ZIBELINE INTERNATIONAL™ PUBLISHING

ISSN: 2521-0831 (Print)
ISSN: 2521-084X(Online)
CODEN: MSMAD

Matrix Science Mathematic (MSMK)

DOI: http://doi.org/10.26480/msmk.01.2023.22.26

CrossMark

RESEARCH ARTICLE

AN EFFECTIVE METHOD FOR PREDICTION OF THE FIELD DISTRIBUTION OF LP_{11} MODE OF DISPERSION-CONTROLLED FIBERS HAVING THIRD ORDER NONLINEARITY

Biplab Kumar Ray, Angshuman Majumdar, Sankar Gangopadhyay*

Department of Electronics and Communication Engineering, Brainware University, Barasat, Kolkata, West Bengal, India
****Corresponding Author Email: Sankar Gangopadhyay, sankar.gangopadhyay@yahoo.co.in***

ARTICLE DETAILS

Article History:

Received 17 March 2023
Revised 19 April 2023
Accepted 22 May 2023
Available online 29 May 2023

ABSTRACT

Field distribution in respect of LP_{11} mode in the dispersion controlled fibers in the appearance and in disappearance of Kerr category nonlinearity. The estimation involves totally the mathematical expression for the field of LP_{11} mode as prescribed in the Chebyshev method. Mathematical solution in connection with the relevant characteristics are described. The method of iterative repetition is applied for the required estimation at the state of said class of nonlinearity. The connected assessment using the prescribed mathematical exercise needs a little computations. But, the derived results exhibit extremely close proximity to the actual values obtained by a methodology namely finite element. As a result, our simple and precise formalism provides ample scope for analyzing various Kerr class nonlinear dual-mode optical fiber properties.

KEYWORDS

Dispersion shifted and flattened fibers, First higher order mode, modal field, Kerr class nonlinearity, Chebyshev methodology

1. INTRODUCTION

Minimum absorption loss measuring nearly 0.2 dB/km is achieved when silica made optical fiber is operated at 1.55 µm wavelength and the material dispersion of these fibers vanishes when the operating wavelength is 1.3 µm (Neumann, 1998; Ghatak and Thyagarajan, 1999). The dispersion parameters related to material and waveguide are of opposite signs at wavelengths larger than the wavelength where there is no material dispersion and can thus be designed to neutralize each other at a larger wavelength. So the operating window of zero first order chromatic dispersion is displaced to 1.55 µm for silica glass fibers to achieve simultaneously the lowest dispersion and the lowest attenuation loss optical medium. These fibers are called dispersion-shifted fibers (DSF) (Neumann, 1998; Ghatak and Thyagarajan, 1999; Ainslie and Day, 1986; Tewari et al., 1992). An alternative modification of the dispersion properties of a singlemode fiber (SMF) involves the availability of a low loss wide spectral window between 1.3 µm and 1.6 µm in which oppositely signed material dispersion and waveguide dispersion components are mutually neutralized. Such fibers, which offers the spectral freedom for laser sources and facilitate suitable wavelength division multiplexing are termed by dispersion-fattened fibers (DFF). These fibers increase the information-carrying capacity in optical communication systems (Olsson et al., 1985).

The silhouette of index of refraction (RI) of a nonlinear optical fiber (NOF) depends on the signal intensity. Hence its propagation parameters differ from those in the linear zone (Tomlinson et al., 1984; Tai et al., 1986; Snyder et al., 1990; Goncharenko, 1990; Sammut and Pask, 1990;Agrawal and Boyd, 1992; Burdin et al., 2018; Nesrallah et al., 2018; Agrawal, 2013). Again, different types of nonlinearity, like third order, fifth order and so on, are originated based on the signal intensity and the type of doping element used in the NOF (Agrawal, 2013). Kerr nonlinearity refers to third-order optical nonlinearity. Simultaneous actions like optical pulse minimization due to nonlinearity and also its widening owing to dispersion in a nonlinear fiber create optical soliton propagation (Agrawal, 2013).

Kerr nonlinearity's impact on opto-mechanical ring resonators has already been explored and documented (Yu et al., 2012). Influence of the Kerr nonlinearity on cutoff frequency of initial higher-order modes in DSF and DFF has been reported (Mondal and Sarkar, 1996). The literature also contains a method for getting a solution for a nonlinear waveguide by finite element method (Hayata et al., 1987). In this back ground, it is significant to note that the finite element technique employed in nonlinearity investigations necessitates lengthy computations. Furthermore, in the case of NOF, a formalism based on the Chebyshev method for evaluating the cut-off frequency of the LP_{11} mode has been reported (Royand and Sarkar, 2013). From the perspective of execution, the formalism is quite simple, yet the results are extremely accurate.

