Communication in Mathematical Modeling and Applications*, 4 (9).
- World Health Organization. 2021. <https://www.who.int/news-room/factsheets/detail/malaria>. Accessed 20/January.
- Yibeltal, A., Abeba, M., Abebaw, B., Bekalu, K., Asmare, T., 2018. Prevalence of malaria and associated risk factors among asymptomatic migrant laborers in West Armachiho District, Northwest Ethiopia. *Research and Reports in Tropical Medicine*, 9, Pp. 95 –101.
- Zhou, J.K., 1986. *Differential transformation and its application for electrical circuits*. Huazhong University Press, Wuhan, China.

