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RESEARCH ARTICLE

THE SOLUTION OF ONE-PHASE STEFAN-LIKE PROBLEMS WITH A FORCING TERM BY MOVING TAYLOR SERIES

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ABSTRACT

In this article, we construct an approximate series solution for the one-dimensional one-phase Stefan-like problems with a forcing term. An algorithm is proposed to represent the nonhomogeneous forcing term in a moving series form to incorporate it into the moving Taylor series method. Numerical examples are solved by the proposed algorithm and the results show that with a suitable number of terms in the utilized moving series, the approximate solution is in good agreement with the exact solution.

KEYWORDS

Moving boundary problems, Series solution, Non-homogeneous Stefan-like problems, Moving Taylor series

1. INTRODUCTION

Many transient heat conduction problems involve one or more boundary that changes its size and shape in time. For example, freezing, solidification, melting, casting, and so on. Such problems have become well known as moving boundary problems or Stefan problems (Pegler and Wykes, 2021; HyeonSon and Park, 2021; Kumar and Rajeev, 2020; Gupta, 2017). We deal with the direct Stefan problems, where we seek the distribution of the temperature field and the equation of moving boundary while the boundary conditions, initial conditions, and the thermal properties of the considered material are given. This type of problems is naturally classified as nonlinear problem as the moving interface is not known a priori and is obtained as a part of the solution.

Because of the nonlinearity of the moving boundary, analytical solutions are difficult to obtain except in some cases. As a result, numerical and semi-analytical techniques are employed to solve different types of Stefan problems. Recent articles in this field include, for example, finite difference method where the author utilized the boundary immobilization method, with a Keller box finite difference scheme, for solving ablation-type Stefan problems in one dimension (Mitchel and Vynnycky, 2012). Another finite difference approach is to use a fixed grid local method for the solution of one-dimensional free boundary problems (Tadi, 2010). The finite element method was utilized where the solution of the Stefan problem was obtained according to the explicit-implicit scheme of the matrixless finite element method (Burago and Fedyushkin, 2021). A cut finite element method was employed to solve one-phase Stefan problems with applications in laser manufacturing (Claus et al., 2018). Another method is proposed as conventional and refined heat balance integral methods were employed to solve several phase-change problems (Mitchell and Myers, 2010). Mixed finite element/finite difference approach is reported where the variable-space grid technique based on the mixed finite element/finite difference approach was applied on a one-dimensional Stefan problem that describes a melting process (Ivanovic et al., 2017). A deep learning approach is used to tackle a general class of Stefan problems (Wang and Perdikaris, 2021). Numerical solution was prposed for moving boundary problems, including the Stefan problems, using a new meshless technique for one-dimensional problems (Reutskiy, 2011). The existence of similarity solutions was proved for a Stefan one-phase problem with temperature-dependent thermal conductivity and a Robin condition at the fixed face (Ceretani et al., 2018). The immersed boundary smooth extension method was employed to solve the bulk advection-diffusion and fluid equations (MacHuang et al., 2021). Semi-analytical techniques include for example the Adomian decomposition method which was used for solution of one-phase Stefan problem in (Grzymkowski et al., 2005). The homotopy analysis method was utilized to solve the inverse one-phase Stefan problem that required the determination of the heat flux on one of boundaries of the region in (Hetmaniok et al., 2015). The variational iteration method was compared with the Adomian decomposition method when used for solving the moving boundary problems in (Hetmaniok et al., 2011).

Taylor series has been recently used to tackle different types of mathematical problems in the form of the differential transform method (DTM) which iteratively determines the coefficients of Taylor series solution to different types of equations (Zhou, 1986). The DTM has also been utilized to solve PDEs in higher dimensions (Jang et al., 2001; Kurnaz et al., 2005). The moving Taylor series (MTS), was proposed to solve the homogeneous Stefan problems in one dimension in (Elsaid and Helal, 2022).

In this paper, the MTS is generalized to obtain approximate solution to one-phase one-dimensional Stefan-like problems with a forcing term. The proposed series is formulated to incorporate the forcing term into the recurrence relation employed to compute the series' coefficients. The rest of this paper is organized as follows. In Section 2, the statement of the direct Stefan problem with forcing term is presented. In Section 3, the theorems that formulate coefficients of the MTS are mentioned, the forcing term is formulated in a moving series form and the proposed solution technique is given. We have considered two problems with given exact solutions, to make comparisons of results in Section 4. Section 5 summarizes the conclusion of this work.

