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RESEARCH ARTICLE

ON COMMUTATIVITY OF PRIME NEAR RINGS

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ABSTRACT

In this paper, we prove commutativity of prime near rings by using the notion of β-derivations. Let M be a zero symmetric prime near ring. If there exist $p \geq 0$, $q \geq 0$ and a nonzero two sided β-derivation d on M, where $\beta : M \to M$ is a homomorphism, such that d satisfy one of the following conditions:

(i) $[\beta(s),d(t)] = s^p(\beta(s)o\beta(t))s^q \; \forall \; s, t \in M$

(ii) $[\beta(s),d(t)] = -s^p(\beta(s)o\beta(t))s^q \; \forall \; s, t \in M$

(iii) $[d(s),\beta(t)] = t^p(\beta(s)o\beta(t))t^q \; \forall \; s, t \in M$

(iv) $[d(s),\beta(t)] = -t^p(\beta(s)o\beta(t)t^q \; \forall \; s, t \in M$

Then M is a commutative ring.

KEYWORDS

Two sided β-Derivation, Prime Near Rings.

1. INTRODUCTION

In 1905, Dikson initiated the idea of near-rings. Later, in 1987, Bell and Gordon Mason did their work on derivation and prove some commutativity theorem for near-rings [Howard and Gordon, 1987]. After that, Bell introduced derivations and generalized derivations in near-rings. Moreover, Wang work on derivations on prime near-ring [Xue, 1994]. Yilun Shang did his work on derivation in prime near-rings [Yilun, 2011]. He shows two results for M to be a commutative ring. Later on, Ahmed and Khalid construct new examples of near-ring which are not ring and discuss theorems on commutativity by using derivation d for non-necessarily 3-prime near-ring [Ahmed and Khalid, 2012; Ahmed and Khalid, 2014].

A non-empty set M with two binary operations namely addition and multiplication is said to be a right near ring if M under addition is a group, M under multiplication is a semi group and $(M,+,.)$ satisfies right distributive law.

Let $M_o = \{s \in M : 0s = 0\}$ and $M_c = \{s \in M : 0s = s\}$, where M_o is known as zero symmetric near ring and M_c is known as constant near ring. If M_o and M_c are near rings itself. If $2s = 0$ this implies that $s = 0$ for all $s \in M$, then M is called $2-$*torsion* free. Let M be a near ring whose centre Z(M) is defined as:

$Z(N) = \{c \in M : cm = mc \; forall \; m \in M\}$. For any $s,t \in M$, $sot = st + ts$ and $[s,t] = st - ts$ is known as Jordan product and Lie product, respectively. A homomorphism near ring is called derivation on M if $d(st) = d(s)t + sd(t)$ for all $s,t \in M$ or equivalently $d(st) = sd(t) + d(s)t$ for all $s,t \in M$. A mapping $d : M \longrightarrow M$ is known as two sided β- derivation if there is a function $\beta : M \longrightarrow M$ such that $d(st) = d(s)\beta(t) + \beta(s)d(t)$ and $d(st) = \beta(s)d(t) + d(s)\beta(t)$, for all $s,t \in M$

Remark 1.1. Some identities are defined as below:

(a) [s,tu] = t[s,u]+[s,t]u

(b) [st,u] = s[t,u]+[s,u]t

(c) so (tu) = (sot)u-t[s,u] = t(sou)+[s,t]u

(d) (st) o u = s(tou)-[s,u]t = (sou)t+s[t,u]

2. RESULTS

In this section, we prove some results by using the idea of two sided β-derivations on prime near rings.

Lemma 2.1. *Let M be a near ring and d be a two sided β-derivation on M, where $\beta : M \to M$ is a homomorphism, then for all $s,t,u \in M$ satisfies:*

(i) $\beta(u)(\beta(s)d(t) + d(s)\beta(t)) = \beta(u)\beta(s)d(t) + \beta(u)d(s)\beta(t)$

(ii) $\beta(u)(d(s)\beta(t) + \beta(s)d(t)) = \beta(u)d(s)\beta(t) + \beta(u)\beta(s)d(t)$

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Proof. (i) Consider

$$d(u(st)) = \beta(u)d(st) + d(u)\beta(st)$$

this gives

(2.1) $d(u(st)) = \beta(u)(\beta(s)d(t) + d(s)\beta(t)) + d(u)\beta(s)\beta(t)$

consider again $d((us)t) = \beta(us)d(t) + d(us)\beta(t)$

this implies

$$d((us)t) = \beta(u)\beta(s)d(t) + (\beta(u)d(s) + d(u)\beta(s))\,\beta(t)$$

from this we get

(2.2) $d((us)t) = \beta(u)\beta(s)d(t) + \beta(u)d(s)\beta(t) + d(u)\beta(s)\beta(t)$

Comparing (2.1) and (2.2), we have

$$\beta(u)(\beta(s)d(t) + d(s)\beta(t)) = \beta(u)\beta(s)d(t) + \beta(u)d(s)\beta(t)$$

(ii) Consider $d(u(st)) = d(u)\beta(st) + \beta(u)d(st)$

this gives

(2.3) $d(u(st)) = d(u)\beta(s)\beta(t) + \beta(u)(d(s)\beta(t) + \beta(s)d(t))$

consider again $d((us)t) = d(us)\beta(t) + \beta(us)d(t)$

this implies

$$d((us)t) = (d(u)\beta(s) + \beta(u)d(s))\beta(t) + \beta(u)\beta(s)d(t)$$

from this we get

(2.4) $d((us)t) = d(u)\beta(s)\beta(t) + \beta(u)d(s)\beta(t) + \beta(u)\beta(s)d(t)$

Comparing equation (2.3) and (2.4), we have

$$\beta(u)(d(s)\beta(t) + \beta(s)d(t)) = \beta(u)d(s)\beta(t) + \beta(u)\beta(s)d(t)$$

Lemma 2.2. *Let d be a two sided β-derivation on M and M be a zero symmetric near ring. If $s \in Z(M)$, then $d(s) \in Z(M)$, where β: $M \rightarrow M$ is a mapping.*

Proof. For any $s \in M$, consider $td(s) + sd(t) = td(s) + d(t)s = d(ts) = d(st) = d(s)t + sd(t)$

From this we get $td(s) = d(s)t$

this implies

$$d(s) \in Z(M)$$

Lemma 2.3. *Let M be a zero symmetric prime near ring. If there is a nonzero two sided β-derivation d in such a way that $d(M) \subset Z(M)$, where β: $M \rightarrow M$ is a mapping, then M is a commutative ring.*

Proof. Since $d(M) \subset Z(M)$ and $d \neq 0$. Consider an element $s \in M$ implies that 0 6= d(s) ∈ Z(M as $0 \neq d(s) \in Z(M)$ this means that s is not a zero divisor in M. As we know that if s is not a zero divisor then M is a commutative ring.

