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RESEARCH ARTICLE

A NOTE ON COMMUTATIVITY OF PRIME NEAR RING WITH GENERALIZED β-DERIVATION

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ABSTRACT

In this paper, we prove commutativity of prime near rings by using the notion of β-derivations. Let M be a prime near ring. If there exist $u_1, v_1 \epsilon M$ and two sided generalized β-derivation G associated with the non-zero two sided β-derivation g on M, where $\beta: M \to M$ is a homomorphism, satisfying the following conditions:

i. $G([p_1, q_1]) = {p_1}^{u_1}[\beta(p_1), \beta(q_1)]{p_1}^{v_1} \;\; \forall \;\; p_1, q_1 \epsilon M$

ii. $G([p_1, q_1]) = {p_1}^{u_1}[\beta(p_1), \beta(q_1)]{p_1}^{v_1} \;\; \forall \;\; p_1, q_1 \epsilon M$

Then M is a commutative ring.

KEYWORDS

β-derivations, commutativity, homomorphism.

1. Introduction

A non-empty set M with two binary operations namely addition and multiplication is said to be a right near ring if M under addition is a group, M under multiplication is a semi group and (M,+,.) satisfies right distributive law. If $M_o = \{p_1 \epsilon M : 0p_1 = 0\}$ then M_o is called zero symmetric near ring. For all $p_1, q_1 \epsilon M, p_1 M q_1 = 0$, this implies that $p_1 = 0$ or $q_1 = 0$, then M is called prime near ring (Bell and Mason, 1987). A mapping $g: M \to M$ is known as two sided β-derivation if there is a function $\beta: M \to M$ such that $g(p_1 q_1) = g(p_1)\beta(q_1) + \beta(p_1) g(q_1)$ and $g(p_1 q_1) = \beta(p_1) g(q_1) + g(p_1)\beta(q_1)$ for all $p_1, q_1 \epsilon M$. A mapping $G: M \to M$ is said to be two sided generalized β-derivation if there is a function $\beta: M \to M$ such that $G(p_1 q_1) = G(p_1)\beta(q_1) + \beta(p_1) g(q_1)$ or equivalently $G(p_1 q_1) = g(p_1)\beta(q_1) + \beta(p_1) G(q_1)$ for all $p_1, q_1 \epsilon M$. Let M be a near ring whose center $Z(M)$ is defined as: $Z(M) = \{c \epsilon M : cm = mc, \; for all \; m \epsilon M\}$. For any $p_1, q_1 \epsilon M$, $(p_1 o q_1) = p_1 q_1 + q_1 p_1$ and $[p_1, q_1] = p_1 q_1 - q_1 p_1$ is known as Jordan and lie product, respectively. In recent years, various mathematicians have studied derivation and generalized derivation for commutativity of prime, semi prime ring and Γ rings (Albas and Argac, 2004; Basudeb, 2010; Daif and Bell, 1992; De Filippis and Rehman, 2010; Golbasi and Koc, 2011; Golbasi and Koc, 2009; Khan et al., 2016; Khan et al., 2013; Khan et al., 2013; Quadri et al., 2003). On near ring, some comparable result has also been derived (Ashraf and Shakir, 2008; Ashraf et al., 2004; Beidar et al., 1996; Bell and Mason, 1992; Boua and Oukhtile, 2011; Kamal, 2001; Raina et al., 2009; Vukman, 2007; Khan et al., 2021).

In this paper, we investigate results for prime near ring involving two sided generalized β-derivation.

2. Main Results

Theorem 2.1. Let M be a prime near ring. If there exist $u_1, v_1 \epsilon M$ and two sided generalized β-derivation G associated with the non-zero two sided β-derivation g on M, where $\beta: M \to M$ is a homomorphism, satisfying the following conditions:

i. $G([p_1,q_1]) = {p_1}^{u_1}[\beta(p_1), \beta(q_1)]{p_1}^{v_1} \quad \forall \;\; p_1, q_1 \epsilon M$

ii. $G([p_1,q_1]) = -{p_1}^{u_1}[\beta(p_1), \beta(q_1)]{p_1}^{v_1} \quad \forall \;\; p_1, q_1 \epsilon M$

Then M is a commutative ring.