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2. Statement of the Problem

Consider the moving boundary problem described by the following equations

$$\frac{\partial u(x,t)}{\partial t}-\alpha\frac{\partial^2 u(x,t)}{\partial x^2}=f(x,t), 0<x<s(t), t>0, \quad (1)$$

where u, α, t, x and s refer to temperature, thermal diffusivity, time, spatial location and position of the moving boundary, respectively.

The initial and boundary conditions are given by

$$u(x,0)=g(x), 0<x<s(0), \quad (2)$$

$$u(0,t)=h(t), t>0, \quad (3)$$

$$u(s(t),t)=u^*, t>0, \quad (4)$$

where $f(x,t)$, $g(x)$ and $h(t)$ are analytic functions. u^* is the phase-change temperature.

The Stefan condition is given by

$$-k\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=s(t)}=L\frac{ds(t)}{dt}, t>0, \quad (5)$$

where L is the latent heat of fusion per unit volume.

3. The Method of Solution

The following two theorems illustrate the formulation of the MTS.

Theorem 1 *Let $s(t)$ and $u(x,t)$ be analytic functions at t_0 and at the line $(s(t),t_0)$, respectively. Then, $u(x,t)$ can be represented by the following moving Taylor series*

$$u(x,t)=\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}U(i,j)(x-s(t))^i(t-t_0)^j, \quad (6)$$

where the coefficient $U(r,k)$ is defined for non-negative integers m and n by

$$U(r,k)=\frac{1}{r!k!}\left[\frac{\partial^{r+k}u(x,t)}{\partial x^r\partial t^k}\right]_{(s(t_0),t_0)}-\sum_{j=0}^{k}\sum_{i=1}^{k-j}\frac{(i+r)!k!}{(k-j)!}U(i+r,j)\times B_{k-j,i}(-s'(t_0),-s''(t_0),\dots,-s^{(k-j)}(t_0)), \quad (7)$$

where $B_{k,n}(x_1,x_2,\dots,x_n)$ denotes the exponential partial Bell polynomial defined by

$$B_{k,n(x_1,x_2,\dots,x_k)}=\sum_{\substack{a_1+a_2+\dots+a_k=n\\ a_1+2a_2+\dots+na_k=k}}\frac{k!}{a_1!a_2!\dots a_k!}\left(\frac{x_1}{1!}\right)^{a_1}\left(\frac{x_2}{2!}\right)^{a_2}\dots\left(\frac{x_k}{n!}\right)^{a_k} \quad (8)$$

Theorem 2 *Let $u(x,t)$ and $s(t)$ satisfy Theorem 1 and let $u(x,t)$ have MTS representation (6). Let $w(x,t)=\frac{\partial u(x,t)}{\partial t}$, then $w(x,t)$ has the MTS representation of the form*

$$w(x,t)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}W(m,n)(x-s(t))^m(t-t_0)^n, \quad (9)$$

where the coefficient of *MTS* $W(m,n)$ is given by

$$W(m,n)=(n+1)U(m,n+1)-(m+1)\sum_{r=0}^{n}(r+1)S(r+1)U(m+1,n-r), \quad (10)$$

where $S(.)$ denotes the coefficients of Taylor series of $s(t)$ about $t=t_0$.

The proofs of the previous theorems are detailed in (Elsaid and Helal, 2022).

Now, the MTS is extended to be applied to the nonhomogeneous Stefan-like problem with the given conditions. From the conditions of Theorem 1 and Theorem 2, we can write

$$s(t)=\sum_{j=0}^{\infty}S(j)(t-t_0)^j. \quad (11)$$

The forcing term $f(x,t)$ is also an analytic function, so it can be represented by the series

$$f(x,t)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A(m,n)(x)^m(t-t_0)^n. \quad (12)$$

By adding and subtracting $s(t)$, we get

$$f(x,t)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A(m,n)(x-s(t)+s(t))^m(t-t_0)^n. \quad (13)$$

From binomial theorem

$$f(x,t)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A(m,n)(t-t_0)^n\sum_{r=0}^{m}\binom{m}{r}(x-s(t))^r(s(t))^{m-r}. \quad (14)$$

If $s(t)$ is substituted by (11), then $(s(t))^l$ can be represented in series form as following

$$(s(t))^l=\sum_{k=0}^{\infty}B_l(k)(t-t_0)^k, \quad (15)$$

where coefficients $B_l(k)$ are computed using Adomian polynomials as formulated in (Elsaid, 2012). Then,

$$f(x,t)=\sum_{n=0}^{\infty}(t-t_0)^n\sum_{m=0}^{\infty}A(m,n)\sum_{r=0}^{m}\binom{m}{r}(x-s(t))^r\sum_{k=0}^{\infty}B_{m-r}(k)(t-t_0)^k. \quad (16)$$