Lemma 2.4. *Let M be a near ring then for all $s, t, k \in M$ then the following holds:*

(i) $[\beta(s), \beta(ts^k)] = [\beta(s), \beta(t)]\,\beta(s^k)$

(ii) $[\beta(st^k), \beta(t)] = [\beta(s), \beta(t)]\,\beta(t^k)$

(iii) $(\beta(s)o\beta(ts^k)) = (\beta(s)o\beta(t))\,\beta(s^k)$

(iv) $(\beta(st^k)o\beta(t)) = (\beta(s)o\beta(t))\,\beta(t^k)$

Proof. (i) Let M be a prime near ring. We have to prove that

$$[\beta(s), \beta(ts^k)] = [\beta(s), \beta(t)\beta(s^k)] \quad \forall\, s,t,k \in M$$

Taking L.H.S

$$[\beta(s), \beta(ts^k)] = [\beta(s), \beta(t)\beta(s^k)].$$

This gives

$$[\beta(s), \beta(ts^k)] = \beta(t)[\beta(s), \beta(s^k)] + [\beta(s), \beta(t)]\,\beta(s^k)$$

from this, we obtain

$$[\beta(s), \beta(ts^k)] = [\beta(s), \beta(t)]\,\beta(s^k) \quad \forall\, s,t,k \in M$$

(ii) We have to show that

$$[\beta(st^k), \beta(t)] = [\beta(s), \beta(t)]\,\beta(t^k) \quad \forall\, s,t,k \in M$$

Taking L.H.S

$$[\beta(st^k), \beta(t)] = [\beta(s)\beta(t^k), \beta(t)]$$

this implies

$$[\beta(st^k), \beta(t)] = \beta(s)[\beta(t^k), \beta(t)] + [\beta(s), \beta(t)]\,\beta(t^k)$$

from this we get

$$[\beta(st^k), \beta(t)] = [\beta(s), \beta(t)]\,\beta(t^k) \quad \forall\, s,t,k \in M$$

(iii) We have to show that

$$(\beta(s)o\beta(ts^k)) = (\beta(s)o\beta(t))\,\beta(s^k) \quad \forall\, s,t,k \in M$$

Taking L.H.S

$$(\beta(s)o\beta(ts^k)) = (\beta(s)o\beta(t)\beta(s^k))$$

This gives

$$(\beta(s)o\beta(ts^k)) = (\beta(s)o\beta(t))\,\beta(s^k) - \beta(t)[\beta(s), \beta(s^k)]$$

from this we get

$$(\beta(s)o\beta(ts^k)) = (\beta(s)o\beta(t))\,\beta(s^k)$$

(iv) We have to show that

$$(\beta(st^k)o\beta(t)) = (\beta(s)o\beta(t))\,\beta(t^k) \quad \forall\, s,t,k \in M$$

Taking L.H.S

$$(\beta(st^k)o\beta(t)) = (\beta(s)\beta(t^k)o\beta(t))$$

this implies

$$(\beta(st^k)o\beta(t)) = (\beta(s)o\beta(t))\,\beta(t^k) + \beta(s)[\beta(t^k), \beta(t)]$$

from this we get

$$(\beta(st^k)o\beta(t)) = (\beta(s)o\beta(t))\,\beta(t^k) \quad \forall\, s,t,k \in M$$

Theorem 2.5. *Let M be a zero symmetric prime near ring. If there exist $p \geq 0$, $q \geq 0$ and a nonzero two sided β-derivation d on M, where β: $M \rightarrow M$ is a homomorphism, satisfying one of the following conditions:*

(i) $[\beta(s), d(t)] = s^p(\beta(s)o\beta(t))\, s^q \;\forall\, s, t \in M$

(ii) $[\beta(s), d(t)] = -s^p(\beta(s)o\beta(t))\, s^q \;\forall\, s, t \in M$

(iii) $[d(s),\beta(t)] = t^p(\beta(s)o\beta(t))t^q \;\forall\, s, t \in M$

(iv) (iv) $[d(s),\beta(t)] = -t^p(\beta(s)o\beta(t)t^q \;\forall\, s, t \in M$

Then M is a commutative ring.

Proof. (i) Since

(2.5) $[\beta(s), d(t)] = s^p(\beta(s)o\beta(t))s^q \;\forall s, t \in M$

We will prove that M is a commutative ring. For this we will show that $d(M) \subset Z(M)$. Since $\beta(s)o(\beta(t)\beta(s)) = (\beta(s)o\beta(t))\,\beta(s)$ by Lemma 2.4. Replacing t by ts in equation (2.5), we get

$$[\beta(s),d(ts)] = s^p(\beta(s)o\beta(ts))s^q$$

Since β is homomorphism, therefore

$$[\beta(s), d(ts)] = s^p(\beta(s)o\beta(t)\beta(s))s^q$$

we get $$[\beta(s), d(ts)] = s^p(\beta(s)o\beta(t))s^q\beta(s)$$

from this we arrive at

(2.6) $[\beta(s),d(ts)] = [\beta(s),d(t)]\beta(s)$

By using the lie product, we obtain

$$\beta(s)d(ts) - d(ts)\beta(s) = (\beta(s)d(t) - d(t)\beta(s))\beta(s)$$

this implies

$\beta(s)d(ts) - d(ts)\beta(s) = \beta(s)d(t)\beta(s) - d(t)(\beta(s))^2$

this gives

$\beta(s)(d(t)\beta(s) + \beta(t)d(s)) - (d(t)\beta(s) + \beta(t)d(s))\beta(s) = \beta(s)d(t)\beta(s) - d(t)(\beta(s))^2$

By using Lemma 2.1 and $-(\beta(s) + \beta(t)) = -\beta(t) - \beta(s)$, we get

$\beta(s)d(t)\beta(s)+\beta(s)\beta(t)d(s)-\beta(t)d(s)\beta(s)-d(t)(\beta(s))^2 = \beta(s)d(t)\beta(s)-d(t)(\beta(s))^2$

this implies

$\beta(s)\beta(t)d(s) - \beta(t)d(s)\beta(s) = 0$

we get

(2.7) $\beta(s)\beta(t)d(s) = \beta(t)d(s)\beta(s)$

Left multiplication by $\beta(u)$ from last equation, we get

(2.8) $\beta(u)\beta(s)\beta(t)d(s) = \beta(u)\beta(t)d(s)\beta(s)$

Replacing t by ut in equation (2.7), we have

$\beta(s)\beta(ut)d(s) = \beta(ut)d(s)\beta(s)$

Since β is homomorphism, so we have

(2.9) $\beta(s)\beta(u)\beta(t)d(s) = \beta(u)\beta(t)d(s)\beta(s)$

By comparing equation (2.8) and equation (2.9), we get

$$\beta(u)\beta(s)\beta(t)d(s) = \beta(s)\beta(u)\beta(t)d(s)$$

this gives

$\beta(u)\beta(s)\beta(t)d(s) - \beta(s)\beta(u)\beta(t)d(s) = 0$

this implies

$$(\beta(u)\beta(s) - \beta(s)\beta(u))\beta(t)d(s) = 0$$

from this we get

$$[\beta(u),\beta(s)]\beta(t)d(s) = 0 \quad \forall\, t,\beta(t) \in M$$

From the last relation, we have

(2.10) $[\beta(u),\beta(s)]Md(s) = 0$

Since M is prime then for each $s \in M$, we get $d(s) = 0$ or $[\beta(u),\beta(s)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(s)] = 0$ or $s \in Z(M)$, by using Lemma 2.2, we have $d(s) \in Z(M)$ this alongwith Lemma 2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring.