Proof. i. Since

$$G([p_1,q_1]) = {p_1}^{u_1}[\beta(p_1), \beta(q_1)]{p_1}^{v_1} \quad \forall \;\; p_1, q_1 \epsilon M \qquad (1)$$

Also $[p_1, q_1 p_1] = [p_1, q_1]p_1$, replacing q_1 by $q_1 p_1$ in equation (1), we obtain

$$G([p_1, q_1]p_1) = G([p_1, q_1 p_1]).$$

This gives

$$G([p_1, q_1]p_1) = {p_1}^{u_1}[\beta(p_1), \beta(q_1 p_1)]{p_1}^{v_1}.$$

Since β is a homomorphism, so the last relation implies

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$G([p_1,q_1]p_1) = {p_1}^{u_1}[\beta(p_1),\beta(q_1)\beta(p_1)]{p_1}^{v_1}$.

This gives

$$G([p_1,q_1]p_1) = {p_1}^{u_1}[\beta(p_1),\beta(q_1)]{p_1}^{v_1}\beta(p_1) \quad \forall \quad p_1,q_1 \in M \tag{2}$$

By using definition of generalized β-derivation, we have

$$G([p_1,q_1]p_1) = G[p_1,q_1]\beta(p_1) + \beta([p_1,q_1])g(p_1)$$

By using equation (1) and equation (2), we obtain

$${p_1}^{u_1}[\beta(p_1),\beta(q_1)]{p_1}^{v_1}\beta(p_1) = {p_1}^{u_1}[\beta(p_1),\beta(q_1)]{p_1}^{v_1}\beta(p_1) + [\beta(p_1),\beta(q_1)]g(p_1)$$

From this we arrive at

$$[\beta(p_1),\beta(q_1)]g(p_1) = 0$$

Replacing q_1 by r_1q_1, we get

$$[\beta(p_1),\beta(r_1q_1)]g(p_1) = 0$$

Since β is a homomorphism, we have

$$[\beta(p_1),\beta(r_1)\beta(q_1)]g(p_1) = 0$$

This implies

$$[\beta(p_1),\beta(r_1)]\beta(q_1)g(p_1) = 0 \quad \forall \quad p_1,q_1,r_1 \in M$$

The last relation gives

$$[\beta(p_1),\beta(r_1)]Mg(p_1) = 0 \quad \forall \quad p_1,r_1 \in M \tag{3}$$

Since M is prime then for each $p_1 \in M$, we get $g(p_1) = 0$ or $[\beta(p_1),\beta(r_1)] = 0$. Since g is a nonzero two sided β-derivation, so we have $[\beta(p_1),\beta(r_1)] = 0$ or $p_1 \in Z(M)$, by using Lemma [2, khan et.al., 2021], we have $g(p_1) \in Z(M)$. This along with Lemma [3, khan et.al., 2021], we get

$$g(M) \subset Z(M)$$

Hence M is a commutative ring.

ii. Since

$$G([p_1,q_1]) = -{p_1}^{u_1}[\beta(p_1),\beta(q_1)]{p_1}^{v_1} \; \forall \quad p_1,q_1 \in M \tag{4}$$

Also $[p_1,q_1p_1] = [p_1,q_1]p_1$, replacing q_1 by q_1p_1 in equation (4), we obtain

$$G([p_1,q_1]p_1) = G([p_1,q_1p_1]).$$

This gives

$$G([p_1,q_1]p_1) = -{p_1}^{u_1}[\beta(p_1),\beta(q_1p_1)]{p_1}^{v_1}.$$

Since β is a homomorphism, so the last relation implies

$$G([p_1,q_1]p_1) = -{p_1}^{u_1}[\beta(p_1),\beta(q_1)\beta(p_1)]{p_1}^{v_1} .$$

This gives

$$G([p_1,q_1]p_1) = -{p_1}^{u_1}[\beta(p_1),\beta(q_1)]{p_1}^{v_1}\beta(p_1) \quad \forall \quad p_1,q_1 \in M \tag{5}$$

By using definition of generalized β-derivation, we have

$$G([p_1,q_1]p_1) = G[p_1,q_1]\beta(p_1) + \beta([p_1,q_1])g(p_1)$$

By using equation (4) and equation (5), we obtain

$$-{p_1}^{u_1}[\beta(p_1),\beta(q_1)]{p_1}^{v_1}\beta(p_1) = -{p_1}^{u_1}[\beta(p_1),\beta(q_1)]{p_1}^{v_1}\beta(p_1) + [\beta(p_1),\beta(q_1)]g(p_1)$$