Obtaining the coefficient of $(x-s(t))^i(t-t_0)^j$ in the previous expansion, we find

$$F(i,j)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A(m,n)\binom{m}{i}B_{m-i}(j-n). \quad (17)$$

When (6) and (17) are substituted in (1), we obtain

$$(j+1)U(i,j+1)-(i+1)\sum_{r=0}^{j}(r+1)S(r+1)U(i+1,j-r)-\alpha(i+2)(i+1)U(i+2,j)=F(i,j), \quad (18)$$

for which the recurrence relation is formulated as

$$U(i,j+1)=\frac{1}{j+1}\left(\alpha(i+2)(i+1)U(i+2,j)+(i+1)\sum_{r=0}^{j}(r+1)S(r+1)U(i+1,j-r)+F(i,j)\right). \quad (19)$$

Let the initial condition $g(x)$ be analytic about $s(0)$. Then we can write

$$g(x)=\sum_{i=0}^{\infty}G(i)(x-s(0))^i, \quad (20)$$

and by substituting in (2), we obtain

$$U(i,0)=G(i), i=0,1,2,.. \quad (21)$$

Consider the case where u^* is constant, then (4) yields

$$\begin{cases}U(0,0)=u^*,\\ U(0,j)=0, j\neq 0.\end{cases} \quad (22)$$

Finally, Stefan condition assumes the series form

$$-k\sum_{j=0}^{\infty}U(1,j)(t-t_0)^j=L\sum_{j=0}^{\infty}(j+1)S(j+1)(t-t_0)^j, \quad (23)$$

which yields

$$S(j+1)=\frac{-k}{(j+1)L}U(1,j), j=0,1,2,.. \quad (24)$$

If any of the assumed constants is given as a function, its Taylor series is substituted into the corresponding equation.

4. Numerical Examples

To illustrate efficiency of the proposed technique, two examples of Stefan-like problems with forcing term are solved using the MTS. We denote by $MTS_{n,m}$ the MTS solution when using n terms and m terms in u-series and s-series, respectively. We present tables of the relative error percentage defined in L_2 norms as follows

$$e_s=\frac{\sqrt{\int_0^1(s_{exc}(t)-s_T(t))^2dt}}{\sqrt{\int_0^1(s_{exc}(t))^2dt}}\times 100\%, \quad (25)$$

$$e_u=\frac{\sqrt{\int_0^1\int_0^{s(t)}(u_{exc}(x,t)-u_{MTS}(x,t))^2dxdt}}{\sqrt{\int_0^1\int_0^{s(t)}(u_{exc}(x,t))^2dxdt}}\times 100\%. \quad (26)$$

The software package Mathematica was used for the symbolic computations presented in this section.

Example 1 *Consider problem (1)-(5) where we set* (Cho, 2002)*:* $t_0 = 0, \alpha = 1, k = 1, L = 1, u^* = 0, s(0) = 1, f(x,t) = xe^t + 2,$ *and* $g(x) = x(1-x)$.

By expanding $g(x)$ in series form about $s(0) = 1$, we have $U(0,0) = 0, U(1,0) = -1, U(2,0) = -1$, and

$$U(i,0) = 0, i > 2, \tag{27}$$

and as $u^* = 0$ we get

$$U(0,j) = 0, j = 0,1,2,.. \tag{28}$$

By adding and subtracting $s(t)$ in $f(x,t)$ we have

$$f(x,t) = (x - s(t))e^t + s(t)e^t + 2. \tag{29}$$

Then from the Taylor series of e^t and (11) we get

$$f(x,t) = \sum_{j=0}^{\infty} (x - s(t))\frac{t^j}{j!} + \sum_{j=0}^{\infty}\sum_{r=0}^{j} S(r)\frac{t^j}{(j-r)!} + 2. \tag{30}$$

By expanding we obtain

$$\begin{cases} F(0,0) = 3, \\ F(0,j) = \sum_{r=0}^{j} \frac{S(r)}{(j-r)!}, j = 1,2,\dots, \\ F(1,j) = \frac{1}{j!}, j = 0,1,2,\dots, \\ F(i,j) = 0, \forall j, i > 1. \end{cases} \tag{31}$$

From the given values for the parameters, we get

$$\begin{cases} S(0) = 1, \\ S(j+1) = \frac{-1}{(j+1)} U(1,j), j = 0,1,2,\dots. \end{cases} \tag{32}$$

By solving (18) and (31) we obtain $S(1) = 1, S(2) = 0.5, S(3) = 0.166667$, $S(4) = 0.0416667, \ldots$ and so on. Whereas, for the MTS coefficients, we obtain $U(1,1) = -1$, $U(1,2) = -0.5$, $U(1,3) = -0.166667$, … and so on. We also find that

$$U(i,j) = 0, i > 1, j > 0. \tag{33}$$

By substituting the values of the coefficients in (6) and (11), we obtain the approximate solution for the temperature and moving interface, respectively. The exact solutions for this problem are $s(t) = e^t$ and $u(x,t) = x(e^t - x)$ as stated in (Cho, 2002).