(ii) Since

(2.11) $[\beta(s),d(t)] = -s^p(\beta(s)o\beta(t))s^q \;\forall\, s, t \in M$

Also $\beta(s)o(\beta(t)\beta(s)) = (\beta(s)o\beta(t))\beta(s)$ by Lemma 2.4. Replacing t by ts in equation (2.11), we get

$[\beta(s),d(ts)] = -s^p(\beta(s)o\beta(ts))s^q$ Since β is a homomorphism, therefore

$$[\beta(s),d(ts)] = -s^p(\beta(s)o(\beta(t)\beta(s)))s^q$$

we get

$$[\beta(s),d(ts)] = -s^p(\beta(s)o\beta(t))s^q\beta(s)$$

from this we arrive at

(2.12) $[\beta(s),d(ts)] = [\beta(s),d(t)]\beta(s)$

By using the lie product, we obtain

$$\beta(s)d(ts) - d(ts)\beta(s) = (\beta(s)d(t) - d(t)\beta(s))\beta(s)$$

this implies

$\beta(s)d(ts) - d(ts)\beta(s) = \beta(s)d(t)\beta(s) - d(t)(\beta(s))^2$

this gives

$\beta(s)(d(t)\beta(s) + \beta(t)d(s)) - (d(t)\beta(s) + \beta(t)d(s))\beta(s) = \beta(s)d(t)\beta(s) - d(t)(\beta(s))^2$ By using Lemma 2.1 and $-(\beta(s) + \beta(t)) = -\beta(t) - \beta(s)$, we get

$\beta(s)d(t)\beta(s)+\beta(s)\beta(t)d(s)-\beta(t)d(s)\beta(s)-d(t)(\beta(s))^2 = \beta(s)d(t)\beta(s)-d(t)(\beta(s))^2$

this implies

$\beta(s)\beta(t)d(s) - \beta(t)d(s)\beta(s) = 0$

we get

(2.13) $\beta(s)\beta(t)d(s) = \beta(t)d(s)\beta(s)$

Left multiplication by $\beta(u)$ from last equation, we get

(2.14) $\beta(u)\beta(s)\beta(t)d(s) = \beta(u)\beta(t)d(s)\beta(s)$

Replacing t by ut in equation (2.13), we have

$\beta(s)\beta(ut)d(s) = \beta(ut)d(s)\beta(s)$

Since β is homomorphism, so we have

(2.15) $\beta(s)\beta(u)\beta(t)d(s) = \beta(u)\beta(t)d(s)\beta(s)$

By comparing equation (2.14) and equation (2.15), we get

$$\beta(u)\beta(s)\beta(t)d(s) = \beta(s)\beta(u)\beta(t)d(s)$$

this gives

$\beta(u)\beta(s)\beta(t)d(s) - \beta(s)\beta(u)\beta(t)d(s) = 0$

this implies

$$(\beta(u)\beta(s) - \beta(s)\beta(u))\, \beta(t)d(s) = 0$$

from this we get

$$[\beta(u),\beta(s)]\beta(t)d(s) = 0 \quad \forall\, t,\beta(t) \in M$$

From the last relation, we have

(2.16) $[\beta(u),\beta(s)]Md(s) = 0$

Since M is prime then for each $s \in M$ we get $d(s) = 0$ or $[\beta(u),\beta(s)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(s)] = 0$ or $s \in Z(M)$, by using Lemma 2.2, we have $d(s) \in Z(M)$ this alongwith Lemma 2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring.

(iii) Since

(2.17) $[d(s),\beta(t)] = t^p(\beta(s)o\beta(t))t^q \;\forall\, s, t \in M$

Also $(\beta(s)\beta(t)) = (\beta(s)o\beta(t))\beta(t)$ by Lemma 2.4, replacing s by st in equation

we get

$[d(st),\beta(t)] = t^p(\beta(st)o\beta(t))t^q$

Since β is a homomorphism, therefore

$$[d(st), \beta(t)] = t^p(\beta(s)\beta(t)o\beta(t))t^q$$

we get

$$[d(st), \beta(t)] = t^p(\beta(s)o\beta(t))t^q\beta(t)$$

this implies

(2.18) $[d(st), \beta(t)] = [d(s), \beta(t)]\ \beta(t)$

By using the lie product, we obtain

$$d(st)\beta(t) - \beta(t)d(st) = (d(s)\beta(t) - \beta(t)d(s))\ \beta(t)$$

this implies

$d(st)\beta(t) - \beta(t)d(st) = d(s)\beta^2(t) - \beta(t)d(s)\beta(t)$

this gives

$(d(s)\beta(t) + \beta(s)d(t))\beta(t) - \beta(t)(d(s)\beta(t) + \beta(s)d(t)) = d(s)(\beta(t))^2 - \beta(t)d(s)\beta(t)$ By using Lemma 2.1 and $-(\beta(s) + \beta(t)) = -\beta(t) - \beta(s)$, we get

$d(s)(\beta(t))^2+\beta(s)d(t)\beta(t)-\beta(t)\beta(s)d(t)-\beta(t)d(s)\beta(t)) = d(s)(\beta(t))^2-\beta(t)d(s)\beta(t)$

this implies

$\beta(s)d(t)\beta(t) - \beta(t)\beta(s)d(t) = 0$

we get

(2.19) $\beta(s)d(t)\beta(t) = \beta(t)\beta(s)d(t)$

Left multiplication by $\beta(u)$ from last equation, we get

(2.20) $\beta(u)\beta(s)d(t)\beta(t) = \beta(u)\beta(t)\beta(s)d(t)$

Replacing s by us in equation (2.19), we have

$\beta(us)d(t)\beta(t) = \beta(t)\beta(us)d(t)$

Since β is homomorphism, so we have

(2.21) $\beta(u)\beta(s)d(t)\beta(t) = \beta(t)\beta(u)\beta(s)d(t)$

By comparing equation (2.20) and equation (2.21), we get

$$\beta(u)\beta(t)\beta(s)d(t) = \beta(t)\beta(u)\beta(s)d(t)$$

this implies

$\beta(u)\beta(t)\beta(s)d(t) - \beta(t)\beta(u)\beta(s)d(t) = 0$

this gives

$$(\beta(u)\beta(t) - \beta(t)\beta(u))\beta(s)d(t) = 0$$

from this we get

$$[(\beta(u),\beta(t)]\beta(s)d(t) = 0 \ \ \forall\, s,\beta(s) \in M$$

From the last relation, we have

(2.22) $[(\beta(u),\beta(t)]Md(t) = 0$

Since M is prime then for each $t \in M$ we get $d(t) = 0$ or $[\beta(u),\beta(t)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(t)] = 0$ or $t \in Z(M)$, by using Lemma 2.2, we have $d(t) \in Z(M)$ this alongwith Lemma2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring.

(iv) Since

(2.23) $[d(s),\beta(t)] = -t^p(\beta(s)o\beta(t))t^q\ \forall\ s, t \in M$

Since $(\beta(s)\beta(t)) = (\beta(s)o\beta(t))\beta(t)$ by Lemma 2.4, replacing s by st in equation (2.23), we get

$[d(st),\beta(t)] = -t^p(\beta(st)o\beta(t))t^q$

Since β is a homomorphism, therefore

$$[d(st),\beta(t)] = -t^p(\beta(s)\beta(t)o\beta(t))t^q$$

we get

$$[d(st),\beta(t)] = -t^p(\beta(s)o\beta(t))t^q\beta(t)$$

this implies

(2.24) $[d(st),\beta(t)] = [d(s),\beta(t)]\beta(t)$

By using the lie product, we obtain

$$d(st)\beta(t) - \beta(t)d(st) = (d(s)\beta(t) - \beta(t)d(s))\beta(t)$$

this implies

$d(st)\beta(t) - \beta(t)d(st) = d(s)\beta^2(t) - \beta(t)d(s)\beta(t)$

this gives

$(d(s)\beta(t) + \beta(s)d(t))\beta(t) - \beta(t)(d(s)\beta(t) + \beta(s)d(t)) = d(s)(\beta(t))^2 - \beta(t)d(s)\beta(t)$ By using Lemma 2.1 and $-(\beta(s) + \beta(t)) = -\beta(t) - \beta(s)$, we get