From this we arrive at

$$[\beta(p_1),\beta(q_1)]g(p_1) = 0$$

Replacing q_1 by r_1q_1, we get

$$[\beta(p_1),\beta(r_1q_1)]g(p_1) = 0$$

Since β is a homomorphism, we have

$$[\beta(p_1),\beta(r_1)\beta(q_1)]g(p_1) = 0$$

This implies

$$[\beta(p_1),\beta(r_1)]\beta(q_1)g(p_1) = 0 \quad \forall \quad p_1,q_1,r_1 \in M$$

The last relation gives

$$[\beta(p_1),\beta(r_1)]Mg(p_1) = 0 \quad \forall \quad p_1,r_1 \in M \tag{6}$$

Since M is prime then for each $p_1 \in M$, we get $g(p_1) = 0$ or $[\beta(p_1),\beta(r_1)] = 0$. Since g is a nonzero two sided β-derivation, so we have $[\beta(p_1),\beta(r_1)] = 0$ or $p_1 \in Z(M)$, by using Lemma [2, khan et.al., 2021], we have $g(p_1) \in Z(M)$. This alongwith Lemma [3, khan et.al., 2021], we get

$$g(M) \subset Z(M)$$

Hence M is a commutative ring.

Example 1. Let X be a commutative ring and

$M = \{\begin{pmatrix} 0 & p_1 & q_1 \\ 0 & 0 & r_1 \\ 0 & 0 & 0 \end{pmatrix} : p_1,q_1,r_1 \in X\}$. We define the following mappings on M.

Let $\beta: M \to M$ is a mapping defined by:

$$\beta\begin{pmatrix} 0 & p_1 & q_1 \\ 0 & 0 & r_1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & p_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and $g: M \to M$ is a mapping defined by:

$$g\begin{pmatrix} 0 & p_1 & q_1 \\ 0 & 0 & r_1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & r_1 - p_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

Then g is nonzero β-derivation on M. If A= $\begin{pmatrix} 0 & 0 & r_1 - p_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, then $AMA = 0$ which proves that M is not prime. Moreover, if $G\begin{pmatrix} 0 & p_1 & q_1 \\ 0 & 0 & r_1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_1 \end{pmatrix}$, then G is a generalized β-derivation on M which satisfies the condition $G([A,B]) = [A,B] \quad \forall \; A,B \in M$.

Theorem 2.2. Let M be a prime near ring and and G a two-sided generalized β-derivation associated with the non-zero two sided β-derivation g on M, where $\beta: M \to M$ is homomorphism. If there exist $u_1, v_1 \in M$, then the following hold:

i. $G(p_1 o q_1) = {p_1}^{u_1}(\beta(p_1) o \beta(q_1)){p_1}^{v_1} \quad \forall \; p_1,q_1 \in M$

ii. $G(p_1 o q_1) = -{p_1}^{u_1}(\beta(p_1) o \beta(q_1)){p_1}^{v_1} \quad \forall \; p_1,q_1 \in M$

Then M is a commutative ring.

Proof. i. Since

$$G(p_1 o q_1) = {p_1}^{u_1}(\beta(p_1) o \beta(q_1)){p_1}^{v_1} \tag{7}$$

Also $p_1 o(q_1p_1) = (p_1 o q_1)p_1$, replacing q_1 by q_1p_1 in equation (7), we obtain

$$G\big((p_1 o q_1)p_1\big) = G(p_1 o(q_1p_1))$$

This gives

$$G\big((p_1 o q_1)p_1\big) = {p_1}^{u_1}\big(\beta(p_1) o \beta(q_1p_1)\big){p_1}^{v_1}$$

Since β is a homomorphism, so the last relation implies

$$G\big((p_1 o q_1)p_1\big) = {p_1}^{u_1}\big(\beta(p_1) o \beta(q_1)\beta(p_1)\big){p_1}^{v_1}$$

This gives

$$G\big((p_1 o q_1)p_1\big) = {p_1}^{u_1}(\beta(p_1) o \beta(q_1)){p_1}^{v_1}\beta(p_1) \quad \forall \; p_1,q_1 \in M \tag{8}$$

By using definition of generalized β-derivation, we have

$$G\big((p_1 o q_1)p_1\big) = G(p_1 o q_1)\beta(p_1) + \beta(p_1 o q_1)g(p_1)$$

By using equation (7) and equation (8), we obtain

$$p_1{}^{u_1}(\beta(p_1)o\beta(q_1))p_1{}^{v_1}\,\beta(p_1) = p_1{}^{u_1}(\beta(p_1)o\beta(q_1))p_1{}^{v_1}\,\beta(p_1) + (\beta(p_1)o\beta(q_1))g(p_1)$$