Chebyshev formalism has already been used to do a basic but accurate analysis of the propagation properties of NOF (Sadhu et al., 2013). The use of the Chebyshev power series technique to estimate the properties of fibers accurately, has been reported already (Chen, 1982; Shijun, 1987; Patra, et al., 2008; Patra et al., 2008; Gangopadhyay and Sarkar, 1998;Bose et al., 2012).The Chebyshev formalism has been used to accurately estimate the LP_{01} field for third order nonlinear power law fibers (GIF) (Chakraborty et al., 2017).The literature also includes predictions of the LP_{11} modal field for GIF using the same formalism in the appearance and

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Cite The Article: Biplab Kumar Ray, Angshuman Majumdar, Sankar Gangopadhyay (2023). An Effective Method for Prediction of The Field Distribution of Lp11 Mode of Dispersion-Controlled Fibers Having Third Order Nonlinearity. *Matrix Science Mathematic, 7(1): 22-26.*

omission of Kerr nonlinearity (Chakraborty et al., 2017).The accuracy of Chebyshev formalism to the analysis of mono-mode Kerr category nonlinear DSF and DFF has been added recently to the literature (Ray et al., 2020).

We used the Chebyshev formalism to estimate LP_{11} modal fields in some typical DSF and DFF both when Kerr class nonlinearity is present as well as absent. Our approach needs little work to implement, but our results are extremely close to the exact ones in both cases. Moreover, no such correct approach involving simplicity for predicting the modal field of such specific types of Kerr class nonlinear DSF and DFF has been described till date. The current study is unquestionably innovative and system engineers will find it to be user-friendly.

2. Theory

The RI distribution n (R) of an optical fiber is mathematically written as

$$\text{“ } n^2(R) = n_1^2(1-2\delta f(R)), \qquad 0 < R \le 1$$
$$= n_2^2 \qquad R > 1\text{"} \tag{1}$$

Here, $\delta = (n_1^2 - n_2^2)/2n_1^2$, R = r/a, where, a is the core- radius

The grading parameter is δ ,$n_1$is the refractive index (RI) of the core-axis and n_2 is the RI of the clad. f(R) defines the RI distribution in the fiber . f(R) is written as follows for different dispersion controlled fibers

(I) For Trapezoidal fiber [28]

$$\text{“}f(R) = 0, \qquad 0 < R \le S$$
$$f(R) = \frac{R-S}{1-S}, \qquad S < R \le 1\text{"} \tag{2}$$

(II) For Power law W fiber [29]

$$\text{“}f(R) = \rho R^q, \qquad R \le 1/C$$
$$f(R) = \rho, \qquad 1/C < R \le 1\text{"} \tag{3}$$

(III) For Step W fiber [30]

$$\text{“}f(R) = 0, \qquad R \le 1/C$$
$$f(R) = \rho, \qquad 1/C < R \le 1\text{"} \tag{4}$$

Here, S is the aspect ratio of the trapezoidal fibre and 1/C is the normalized radial distance. Again, q denotes the profile exponent. Also, ρ is the relative RI-depth of the inner clad with RI n_i and it is defined by

$$\text{“}\rho = \frac{n_1^2 - n_i^2}{n_1^2 - n_2^2}\text{"}$$

The silhouette of index of refraction *n(R)* of Kerr category NOF is

$$\text{“ } n^2(R) = n_L^2(R) + \frac{n_2^2 n_{NL}(R)}{\eta_0}\psi^2(R)\text{ "} \tag{5}$$

Here, $n_L(R)$depicts the RI profile when there is no nonlinearity and $n_{NL}(R)$ represents the range of non-linear Kerr parameter. The following scalar equation expresses the LP_{11} modal field ψ(R) for a Kerr class nonlinear fibre (Mondal and Sarkar, 1996)

$$\text{“}\frac{d^2\psi(R)}{dR^2} + \frac{1}{R}\frac{d\psi(R)}{dR} + [V^2\{1-f(R)\} - W^2]\psi(R) - \frac{\psi(R)}{R^2} + V^2 g\psi^3(R) = 0\text{ "} \tag{6}$$

Where, mathematical expression of g is

$$\text{“ } g = \frac{n_2 n_{NL} P}{\pi a^2(n_1^2 - n_2^2)}\text{"}$$

In Eq.(6), W and V are the clad decay parameter and V-number respectively.

For continuity of the wave function at the core-clad interface following condition has to be satisfied

$$\text{“ }\left[\frac{1}{R}\frac{d\psi}{dR}\right]_{R=1} = -\left[1 + \frac{WK_0(W)}{K_1(W)}\right]\text{ "} \tag{7}$$

Where, the terms $K_0(W)$,$K_1(W)$ denote the modified Bessel functions of W [31-33].

Again, the field in the clad can be written by “ $\psi(R) \sim K_1(WR)$ " when R > 1

Following Chebyshev methodology approximated field ψ(R) in respect of LP_{11}mode in dispersion controlled fibers is (Chen, 1982; Shijun, 1987; Patra et al., 2008; Chakraborty et al., 2017).

$$\text{“ } \psi(R) = a_1R + a_3R^3 + a_5R^5 \qquad R \le 1$$
$$= (a_1 + a_3 + a_5)\frac{K_1(WR)}{K_1(W)}\text{ "} \qquad R > 1 \tag{8}$$

a_1, a_3 and a_5 being numerically constants.