$d(s)(\beta(t))^2+\beta(s)d(t)\beta(t)-\beta(t)\beta(s)d(t)-\beta(t)d(s)\beta(t)) = d(s)(\beta(t))^2-\beta(t)d(s)\beta(t)$

this implies

$\beta(s)d(t)\beta(t) - \beta(t)\beta(s)d(t) = 0$

we get

(2.25) $\beta(s)d(t)\beta(t) = \beta(t)\beta(s)d(t)$

Left multiplication by $\beta(u)$ from last equation, we get

(2.26) $\beta(u)\beta(s)d(t)\beta(t) = \beta(u)\beta(t)\beta(s)d(t)$

Replacing s by us in equation (2.25) we have

$\beta(us)d(t)\beta(t) = \beta(t)\beta(us)d(t)$

Since β is homomorphism, so we have

(2.27) $\beta(u)\beta(s)d(t)\beta(t) = \beta(t)\beta(u)\beta(s)d(t)$

By comparing equation (2.26) and equation (2.27), we get

$$\beta(u)\beta(t)\beta(s)d(t) = \beta(t)\beta(u)\beta(s)d(t)$$

this implies

$\beta(u)\beta(t)\beta(s)d(t) - \beta(t)\beta(u)\beta(s)d(t) = 0$

this gives

$$(\beta(u)\beta(t) - \beta(t)\beta(u))\beta(s)d(t) = 0$$

from this we get

$$[(\beta(u),\beta(t)]\beta(s)d(t) = 0 \ \ \forall\ s \in M,\beta(s) \in M$$

From the last relation, we have

(2.28) $[(\beta(u),\beta(t)]Md(t) = 0$

Since M is prime then for each $t \in M$ we get $d(t) = 0$ or $[\beta(u),\beta(t)] = 0$. But we know that d is the nonzero two sided β-derivation this

$[\beta(u),\beta(t)] = 0$ or $t \in Z(M)$, by using Lemma 2.2, we have $d(t) \in Z(M)$ this along with Lemma2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring.

Theorem 2.6. *Let M be a zero symmetric prime near ring. If there exist $p \geq 0$, $q \geq 0$ and a nonzero two sided β-derivation d on M, where $\beta : M \rightarrow M$ is a homomorphism, satisfying one of the following conditions:*

(i) $\beta(s)od(t) = s^p[\beta(s),\beta(t)]s^q \; \forall \; s, t \in M$

(ii) $\beta(s)od(t) = -s^p[\beta(s),\beta(t)]s^q \; \forall \; s, t \in M$

(iii) $d(s)o\beta(t) = t^p[\beta(s),\beta(t)]t^q \; \forall \; s, t \in M$

(iv) $d(s)o\beta(t) = t^p[\beta(s),\beta(t)]t^q \; \forall \; s, t \in M$

Then M is a commutative ring.

Proof. (i) Since

(2.29) $\beta(s)od(t) = s^p[\beta(s),\beta(t)]s^q$

Also $[\beta(s),\beta(t)\beta(s)] = [\beta(s),\beta(t)]\beta(s)$ by Lemma 2.4, replacing t by ts in equation(2.29), we have $\beta(s)od(ts) = s^p[\beta(s),\beta(ts)]s^q$

since β is a homomorphism, therefore

$\beta(s)od(ts) = s^p[\beta(s),\beta(t)\beta(s)]s^q$

this implies

$\beta(s)od(ts) = s^p[\beta(s),\beta(t)]s^q\beta(s)$

from this we arrive at

(2.30) $\beta(s)od(ts) = (\beta(s)od(t))\beta(s)$

By using the Jordan product, we get

$\beta(s)d(ts) + d(ts)\beta(s) = (\beta(s)d(t) + d(t)\beta(s))\beta(s)$

this gives

$\beta(s)(d(t)\beta(s) + \beta(t)d(s)) + (\beta(t)d(s) + d(t)\beta(s))\beta(s) = \beta(s)d(t)\beta(s) + d(t)(\beta(s))^2$ By using Lemma 2.1, we have

$\beta(s)d(t)\beta(s)+\beta(s)\beta(t)d(s)+\beta(t)d(s)\beta(s)+d(t)(\beta(s))^2 = \beta(s)d(t)\beta(s)+d(t)(\beta(s))^2$ this implies $\beta(s)\beta(t)d(s) + \beta(t)d(s)\beta(s) = 0$

we obtain

(2.31) $\beta(s)\beta(t)d(s) = -\beta(t)d(s)\beta(s)$

Replacing t by ut in equation (2.31) and using $-\beta(s)\beta(t) = (-\beta(s))\beta(t) = (\beta(-s))\beta(t)$, we get

$\beta(s)\beta(ut)d(s) = -\beta(ut)d(s)\beta(s)$

Since β is a homomorphism, so we have

$$\beta(s)\beta(u)\beta(t)d(s) = -\beta(u)\beta(t)d(s)\beta(s)$$

this gives

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)\beta(t)d(s)\beta(s)$

From equation (2.31), we get

$$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)(-\beta(s)\beta(t)d(s))$$

Since β is homomorphism, therefore

$$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)\beta(-s)\beta(t)d(s)$$

from this we arrive at

(2.32) $\beta(s)\beta(u)\beta(t)d(s) - \beta(-u)\beta(-s)\beta(t)d(s) = 0$

Replacing s by $-s$ in equation (2.32), we obtain

$$\beta(-s)\beta(u)\beta(t)d(-s) - \beta(-u)\beta(s)\beta(t)d(-s) = 0$$

this implies

$$-\beta(s)\beta(u)\beta(t)d(-s) + \beta(u)\beta(s)\beta(t)d(-s) = 0$$

this gives

$\beta(u)\beta(s)\beta(t)d(-s) = \beta(s)\beta(u)\beta(t)d(-s)$

this implies

$\beta(u)\beta(s)\beta(t)d(-s) - \beta(s)\beta(u)\beta(t)d(-s) = 0$

from this we get

$$(\beta(u)\beta(s) - \beta(s)\beta(u))\beta(t)d(-s) = 0$$

this gives

$$[\beta(u),\beta(s)]\beta(t)d(-s) = 0 \;\; \forall \; t,\beta(t) \in M$$

From the last relation, we have

$$[\beta(u),\beta(s)]Md(-s) = 0$$

Since M is prime then for each $s \in M$ we get $d(-s) = 0$ or $[\beta(u),\beta(s)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(s)] = 0$ or $s \in Z(M)$, by Lemma 2.2, we have $d(-s) \in Z(M)$ this along with Lemma 2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring.