From this we arrive at

$$(\beta(p_1)o\beta(q_1))g(p_1) = 0$$

This implies

$$(\beta(p_1)\beta(q_1) + \beta(q_1)\ \beta(p_1))g(p_1) = 0$$

From this we get

$$\beta(p_1)\beta(q_1)g(p_1) + \beta(q_1)\beta(p_1)g(p_1) = 0$$

This gives

$$\beta(p_1)\beta(q_1)g(p_1) = -\beta(q_1)\beta(p_1)g(p_1)\ \ \forall\ p_1, q_1\,\epsilon\,M \quad (9)$$

Replacing q_1 by r_1q_1 in last equation, we get

$$\beta(p_1)\beta(r_1q_1)g(p_1) = -\beta(r_1q_1)\beta(p_1)g(p_1)$$

Since β is a homomorphism, so we have

$$\beta(p_1)\beta(r_1)\beta(q_1)g(p_1) = -\beta(r_1)\beta(q_1)\beta(p_1)g(p_1)$$

By using equation (2.9), we obtain

$$\beta(p_1)\beta(r_1)\beta(q_1)g(p_1) = -\beta(r_1)(-\beta(p_1)\ \beta(q_1)g(p_1))$$

Since β is a homomorphism, so the last relation implies

$$\beta(p_1)\beta(r_1)\beta(q_1)g(p_1) = \beta(-r_1)\beta(-p_1)\ \beta(q_1)g(p_1)$$

This gives

$$\beta(p_1)\beta(r_1)\beta(q_1)g(p_1) - \beta(-r_1)\beta(-p_1)\ \beta(q_1)g(p_1) = 0$$

This implies

$$(\beta(p_1)\beta(r_1) + \beta(r_1)\beta(-p_1))\ \beta(q_1)g(p_1) = 0$$

Replacing p_1 by $-p_1$, we get

$$(\beta(-p_1)\beta(r_1) + \beta(r_1)\beta(p_1))\ \beta(q_1)g(-p_1) = 0$$

Since β is a homomorphism, so we have

$$(-\beta(p_1)\beta(r_1) + \beta(r_1)\beta(p_1))\ \beta(q_1)g(-p_1) = 0$$

From this we get

$$-(\beta(p_1)\beta(r_1) - \beta(r_1)\beta(p_1))\ \beta(q_1)g(-p_1) = 0$$

This gives

$$-[\ \beta(p_1), \beta(r_1)]\ \beta(q_1)g(-p_1) = 0$$

This implies

$$[\ \beta(p_1), \beta(r_1)]\beta(q_1)g(-p_1) = 0 \qquad \forall\ p_1, q_1, r_1\,\epsilon\,M$$

The last relation gives

$$[\ \beta(p_1), \beta(r_1)]Mg(-p_1) = 0 \qquad \forall\ p_1, r_1\,\epsilon\,M \quad (10)$$

Since M is prime then for each $p_1\,\epsilon\,M$, we get $g(-p_1) = 0$ or $[\beta(p_1), \beta(r_1)] = 0$. Since g is a nonzero two sided β-derivation, so we have $[\beta(p_1), \beta(r_1)] = 0$ or $p_1\,\epsilon\,Z(M)$, by using Lemma [2, khan et.al., 2021], we have $g(p_1)\,\epsilon\,Z(M)$. This alongwith Lemma [3, khan et.al., 2021], we get

$$g(M) \subset Z(M)$$

Hence M is a commutative ring.

ii. Since

$$G(p_1oq_1) = -p_1{}^{u_1}(\beta(p_1)o\beta(q_1))p_1{}^{v_1} \quad (11)$$

Also $p_1o(q_1p_1) = (p_1oq_1)p_1$, replacing q_1 by q_1p_1 in equation (11), we obtain

$$G((p_1oq_1)p_1) = G(p_1o(q_1p_1))$$

This gives

$$G((p_1oq_1)p_1) = -p_1{}^{u_1}(\beta(p_1)o\beta(q_1p_1))p_1{}^{v_1}$$

Since β is a homomorphism, so the last relation implies

$$G((p_1oq_1)p_1) = -p_1{}^{u_1}(\beta(p_1)o\beta(q_1)\beta(p_1))p_1{}^{v_1}$$

This gives

$$G((p_1oq_1)p_1) = -p_1{}^{u_1}(\beta(p_1)o\beta(q_1))p_1{}^{v_1}\,\beta(p_1)\ \ \forall\ p_1, q_1\,\epsilon\,M \quad (12)$$