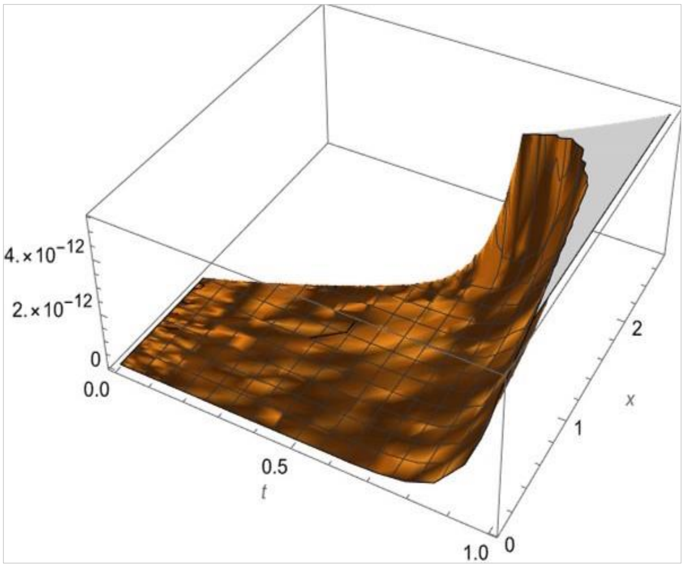

Figure 1: The relative error between exact solution and $MTS_{20,20}$ of temperature for *Example*1.

Figure 1 shows the relative error as defined by (26) when using $MTS_{20,20}$ for Example1. **Figure 2** and **Figure 3** show the effect of number of terms utilized in the moving boundary series and the temperature series solution, respectively, on the accuracy at $t = 1$. These figures demonstrate the good performance of the proposed technique.

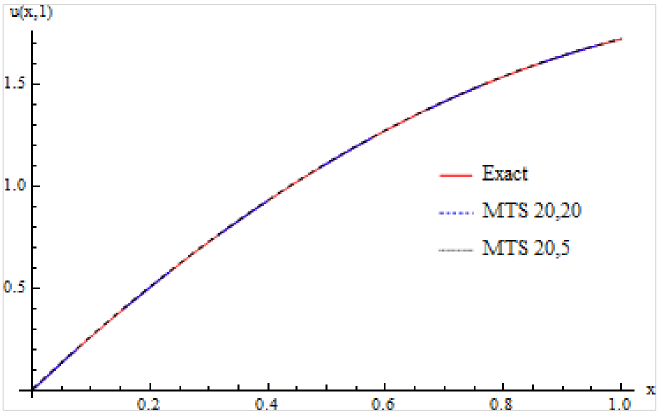

Figure 2: The exact solution vs $MTS_{20,n}$ of temperature for *Example*1 at $t = 1$.

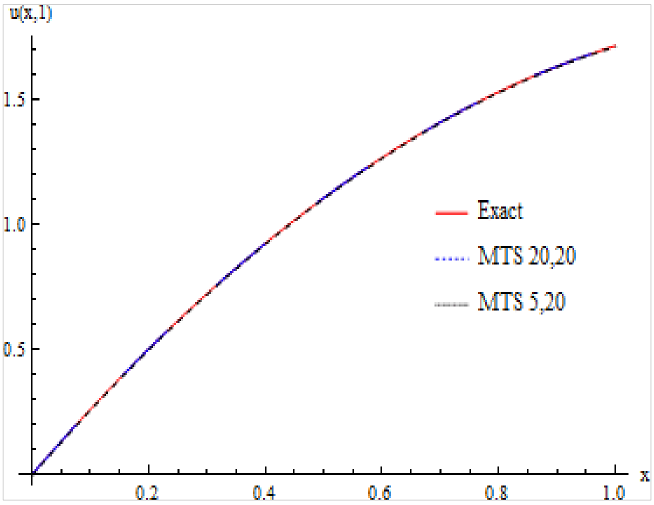

Figure 3: The exact solution vs $MTS_{n,20}$ of temperature for *Example*1 at $t = 1$.