(ii) Since

(2.33) $\beta(s)od(t) = -s^p[\beta(s),\beta(t)]s^q$

Also $[\beta(s),\beta(t)\beta(s)] = [\beta(s),\beta(t)]\beta(s)$ by Lemma 2.4, replacing t by ts in equation

(2.33), we get $\beta(s)od(ts) = -s^p[\beta(s),\beta(ts)]s^q$

Since β is homomorphism, therefore

$$\beta(s)od(ts) = -s^p[\beta(s),(\beta(t)\beta(s))]s^q$$

this implies

$\beta(s)od(ts) = -s^p[\beta(s),\beta(t)]s^q\beta(s)$

from this we arrive at

(2.34) $\beta(s)od(ts) = (\beta(s)od(t))\beta(s)$

By using the Jordan product, we get

$\beta(s)d(ts) + d(ts)\beta(s) = (\beta(s)d(t) + d(t)\beta(s))\beta(s)$

this gives

$\beta(s)(d(t)\beta(s) + \beta(t)d(s)) + (\beta(t)d(s) + d(t)\beta(s))\beta(s) = \beta(s)d(t)\beta(s) + d(t)(\beta(s))^2$ By using Lemma 2.1, we have

$\beta(s)d(t)\beta(s)+\beta(s)\beta(t)d(s)+\beta(t)d(s)\beta(s)+d(t)(\beta(s))^2 = \beta(s)d(t)\beta(s)+d(t)(\beta(s))^2$

this implies

$\beta(s)\beta(t)d(s) + \beta(t)d(s)\beta(s) = 0$

we obtain

(2.35) $\beta(s)\beta(t)d(s) = -\beta(t)d(s)\beta(s)$

Replacing t by ut in equation (2.35) and using $-\beta(s)\beta(t) = (-\beta(s))\beta(t) = (\beta(-s))\beta(t)$, we get

$\beta(s)\beta(ut)d(s) = -\beta(ut)d(s)\beta(s)$

Since β is homomorphism, so we have

$$\beta(s)\beta(u)\beta(t)d(s) = -\beta(u)\beta(t)d(s)\beta(s)$$

this gives

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)\beta(t)d(s)\beta(s)$

From equation (2.35), we get

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)(-\beta(s)\beta(t)d(s))$

Since β is homomorphism, therefore

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)\beta(-s)\beta(t)d(s)$

from this we arrive at

(2.36) $\beta(s)\beta(u)\beta(t)d(s) - \beta(-u)\beta(-s)\beta(t)d(s) = 0$

Replacing s by $-s$ in equation (2.36), we obtain

$\beta(-s)\beta(u)\beta(t)d(-s) - \beta(-u)\beta(s)\beta(t)d(-s) = 0$

this implies

$-\beta(s)\beta(u)\beta(t)d(-s) + \beta(u)\beta(s)\beta(t)d(-s) = 0$

this gives

$\beta(u)\beta(s)\beta(t)d(-s) = \beta(s)\beta(u)\beta(t)d(-s)$

this implies

$\beta(u)\beta(s)\beta(t)d(-s) - \beta(s)\beta(u)\beta(t)d(-s) = 0$

from this we get

$$(\beta(u)\beta(s) - \beta(s)\beta(u))\beta(t)d(-s) = 0$$

this gives

$[\beta(u),\beta(s)]\beta(t)d(-s) = 0 \ \forall\ t \in M, \beta(t) \in M$ From the last relation, we have

$$[\beta(u),\beta(s)]Md(-s) = 0$$

Since M is prime then for each $s \in M$ we get $d(-s) = 0$ or $[\beta(u),\beta(s)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(s)] = 0$ or $s \in Z(M)$, by Lemma 2.2, we have $d(-s) \in Z(M)$ this along with Lemma 2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring.

(iii) Since

(2.37) $d(s)o\beta(t) = t^p[\beta(s),\beta(t)]t^q$

Also $[\beta(s)\beta(t),\beta(t)] = [\beta(s),\beta(t)]\beta(t)$ by Lemma 2.4, replacing s by st in equation

(2.37), we have

$$d(st)o\beta(t) = t^p[\beta(st),\beta(t)]t^q$$

Since β is a homomorphism, therefore

$$d(st)o\beta(t) = t^p[\beta(s)\beta(t),\beta(t)]t^q$$

this implies

$d(st)o\beta(t) = t^p[\beta(s),\beta(t)]t^q\beta(t)$

from this we arrive at

(2.38) $d(st)o\beta(t) = (d(s)o\beta(t))\beta(t)$

By using the Jordan product, we get

$d(st)\beta(t) + \beta(t)d(st) = (d(s)\beta(t) + \beta(t)d(s))\beta(t)$

this gives

$(d(s)\beta(t) + \beta(s)d(t))\beta(t) + \beta(t)(\beta(s)d(t) + d(s)\beta(t) = d(s)(\beta(t))^2 + \beta(t)d(s)\beta(t)$ By using lemma 2.1, we get

$d(s)(\beta(t))^2 + \beta(s)d(t)\beta(t) + \beta(t)\beta(s)d(t) + \beta(t)d(s)\beta(t) = d(s)(\beta(t))^2 + \beta(t)d(s)\beta(t)$

this implies

$\beta(s)d(t)\beta(t) + \beta(t)\beta(s)d(t) = 0$

we obtain

(2.39) $\beta(t)\beta(s)d(t) = -\beta(s)d(t)\beta(t)$

Replacing s by us in equation (2.39) and using $-\beta(s)\beta(t) = (-\beta(s))\beta(t) = (\beta(-s))\beta(t)$, we have

$\beta(t)\beta(us)d(t) = -\beta(us)d(t)\beta(t)$

Since β is homomorphism, so we have

$$\beta(t)\beta(u)\beta(s)d(t) = -\beta(u)\beta(s)d(t)\beta(t)$$

this gives

$\beta(t)\beta(u)\beta(s)d(t) = (\beta(-u))\beta(s)d(t)\beta(t)$

From equation (2.39), we get

$\beta(t)\beta(u)\beta(s)d(t) = (\beta(-u))(-\beta(t)\beta(s)d(t))$

Since β is a homomorphism, therefore

$\beta(t)\beta(u)\beta(s)d(t) = \beta(-u)\beta(-t)\beta(s)d(t)$

from this we arrive at

(2.40) $\beta(t)\beta(u)\beta(s)d(t) - \beta(-u)\beta(-t)\beta(s)d(t) = 0$

Replacing t by $-t$ in equation (2.40), we obtain

$$\beta(-t)\beta(u)\beta(s)d(-t) - \beta(-u)\beta(t)\beta(s)d(-t) = 0$$

this implies

$$-\beta(t)\beta(u)\beta(s)d(-t) + \beta(u)\beta(t)\beta(s)d(-t) = 0$$

we obtain $\beta(u)\beta(t)\beta(s)d(-t) = \beta(t)\beta(u)\beta(s)d(-t)$

this implies

$$\beta(u)\beta(t)\beta(s)d(-t) - \beta(t)\beta(u)\beta(s)d(-t) = 0$$

this gives

$$(\beta(u)\beta(t) - \beta(t)\beta(u))\beta(s)d(-t) = 0$$

from this we get

$$[\beta(u),\beta(t)]\beta(s)d(-t) = 0 \qquad \forall\ s,\beta(s) \in M$$

From the last relation, we have

$$[\beta(u),\beta(t)]Md(-t) = 0$$

Since M is prime then for each $t \in M$ we get $d(-t) = 0$ or $[\beta(u),\beta(t)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(t)] = 0$ or $t \in Z(M)$, by Lemma 2.2, we have $d(-t) \in Z(M)$ this along with

Lemma 2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring.