By using definition of generalized β-derivation, we have

$$G((p_1oq_1)p_1) = G(p_1oq_1)\beta(p_1) + \beta(p_1oq_1)g(p_1)$$

By using equation (11) and equation (12), we obtain

$$-p_1{}^{u_1}(\beta(p_1)o\beta(q_1))p_1{}^{v_1}\,\beta(p_1) = -p_1{}^{u_1}(\beta(p_1)o\beta(q_1))p_1{}^{v_1}\,\beta(p_1) + (\beta(p_1)o\beta(q_1))g(p_1)$$

From this we arrive at

$$(\beta(p_1)o\beta(q_1))g(p_1) = 0$$

This implies

$$(\beta(p_1)\beta(q_1) + \beta(q_1)\ \beta(p_1))g(p_1) = 0$$

From this we get

$$\beta(p_1)\beta(q_1)g(p_1) + \beta(q_1)\beta(p_1)g(p_1) = 0$$

This gives

$$\beta(p_1)\beta(q_1)g(p_1) = -\beta(q_1)\beta(p_1)g(p_1)\ \ \forall\ p_1, q_1\,\epsilon\,M \quad (13)$$

Replacing q_1 by r_1q_1 in last equation, we get

$$\beta(p_1)\beta(r_1q_1)g(p_1) = -\beta(r_1q_1)\beta(p_1)g(p_1)$$

Since β is a homomorphism, so we have

$$\beta(p_1)\beta(r_1)\beta(q_1)g(p_1) = -\beta(r_1)\beta(q_1)\beta(p_1)g(p_1)$$

By using equation (13), we obtain

$$\beta(p_1)\beta(r_1)\beta(q_1)g(p_1) = -\beta(r_1)(-\beta(p_1)\ \beta(q_1)g(p_1))$$

Since β is a homomorphism, so the last relation implies

$$\beta(p_1)\beta(r_1)\beta(q_1)g(p_1) = \beta(-r_1)\beta(-p_1)\ \beta(q_1)g(p_1)$$

This gives

$$\beta(p_1)\beta(r_1)\beta(q_1)g(p_1) - \beta(-r_1)\beta(-p_1)\ \beta(q_1)g(p_1) = 0$$

This implies

$$(\beta(p_1)\beta(r_1) + \beta(r_1)\beta(-p_1))\ \beta(q_1)g(p_1) = 0$$

Replacing p_1 by $-p_1$, we get

$$(\beta(-p_1)\beta(r_1) + \beta(r_1)\beta(p_1))\ \beta(q_1)g(-p_1) = 0$$

Since β is a homomorphism, so we have

$$(-\beta(p_1)\beta(r_1) + \beta(r_1)\beta(p_1))\ \beta(q_1)g(-p_1) = 0$$

From this we get

$$-(\beta(p_1)\beta(r_1) - \beta(r_1)\beta(p_1))\ \beta(q_1)g(-p_1) = 0$$

This gives

$-[\,\beta(p_1),\beta(r_1)]\,\beta(q_1)g(-p_1)=0$

This implies

$[\,\beta(p_1),\beta(r_1)]\beta(q_1)g(-p_1)=0 \quad \forall\ p_1,q_1,r_1\,\epsilon\, M$

The last relation gives

$$[\,\beta(p_1),\beta(r_1)]Mg(-p_1)=0 \quad \forall\ p_1,r_1\,\epsilon\, M \qquad (14)$$

Since M is prime then for each $p_1\,\epsilon\, M$, we get $g(-p_1)=0$ or $[\beta(p_1),\beta(r_1)]=0$. Since g is a nonzero two sided β-derivation, so we have $[\beta(p_1),\beta(r_1)]=0$ or $p_1\,\epsilon\, Z(M)$, by using Lemma [2, khan et.al., 2021], we have $g(p_1)\,\epsilon\, Z(M)$. This along with Lemma [3, khan et.al., 2021], we get

$g(M)\subset Z(M)$

Hence M is a commutative ring.

Example 2. Let X be a commutative ring and $M=\{\begin{pmatrix}0 & p_1 & q_1\\ 0 & 0 & r_1\\ 0 & 0 & 0\end{pmatrix}: p_1,q_1,r_1\,\epsilon\, X\}$. We define the following mappings on M.