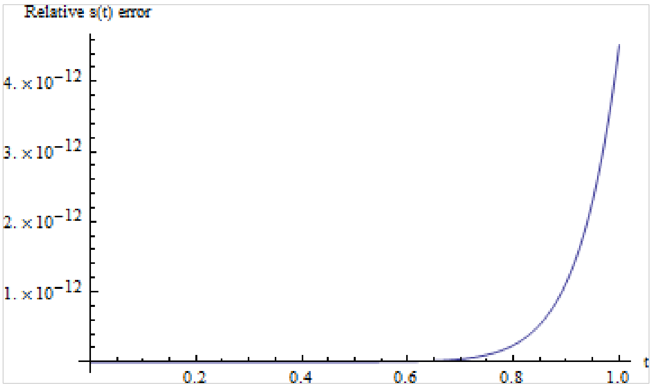

Figure 4: The relative error between exact solution and $20-term$ approximation of $s(t)$ for $Example$ 1.

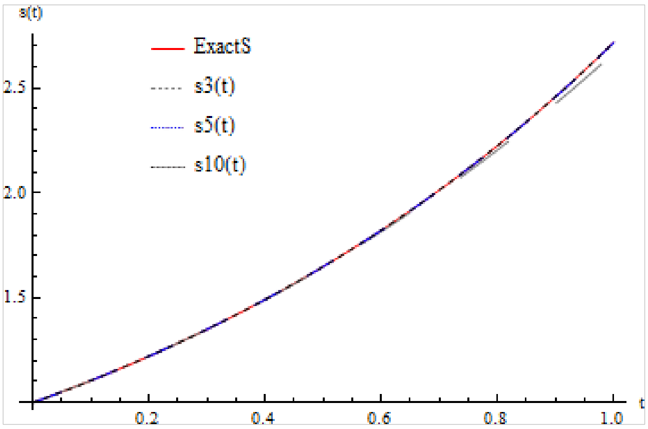

Figure 5: The exact solution and the MTS approximate solution of $s(t)$ for $Example$1.

Figure 4 shows the relative error as defined by (25) when using 20 −term approximation of $s(t)$ for Example1. **Figure 5** shows the improvement of accuracy in the approximation of the moving boundary as the number of terms in the Taylor series increases. Using 5 and more terms, the approximate solution agrees with the exact one whereas when using only 3 terms, it yields lower accuracy especially at $t=1$. These two figures illustrate that the error assumes its highest value when employing insufficient number of terms and mainly at $t=1$ as it is the farthest from the center of the series.

Table 1 and **Table 2** illustrate that with large number of terms in the series, the error assumes smaller values. Yet, from **Table 1,** it is noticed that acceptable number of terms should be used in both series to obtain good results. From these tables we also note that computing more terms of u_series and s_series remarkably yields more accuracy. The values presented in **Tables 1** and **2** are better when compared to those presented by other semi-analytic techniques on similar problems (Kutluay, 2005; Vildan, 2009; Onyejekwe, 2014).

Table 1: Relative Error of Example 1 in L_2 as Defined by (26) for $u(x,t)$ at $t=1$.

Subscripts denote the number of terms utilized in the series solution.

	MTS_3	MTS_5	MTS_{15}
s_3	3.85914%	3.81192%	0%
s_5	3.80474%	0.104997%	0%
s_{20}	0%	0%	0%

Table 2: Relative Error of Example 1 in L_2 Norm as Defined by (25) for s(t) at t = 1.

Subscripts denote the number of terms utilized in the series solution.

s_3	s_5	s_{10}
0.941464%	0.0247844%	0%

Example 2 *Consider problem (1)-(5) with the following choices:* $t_0=0, \alpha=1, k=1, L=4t-\frac{29}{4}, u^*=0, s(0)=\frac{1}{2}(3-\sqrt{29}), f(x,t)=-2,$ *and* $g(x)=3x-x^2-5$.