(iv) Since

(2.41) $d(s)o\beta(t) = -t^p[\beta(s),\beta(t)]t^q$

Also $[\beta(s)\beta(t),\beta(t)] = [\beta(s),\beta(t)]\beta(t)$ by Lemma 2.4, replacing s by st in equation

(2.41), we have $d(st)o\beta(t) = -t^p[\beta(st),\beta(t)]t^q$

Since β is homomorphism, therefore

$d(st)o\beta(t) = -t^p[\beta(s)\beta(t),\beta(t)]t^q$

this implies

$d(st)o\beta(t) = -t^p[\beta(s),\beta(t)]t^q\beta(t)$

from this we arrive at

(2.42) $d(st)o\beta(t) = (d(s)o\beta(t))\beta(t)$

By using the Jordan product, we get

$$d(st)\beta(t) + \beta(t)d(st) = (d(s)\beta(t) + \beta(t)d(s))\beta(t)$$

this gives

$(d(s)\beta(t) + \beta(s)d(t))\beta(t) + \beta(t)(\beta(s)d(t) + d(s)\beta(t) = d(s)(\beta(t))^2 + \beta(t)d(s)\beta(t)$ by using lemma 2.1 we get

$d(s)(\beta(t))^2 +\beta(s)d(t)\beta(t)+\beta(t)\beta(s)d(t)+\beta(t)d(s)\beta(t) = d(s)(\beta(t))^2 +\beta(t)d(s)\beta(t)$

this implies

$\beta(s)d(t)\beta(t) + \beta(t)\beta(s)d(t) = 0$

we obtain

(2.43) $\beta(t)\beta(s)d(t) = -\beta(s)d(t)\beta(t)$

Replacing s by us in equation (2.43) and using $-\beta(s)\beta(t) = (-\beta(s))\beta(t) = (\beta(-s))\beta(t)$, we have

$\beta(t)\beta(us)d(t) = -\beta(us)d(t)\beta(t)$

Since β is homomorphism, so we have

$$\beta(t)\beta(u)\beta(s)d(t) = -\beta(u)\beta(s)d(t)\beta(t)$$

this gives

$\beta(t)\beta(u)\beta(s)d(t) = (\beta(-u))\beta(s)d(t)\beta(t)$

we get $\beta(t)\beta(u)\beta(s)d(t) = (\beta(-u))(-\beta(t)\beta(s)d(t))$

Since β is homomorphism, therefore

$\beta(t)\beta(u)\beta(s)d(t) = \beta(-u)\beta(-t)\beta(s)d(t)$

from this we arrive at

(2.44) $\beta(t)\beta(u)\beta(s)d(t) - \beta(-u)\beta(-t)\beta(s)d(t) = 0$

Replacing t by $-t$ in equation (2.63), we obtain

$\beta(-t)\beta(u)\beta(s)d(-t) - \beta(-u)\beta(t)\beta(s)d(-t) = 0$

this implies

$-\beta(t)\beta(u)\beta(s)d(-t) + \beta(u)\beta(t)\beta(s)d(-t) = 0$

we obtain $\beta(u)\beta(t)\beta(s)d(-t) = \beta(t)\beta(u)\beta(s)d(-t)$

this gives

$\beta(u)\beta(t)\beta(s)d(-t) - \beta(t)\beta(u)\beta(s)d(-t) = 0$

from this we get

$$(\beta(u)\beta(t) - \beta(t)\beta(u))\beta(s)d(-t) = 0$$

we obtain

$$[\beta(u),\beta(t)]\beta(s)d(-t) = 0 \qquad \forall\, s,\beta(s) \in M$$

From the last relation, we get

$$[\beta(u),\beta(t)]Md(-t) = 0$$

Since M is prime then for each $t \in M$ we get $d(-t) = 0$ or $[\beta(u),\beta(t)] = 0$. But we know that d is the nonzero two sided β-derivation this gives $[\beta(u),\beta(t)] = 0$ or $t \in Z(M)$, by Lemma 2.2, we have $d(-t) \in Z(M)$ this along with Lemma 2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring.

Theorem 2.7. *Let M be a prime zero symmetric near-ring. If* $(M,+)$ *is* 2 *torsion free, then there is no non negative integers p, q* ≥ 0 *and a nonzero two sided βderivation d, where* $\beta : M \to M$ *is a homomorphism such that*

(i) $\beta(s)od(t) = s^p[\beta(s),\beta(t)]s^q\ \forall\, s, t \in M$

(ii) $\beta(s)od(t) = -s^p[\beta(s),\beta(t)]s^q\ \forall\, s, t \in M$

(iii) $d(s)o\beta(t) = t^p[\beta(s),\beta(t)]t^q\ \forall\, s, t \in M$

(iv) $d(s)o\beta(t) = t^p[\beta(s),\beta(t)]t^q\ \forall\, s, t \in M$

Proof. (i) Since

(2.45) $\beta(s)od(t) = s^p[\beta(s), \beta(t)]s^q$

Also $[\beta(s), \beta(t)\beta(s)] = [\beta(s), \beta(t)]\, \beta(s)$ by Lemma 2.4, replacing t by ts in equation

(2.45), we have $\beta(s)od(ts) = s^p[\beta(s), \beta(ts)]s^q$

Since β is a homomorphism, therefore

$$\beta(s)od(ts) = s^p[\beta(s),(\beta(t)\beta(s))]s^q$$

this implies $\beta(s)od(ts) = s^p[\beta(s), \beta(t)]s^q\beta(s)$

from this we arrive at

(2.46) $\beta(s)od(ts) = (\beta(s)od(t))\beta(s)$

By using the Jordan product, we get

$\beta(s)d(ts) + d(ts)\beta(s) = (\beta(s)d(t) + d(t)\beta(s))\beta(s)$

this gives

$\beta(s)(d(t)\beta(s) + \beta(t)d(s)) + (\beta(t)d(s) + d(t)\beta(s))\beta(s) = \beta(s)d(t)\beta(s) + d(t)(\beta(s))^2$ By using Lemma 2.1, we have

$\beta(s)d(t)\beta(s)+\beta(s)\beta(t)d(s)+\beta(t)d(s)\beta(s)+d(t)(\beta(s))^2 = \beta(s)d(t)\beta(s)+d(t)(\beta(s))^2$ this implies $\beta(s)\beta(t)d(s) + \beta(t)d(s)\beta(s) = 0$

we obtain

(2.47) $\beta(s)\beta(t)d(s) = -\beta(t)d(s)\beta(s)$

Replacing t by ut in equation (2.47) and using $-\beta(s)\beta(t) = (-\beta(s))\beta(t) = (\beta(-s))\beta(t)$, we get

$\beta(s)\beta(ut)d(s) = -\beta(ut)d(s)\beta(s)$

Since β is homomorphism, so we have

$$\beta(s)\beta(u)\beta(t)d(s) = -\beta(u)\beta(t)d(s)\beta(s)$$

this gives

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)\beta(t)d(s)\beta(s)$

From equation (2.47), we get

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)(-\beta(s)\beta(t)d(s))$

Since β is a homomorphism, therefore

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)\beta(-s)\beta(t)d(s)$

from this we arrive at

(2.48) $\beta(s)\beta(u)\beta(t)d(s) - \beta(-u)\beta(-s)\beta(t)d(s) = 0$

Replacing s by $-s$ in equation (2.48), we obtain

$$\beta(-s)\beta(u)\beta(t)d(-s) - \beta(-u)\beta(s)\beta(t)d(-s) = 0$$

this implies

$$-\beta(s)\beta(u)\beta(t)d(-s) + \beta(u)\beta(s)\beta(t)d(-s) = 0$$

this gives

$\beta(u)\beta(s)\beta(t)d(-s) = \beta(s)\beta(u)\beta(t)d(-s)$

this implies

$\beta(u)\beta(s)\beta(t)d(-s) - \beta(s)\beta(u)\beta(t)d(-s) = 0$

from this we get

$$(\beta(u)\beta(s) - \beta(s)\beta(u))\beta(t)d(-s) = 0$$

this gives

$[\beta(u),\beta(s)]\beta(t)d(-s) = 0 \ \forall\ t,\beta(t) \in M$ From the las relation, we have