Let $\beta: M\to M$ is a mapping defined by:

$$\beta\begin{pmatrix}0 & p_1 & q_1\\ 0 & 0 & r_1\\ 0 & 0 & 0\end{pmatrix}=\begin{pmatrix}0 & 0 & p_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

and $g: M\to M$ is a mapping defined by:

$$g\begin{pmatrix}0 & p_1 & q_1\\ 0 & 0 & r_1\\ 0 & 0 & 0\end{pmatrix}=\begin{pmatrix}0 & 0 & r_1-q_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

Then g is nonzero β-derivation on M. If A$=\begin{pmatrix}0 & 0 & r_1-q_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$, then $AMA=0$ which proves that M is not prime. Moreover, if $G\begin{pmatrix}0 & p_1 & q_1\\ 0 & 0 & r_1\\ 0 & 0 & 0\end{pmatrix}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & r_1\end{pmatrix}$, then G is a generalized β-derivation on M which satisfies the condition $G(AoB)=(AoB) \quad \forall\ A,B\,\epsilon\, M$.

References

Albas, E., Argac, N., 2004. Generalized derivations of prime rings, Algebra Colloq., 11, Pp. 399-410.

Ashraf, M., Ali, A., Ali, S., 2004. (σ; τ)-Derivations on prime near rings. Arch. Math. (Brno) 40, Pp. 281-286.

Ashraf, M., Shakir, A., 2008. On (σ; τ)-derivations of prime near-rings II. Sarajevo J. Math., 4, Pp. 23-30.

Basudeb, D., 2010. Remarks on generalized derivations in prime and semi prime rings. Int. J. Math. Math. Sci., Art. ID 646587, Pp. 6.

Beidar, K.I., Fong, Y., Wang, X.K., 1996. Posner and Herstein theorems for derivations of 3-prime near-rings. Comm. Algebra, 24, Pp. 1581-1589.

Bell, H.E., Mason, G., 1987. On derivations in near-rings, in: Near-Rings and Near-Fields (G. Betsch, ed.), North-Holland, Amsterdam, Pp. 31-35.

Bell, H.E., Mason, G., 1992. On derivations in near-rings and rings. Math. J. Okayama Univ., 34, Pp. 135-144.

Boua, A., Oukhtite, L., 2011. Generalized derivations and commutativity of prime near-rings. J. Adv. Res. Pure Math., 3, Pp. 120-124.

Daif, M.N., Bell, H.E., 1992. Remarks on derivations on semi prime rings. Internat. J. Math. Math. Sci., 15, Pp. 205-206.

De Filippis, V., Rehman, N., 2010. Commutativity and skew-commutativity conditions with generalized derivations. Algebra Colloq., 17, Pp. 841-850.

Golbasi, O., Koc, E., 2009. Notes on commutativity of prime rings with generalized derivation, Commun. Fac. Sci. Univ., Ank. (Ser. A1), 58, Pp. 39-46.

Golbasi, O., Koc, E., 2011. Some commutativity theorems of prime rings with generalized (σ; τ)-derivation. Commun. Korean Math. Soc., 26, Pp. 445-454.

Kamal, A.A.M., 2001. σ-Derivations on prime near-rings, Tamkang J. Math., 32, Pp. 89-93.

Khan, A.R., Javaid, I., and Chaudhry, M.A., 2013. Derivations on semiprime Γ -rings. Utilitas Mathematica, 90, Pp. 171-185.

Khan, A.R., Chaudhry M.A., and Javaid I., 2013. Generalized derivations on prime-rings. World Applied Sciences Journal, 23, no. 12, Pp. 59-63.

Khan, A.R., Chaudhry, M.A., and Javaid, I., 2016. Some results on prime and semiprime Γ -rings. Analele Universitatii Oradea Fasc. Mathematica, Tom XXIII, 1, Pp. 177-187.

Khan, A.R., Mumtaz, K., and Waqas, M. M., 2021. On Commutativity of Prime Near Rings. Matrix Science Mathematics, 5 (1), Pp. 6-15.

Quadri, M.A., Khan, M.S., Rehman, N., 2003. Generalized derivations and commutativity of prime rings. Indian J. Pure Appl. Math., 34, Pp. 1393-1396.

Raina, R., Bhat, V.K., Kumari, N., 2009. Commutativity of prime Γ-near rings with Γ- (σ; τ)-derivation. Acta Math. Acad. Paedagog. Nyhazi, 25, Pp. 165-173.

Vukman, J., 2007. A note on generalized derivations of semi prime rings. Taiwanese J. Math.,11, Pp. 367-370.