For $g(x)$, we can write

$$g(x)=\sum_{i=0}^{\infty} G(i)(x-\frac{1}{2}(3-\sqrt{29}))^i. \tag{34}$$

Then from relation (21) we get $U(0,0)=0, U(1,0)=5.38516, U(2,0)=-1$, and

$$U(i,0)=0, i>2. \tag{35}$$

Because $u^*=0$, we get

$$U(0,j)=0, j=0,1,2,\dots. \tag{36}$$

For the forcing term $f(x,t)$ series representation, we have

$$\begin{cases} F(0,0)=-2, \\ F(i,j)=0, i\neq 0, j\neq 0. \end{cases} \tag{37}$$

As L is a function of t, (23) is rewritten as

$$-\sum_{j=0}^{\infty} U(1,j)t^j=(4t-\frac{29}{4})\sum_{j=0}^{\infty}(j+1)S(j+1)t^j, \tag{38}$$

which yields

$$S(j+1)=\frac{4}{29(j+1)}(U(1,j)+4(j)S(j), j=0,1,2,\dots. \tag{39}$$

By solving (18) and (38) we obtain $S(1) = 0.742781$, $S(2) = 0.102453$, $S(3) = 0.0282628, S(4) = 0.00974579, \ldots$ and so on. For the temperature distribution we obtain $U(1,1) = -1.48556$, $U(1,2) = -0.204905$, $U(1,3) = -0.0565256, \ldots$ and so on. We also find that

$$U(i,j) = 0, i > 1, j > 0. \quad (40)$$

By substituting these values in (6) and (11) we construct the MTS solution and Taylor series solution for the temperature and the moving boundary, respectively. The exact solutions for this problem are $s(t) = \frac{3}{2} - \sqrt{\frac{29}{4} - 4t}$ and $u(x,t) = 3x - x^2 - 4t + 5$.

Figure 6 shows the relative error as defined by (26) when using $MTS_{20,20}$ for Example2. **Figure 7** and **Figure 8** show the effect of number of terms utilized in the moving boundary series and the temperature series solution, respectively, on the accuracy at $t = 1$.

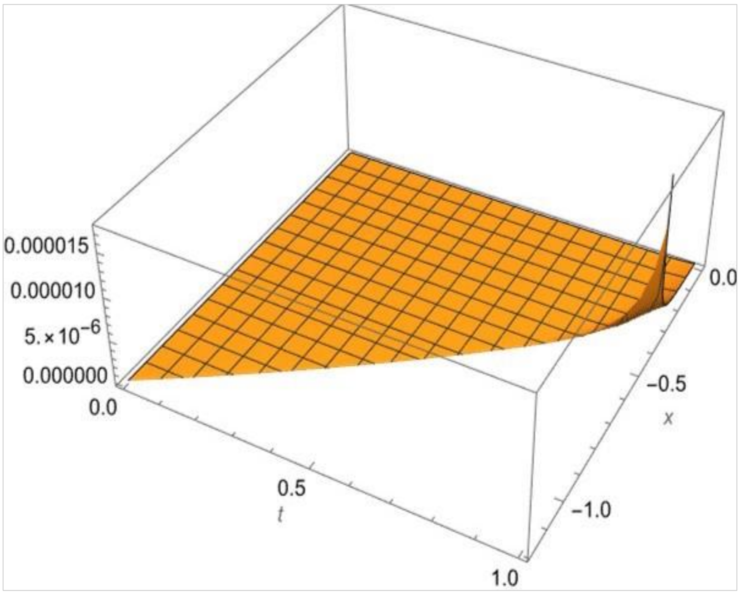

Figure 6: The relative error between exact solution and $MTS_{20,20}$ of temperature for $Example2$.

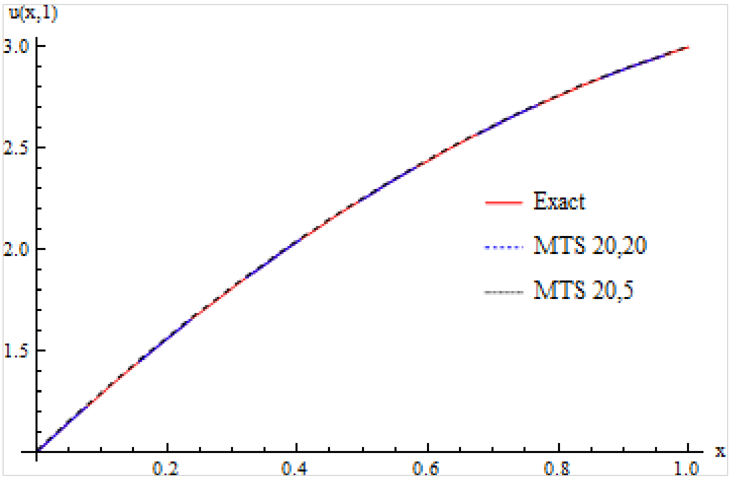

Figure 7: The exact solution vs $MTS_{20,n}$ of temperature for $Example2$ at $t = 1$.