$$[\beta(u),\beta(s)]Md(-s) = 0$$

Since M is prime then for each $s \in M$ we get $d(-s) = 0$ or $[\beta(u),\beta(s)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(s)] = 0$ or $s \in Z(M)$, by Lemma 2.2, we have $d(-s) \in Z(M)$ this along with Lemma 2.3, we get

$$\mathrm{d(M)} \subset \mathrm{Z(M)}$$

Hence M is a commutative ring. From equation (2.47), we have

(2.49) $\quad 2\beta(s)\beta(t)d(s) = 0$

Since M is 2 torsion free, this implies

$$\beta(s)\beta(t)d(s) = 0 \quad \forall\ t,\beta(t) \in M$$

this gives

$$\beta(s)Md(s) = 0$$

Since M is prime then for each $s \in M$ we have $d(s) = 0$ or $\beta(s) = 0$. But d is non zero two sided β-derivation hence we get $\beta(s) = 0$ for every $s \in M$. A

Contradiction, thus there is no such derivation.

(ii) Since

(2.50) $\quad \beta(s)od(t) = -s^p[\beta(s),\beta(t)]s^q$

Also $[\beta(s),\beta(t)\beta(s)] = [\beta(s),\beta(t)]\beta(s)$ by Lemma 2.4, replacing t by ts in equation (2.50), we get $\beta(s)od(ts) = -s^p[\beta(s),\beta(ts)]s^q$

Since β is homomorphism, therefore

$$\beta(s)od(ts) = -s^p[\beta(s),(\beta(t)\beta(s))]s^q$$

this implies $\beta(s)od(ts) = -s^p[\beta(s),\beta(t)]s^q\beta(s)$

from this we arrive at

(2.51) $\quad \beta(s)od(ts) = (\beta(s)od(t))\beta(s)$

By using the Jordan product, we get

$\beta(s)d(ts) + d(ts)\beta(s) = (\beta(s)d(t) + d(t)\beta(s))\beta(s)$

this gives

$\beta(s)(d(t)\beta(s) + \beta(t)d(s)) + (\beta(t)d(s) + d(t)\beta(s))\beta(s) = \beta(s)d(t)\beta(s) + d(t)(\beta(s))^2$ By using Lemma 2.1, we have

$\beta(s)d(t)\beta(s)+\beta(s)\beta(t)d(s)+\beta(t)d(s)\beta(s)+d(t)(\beta(s))^2 = \beta(s)d(t)\beta(s)+d(t)(\beta(s))^2$ this implies $\beta(s)\beta(t)d(s) + \beta(t)d(s)\beta(s) = 0$

we obtain

(2.52) $\quad \beta(s)\beta(t)d(s) = -\beta(t)d(s)\beta(s)$

Replacing t by ut in equation (2.52) and using $-\beta(s)\beta(t) = (-\beta(s))\beta(t) = (\beta(-s))\beta(t)$, we get

$$\beta(s)\beta(ut)d(s) = -\beta(ut)d(s)\beta(s)$$

Since β is homomorphism, so we have

$$\beta(s)\beta(u)\beta(t)d(s) = -\beta(u)\beta(t)d(s)\beta(s)$$

this gives

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)\beta(t)d(s)\beta(s)$

From equation (2.52), we get

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)(-\beta(s)\beta(t)d(s))$

Since β is homomorphism, therefore

$\beta(s)\beta(u)\beta(t)d(s) = \beta(-u)\beta(-s)\beta(t)d(s)$

from this we arrive at

(2.53) $\quad \beta(s)\beta(u)\beta(t)d(s) - \beta(-u)\beta(-s)\beta(t)d(s) = 0$

Replacing s by $-s$ in equation (2.53), we obtain

$\beta(-s)\beta(u)\beta(t)d(-s) - \beta(-u)\beta(s)\beta(t)d(-s) = 0$

this implies

$-\beta(s)\beta(u)\beta(t)d(-s) + \beta(u)\beta(s)\beta(t)d(-s) = 0$

this gives

$\beta(u)\beta(s)\beta(t)d(-s) = \beta(s)\beta(u)\beta(t)d(-s)$

this implies

$\beta(u)\beta(s)\beta(t)d(-s) - \beta(s)\beta(u)\beta(t)d(-s) = 0$

from this we get

$$(\beta(u)\beta(s) - \beta(s)\beta(u))\beta(t)d(-s) = 0$$

this gives

$[\beta(u),\beta(s)]\beta(t)d(-s) = 0 \ \forall\ t,\beta(t) \in M$ From the last relation, we have

$$[\beta(u),\beta(s)]Md(-s) = 0$$

Since M is prime then for each $s \in M$ we get $d(-s) = 0$ or $[\beta(u),\beta(s)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(s)] = 0$ or $s \in Z(M)$, by Lemma 2.2, we have $d(-s) \in Z(M)$ this along with Lemma 2.3, we get

$$\mathrm{d(M)} \subset \mathrm{Z(M)}$$

Hence M is a commutative ring. From equation (2.52), we have

(2.54) $\quad 2\beta(s)\beta(t)d(s) = 0$

Since M is 2 torsion free, this implies

$$\beta(s)\beta(t)d(s) = 0 \quad \forall\ t,\beta(t) \in M$$

this gives

$$\beta(s)Md(s) = 0$$

Since M is prime then for each $s \in M$ we have $d(s) = 0$ or $\beta(s) = 0$. But d is non zero two sided β-derivation hence we get $\beta(s) = 0$ for every $s \in M$. A

Contradiction, thus there is no such derivation.

(iii) Since

(2.55) $\quad d(s)o\beta(t) = t^p[\beta(s),\beta(t)]t^q$

Also $[\beta(s)\beta(t),\beta(t)] = [\beta(s),\beta(t)]\beta(t)$ by Lemma 2.4, replacing s by st in equation

(2.55), we have

$$d(st)o\beta(t) = t^p[\beta(st),\beta(t)]t^q$$

Since β is homomorphism, therefore

$$d(st)o\beta(t) = t^p[\beta(s)\beta(t),\beta(t)]t^q$$

this implies

$d(st)o\beta(t) = t^p[\beta(s),\beta(t)]t^q\beta(t)$

from this we arrive at

(2.56) $d(st)o\beta(t) = (d(s)o\beta(t))\beta(t)$

By using the Jordan product, we get

$d(st)\beta(t) + \beta(t)d(st) = (d(s)\beta(t) + \beta(t)d(s))\beta(t)$

this gives

$(d(s)\beta(t) + \beta(s)d(t))\beta(t) + \beta(t)(\beta(s)d(t) + d(s)\beta(t) = d(s)(\beta(t))^2 + \beta(t)d(s)\beta(t)$ By using lemma 2.1, we get

$d(s)(\beta(t))^2 + \beta(s)d(t)\beta(t) + \beta(t)\beta(s)d(t) + \beta(t)d(s)\beta(t) = d(s)(\beta(t))^2 + \beta(t)d(s)\beta(t)$ this implies $\beta(s)d(t)\beta(t) + \beta(t)\beta(s)d(t) = 0$

we obtain

(2.57) $\beta(t)\beta(s)d(t) = -\beta(s)d(t)\beta(t)$

Replacing s by us in equation (2.57) and using $-\beta(s)\beta(t) = (-\beta(s))\beta(t) = (\beta(-s))\beta(t)$, we have $\beta(t)\beta(us)d(t) = -\beta(us)d(t)\beta(t)$