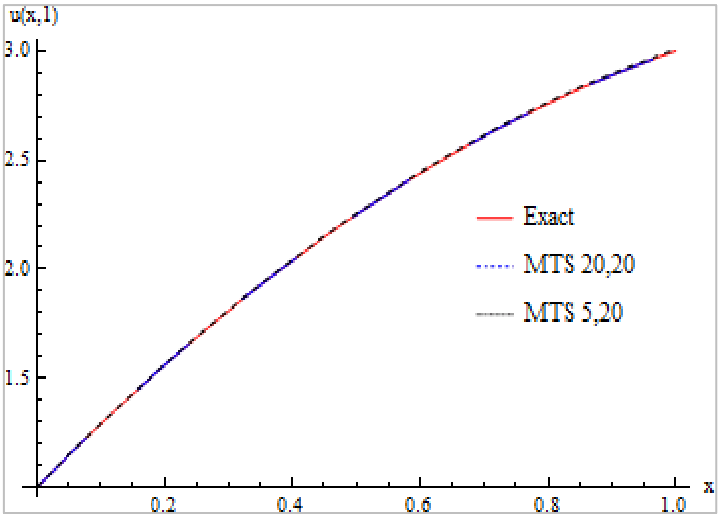

Figure 8: The exact solution vs $MTS_{n,20}$ of temperature for $Example$ 2 at $t = 1$.

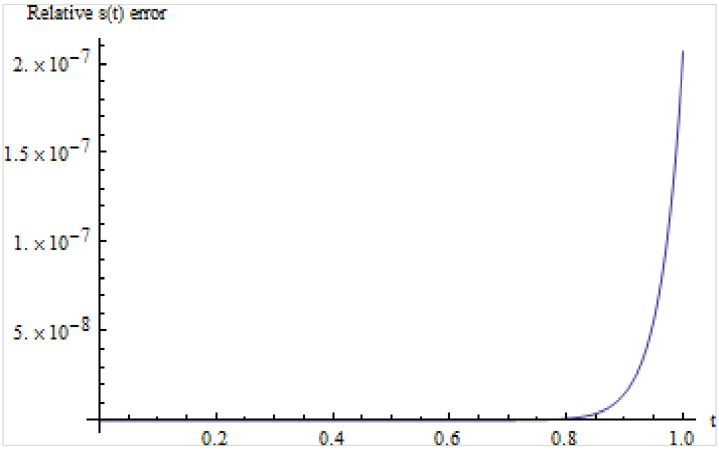

Figure 9: The relative error between exact solution and $20 - term$ approximation of $s(t)$ for $Example$ 2.

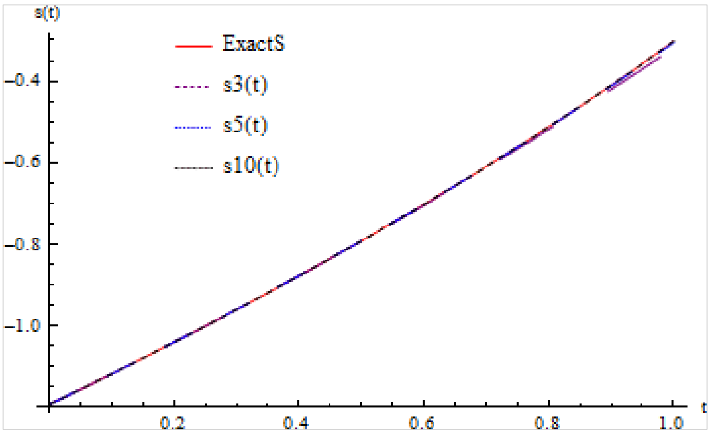

Figure 10: The exact solution and the MTS approximate solution of $s(t)$ for $Example$2.

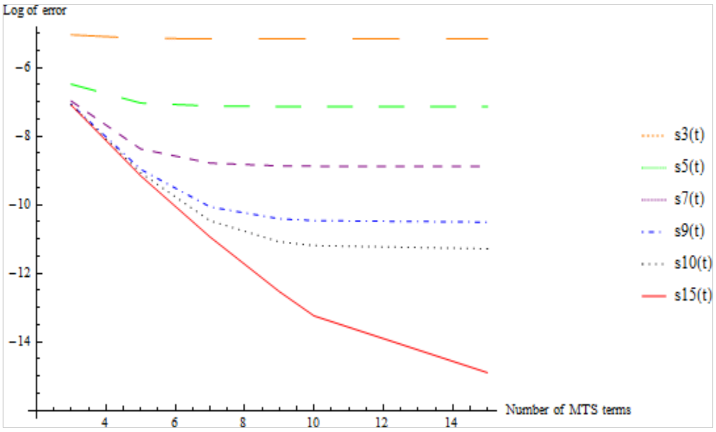

Figure 11: The logarithm of error vs the number of terms of MTS at different numbers of terms of $s(t)$ for $Example$ 2.