Since β is homomorphism, therefore

$$\beta(t)\beta(u)\beta(s)d(t) = -\beta(u)\beta(s)d(t)\beta(t)$$

this gives

$\beta(t)\beta(u)\beta(s)d(t) = (\beta(-u))\beta(s)d(t)\beta(t)$

From equation (2.57), we get

$$\beta(t)\beta(u)\beta(s)d(t) = (\beta(-u))(-\beta(t)\beta(s)d(t))$$

Since β is homomorphism, therefore

$$\beta(t)\beta(u)\beta(s)d(t) = \beta(-u)\beta(-t)\beta(s)d(t)$$

from this we arrive at

(2.58) $\beta(t)\beta(u)\beta(s)d(t) - \beta(-u)\beta(-t)\beta(s)d(t) = 0$

Replacing t by $-t$ in equation (2.58), we obtain

$$\beta(-t)\beta(u)\beta(s)d(-t) - \beta(-u)\beta(t)\beta(s)d(-t) = 0$$

this implies

$$-\beta(t)\beta(u)\beta(s)d(-t) + \beta(u)\beta(t)\beta(s)d(-t) = 0$$

we obtain $\beta(u)\beta(t)\beta(s)d(-t) = \beta(t)\beta(u)\beta(s)d(-t)$

this implies

$$\beta(u)\beta(t)\beta(s)d(-t) - \beta(t)\beta(u)\beta(s)d(-t) = 0$$

this gives

$$(\beta(u)\beta(t) - \beta(t)\beta(u))\beta(s)d(-t) = 0$$

from this we get

$[\beta(u),\beta(t)]\beta(s)d(-t) = 0 \ \forall \ s,\beta(s) \in M$ From the last relation, we have

$$[\beta(u),\beta(t)]Md(-t) = 0$$

Since M is prime then for each $t \in M$ we get $d(-t) = 0$ or $[\beta(u),\beta(t)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(t)] = 0$ or $t \in Z(M)$, by Lemma 2.2, we have $d(-t) \in Z(M)$ this along with Lemma 2.3, we get

$$d(M) \subset Z(M)$$

Hence M is a commutative ring. From equation (2.57), we have

(2.59) $2\beta(t)\beta(s)d(t) = 0$

Since M is 2 torsion free, this implies

$$\beta(t)\beta(s)d(t) = 0 \quad \forall s,\beta(s) \in M$$

this gives

$$\beta(t)Md(t) = 0$$

Since M is prime then for each $t \in M$ we have $d(t) = 0$ or $\beta(s) = 0$. But d is non zero two sided β-derivation hence we get $\beta(t) = 0$ for every $t \in M$. A Contradiction, thus there is no such derivation. (iv) Since

(2.60) $d(s)o\beta(t) = -t^p[\beta(s),\beta(t)]t^q$

Also $[\beta(s)\beta(t),\beta(t)] = [\beta(s),\beta(t)]\beta(t)$ by Lemma 2.4, replacing s by st in equation (2.60), we have $d(st)o\beta(t) = -t^p[\beta(st),\beta(t)]t^q$

Since | B-homomorphism, therefore

$d(st)o\beta(t) = -t^p[\beta(s)\beta(t),\beta(t)]t^q$

this implies

$d(st)o\beta(t) = -t^p[\beta(s),\beta(t)]t^q\beta(t)$

from this we arrive at

(2.61) $d(st)o\beta(t) = (d(s)o\beta(t))\beta(t)$

By using the Jordan product, we get

$d(st)\beta(t) + \beta(t)d(st) = (d(s)\beta(t) + \beta(t)d(s))\beta(t)$

this gives

$(d(s)\beta(t) + \beta(s)d(t))\beta(t) + \beta(t)(\beta(s)d(t) + d(s)\beta(t) = d(s)(\beta(t))^2 + \beta(t)d(s)\beta(t)$ By using lemma 2.1, we get

$d(s)(\beta(t))^2 + \beta(s)d(t)\beta(t) + \beta(t)\beta(s)d(t) + \beta(t)d(s)\beta(t) = d(s)(\beta(t))^2 + \beta(t)d(s)\beta(t)$ this implies $\beta(s)d(t)\beta(t) + \beta(t)\beta(s)d(t) = 0$

we obtain

(2.62) $\beta(t)\beta(s)d(t) = -\beta(s)d(t)\beta(t)$

Replacing s by us in equation (2.62) and using $-\beta(s)\beta(t) = (-\beta(s))\beta(t) = (\beta(-s))\beta(t)$, we have

$$\beta(t)\beta(us)d(t) = -\beta(us)d(t)\beta(t)$$

Since β is a homomorphism, so we have

$$\beta(t)\beta(u)\beta(s)d(t) = -\beta(u)\beta(s)d(t)\beta(t)$$

this gives

$\beta(t)\beta(u)\beta(s)d(t) = (\beta(-u))\beta(s)d(t)\beta(t)$

From equation (62), we get

$\beta(t)\beta(u)\beta(s)d(t) = (\beta(-u))(-\beta(t)\beta(s)d(t))$

Since β is a homomorphism, therefore

$\beta(t)\beta(u)\beta(s)d(t) = \beta(-u)\beta(-t)\beta(s)d(t)$

from this we arrive at

(2.63) $\beta(t)\beta(u)\beta(s)d(t) - \beta(-u)\beta(-t)\beta(s)d(t) = 0$

Replacing t by $-t$ in equation (2.63), we obtain

$\beta(-t)\beta(u)\beta(s)d(-t) - \beta(-u)\beta(t)\beta(s)d(-t) = 0$

this implies

$-\beta(t)\beta(u)\beta(s)d(-t) + \beta(u)\beta(t)\beta(s)d(-t) = 0$

we obtain $\beta(u)\beta(t)\beta(s)d(-t) = \beta(t)\beta(u)\beta(s)d(-t)$

this implies

$\beta(u)\beta(t)\beta(s)d(-t) - \beta(t)\beta(u)\beta(s)d(-t) = 0$

this gives

$$(\beta(u)\beta(t) - \beta(t)\beta(u))\beta(s)d(-t) = 0$$

from this we get

$$[\beta(u),\beta(t)]\beta(s)d(-t) = 0 \quad \forall s,\beta(s) \in M$$

From the last relation, we have

$$[\beta(u),\beta(t)]Md(-t) = 0$$

Since M is prime then for each $t \in M$ we get $d(-t) = 0$ or $[\beta(u),\beta(t)] = 0$. But we know that d is the nonzero two sided β-derivation this gives

$[\beta(u),\beta(t)] = 0$ or $t \in Z(M)$, by Lemma 2.2, we have $d(-t) \in Z(M)$ this along with

Lemma 2.3, we get

$$\mathrm{d(M)} \subset \mathrm{Z(M)}$$

Hence M is a commutative ring. From equation (2.62), we have

$$2\beta(t)\beta(s)d(t) = 0 \tag{2.64}$$

Since M is 2 torsion free, this implies

$$\beta(t)\beta(s)d(t) = 0 \quad \forall s,\beta(s) \in M$$

this gives

$$\beta(t)Md(t) = 0$$

Since M is prime then for each $t \in M$ we have $d(t) = 0$ or $\beta(t) = 0$. But d is nonzero two sided β-derivation hence we get $\beta(t) = 0$ for every $t \in M$. It is a contradiction, thus there is no such derivation.

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