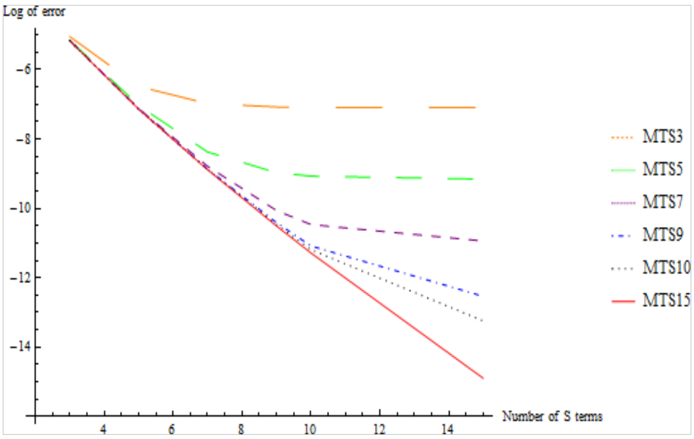

Figure 12: The logarithm of error vs the number of terms of $s(t)$ at different numbers of terms of MTS for $Example$2.

Figure 9 shows the relative error as defined by (25) when using $20-term$ approximation of $s(t)$ for Example2. **Figure 10** illustrates the improvement of accuracy in the approximation of the moving boundary as the number of terms in the series increases. When using 5 or more terms in the series, the approximate solution is in good agreement with the exact one whereas when using 3 terms, the results show less accuracy especially at $t=1$.

Figure 11 shows the logarithm of error versus the number of terms utilized in the MTS solution at different numbers of terms of $s(t)$ for Example2. **Figure 12** shows the logarithm of error versus the number of terms utilized in the $s(t)$ series solution at different numbers of terms of MTS for Example 2.

By analyzing the data in the previous two figures, it is obvious that if small number of terms is utilized in either one of the two series, increasing the number of terms of the other series does not improve the results. We also notice that with increasing the number of terms of both series the error tends to be linear and the order of convergence of the error is approximately 0.75 and 0.83333 in **Figure 11** and **Figure 12,** respectively. **Table 3** shows the change in the values of relative error for the moving boundary series solution with the increase of number of terms at $t=1$.

Table 3: Relative Error of Example 2 in L_2 Norm as Defined by (25) for $s(t)$ at $t=1$.

Subscripts to solutions indicate number of terms used in the series solution.

s_3	s_5	s_{10}
0.623881%	0.0900177%	0.00148927%

In this work we solved nonhomogeneous Stefan-like problems using the MTS by applying the proposed technique to a linear problem with Dirichlet boundary conditions. Some techniques that employed the Taylor series technique to solve PDEs subject to some other types of boundary conditions are shown in (Vazquez-Leal et al., 2014; Xu and Duan, 2011). Also, some algorithms have been implemented to enable this semi-analytical method of dealing with different types of nonlinear terms (Elsaid, 2012; Elsaid and Helal, 2020). Hence, we plan that the future work considers dealing with Stefan problems with nonlinearities and to incorporates different types of boundary conditions in the solution scheme. Another open point related to this work includes applying the multistage technique that divides the time interval into some segments and uses the solution at the end of each segment as an initial condition to the following one. This enlarges the time span where semi-analytical methods present results with good accuracy (Nour et al., 2012; Gokdogan et al., 2012). Also, this idea makes it possible to represent the solution with lower degree series and with variable time step approaches (Hashish et al., 2009; Gupta and Kumar, 1980). Also, the idea of series solution with a moving center point can be considered with other series solution techniques which have been used for non-fixed domain problems like in (AlMdallal et al., 2019). Finally, theoretical aspects regarding the proposed technique are open points for investigations.

5. Conclusion

We extended the moving Taylor series method to be able to solve one-phase one-dimensional Stefan-like problems with a forcing term. This generalized form of MTS enables involving the forcing term in the recurrence relation after representing it in a moving series form then applying the solution technique. The numerical results obtained by MTS are in good agreement with the exact values for the temperature distribution $u(x,t)$ and the moving boundary $s(t)$. So, the MTS method shows good accuracy and efficiency in solving the mentioned type of problems. On the other hand, it is applicable only when the considered functions are analytic. Also, the approximate solution shows lower accuracy at points far from the center of the moving series.

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