The following are the Chebyshev values [20]

$$\text{“}R_m = \cos\left(\frac{2m-1}{2M-1}\frac{\pi}{2}\right)\text{" " } m = 1, 2, \dots, (M-1)\text{"} \tag{9}$$

Using Eqs. (8) and (9) and taking M =3 one can obtain concerned Chebyshev values as below

$$R_1 = 0.9511, \quad R_2 = 0.5878 \tag{10}$$

We get the following two equations by using these two important Chebyshev values and Eqs. (8) and (6).

$$\text{“ } a_1[V^2((1-f(R_1)) - W^2 + V^2 g\psi^2(R_1)] + a_3[8 + R_1^2\{V^2((1-f(R_1)) - W^2 + V^2 g\psi^2(R_1)\}] + a_5[24R_1^2 + R_1^4\{V^2((1-f(R_1)) - W^2 + V^2 g\psi^2(R_1)\}] = 0\text{ "} \tag{11}$$

And

$$\text{“ } a_1[V^2((1-f(R_2)) - W^2 + V^2 g\psi^2(R_2)] + a_3[8 + R_2^2\{V^2((1-f(R_2)) - W^2 + V^2 g\psi^2(R_2)\}] + a_5 \ 0\text{ "} \tag{12}$$

The plotting of $K_1(W)/K_0(W)$ vs 1/W is adequately linear for W between 0.6 and 2.5 [33]. This allows us to create the following relationship in the stated band using the linear least square fitting approach [26].

$$\text{“}\frac{K_1(W)}{K_0(W)} = \alpha + \frac{\beta}{W}\text{“} \quad \text{with } \alpha \text{ is equal to} 1.034623 \text{ and } \beta \text{ is equal to } 0.3890323 \tag{13}$$

Combining Eqs. (8), (13) and (7)

$$\text{“}a_1[2(\alpha W + \beta) + W^2] + a_3[4(\alpha W + \beta) + W^2] + a_5[6(\alpha W + \beta) + W^2] = 0\text{ "} \tag{14}$$

For nontrivial solution for the constants$a_1, a_3$and $a_5$from Eqs.(11),(12) and (14) ,the mandatory condition is

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0 \tag{15}$$

where,

$$\text{“ } A_1 = V^2\{1-f(R_1)\} - W^2 + V^2 g\psi^2(R_1)\text{ ";}$$

$$\text{“ } B_1 = 8 + R_1^2[V^2\{1-f(R_1)\} - W^2 + V^2 g\psi^2(R_1)]\text{ ";}$$

$$\text{“ } C_1 = 24R_1^2 + R_1^4[V^2\{1-f(R_1)\} - W^2 + V^2 g\psi^2(R_1)]\text{ ";}$$

$$\text{“ } A_2 = V^2\{1-f(R_2)\} - W^2 + V^2 g\psi^2(R_2)\text{ ";}$$

$$\text{“ } B_2 = 8 + R_2^2[V^2\{1-f(R_2)\} - W^2 + V^2 g\psi^2(R_2)]\text{ ";}$$

$$\text{“ } C_2 = 24R_2^2 + R_2^4[V^2\{1-f(R_2)\} - W^2 + V^2 g\psi^2(R_2)]\text{ ";}$$

$$\text{“ } A_3 = 2(\alpha W + \beta) + W^2\text{ ";}$$

$$\text{“ } B_3 = 4(\alpha W + \beta) + W^2\text{ ";}$$

$$\text{“ } C_3 = 6(\alpha W + \beta) + W^2\text{ "} \tag{16}$$

Putting g = 0 in Eq. (15) we can obtain W value for a selected V value when there is no nonlinearity. Next, using that W in any two of three Eqs. (11), (12), and (14),the constant values of $a_3$and $a_5$in terms of $a_1$are found out for linear case. We now put the W-value for linear case in Eq. (15) to know the value of W for the chosen fibre for an opted g value and continue the iteration technique for knowing the conjoining value of W . This conjoining W-value is employed in any two of the Eqs. (11), (12), and (14) to know conjoining values of a_3 and a_5 for that specific Kerr-class nonlinear fibre for that unique g value. Hence, in appearance of a specific type of Kerr nonlinearity, the field of the LP_{11} mode for every fibre of this category of the given V -number is approximated from Eq.(8). The same method is followed for all opted fibers for prediction of the field of LP_{11} mode

3. Results and Discussions

For determining the field distribution, we use three trapezoidal profiles fibers with the aspect ratios (S) 0.25, 0.50 and 0.75 all with the same V-value 4.0, in our study. We use Eq. (15) to obtain W- values for the selected fibers in the disappearance of nonlinearity. We put the respective W-values for those fibers to determine suitable W- values for two nonlinear situations as in Ref. (Mondal and Sarkar, 1996). Furthermore, using the methods described in section 2, determine the pertinent values of W in both the disappearance and appearance of nonlinearity and utilise to estimate the fields of the LP_{11} mode. For those fibers with S=0.25, 0.50 and 0.75, the change of LP_{11} mode field ψ(R) with normalized radius path (R) is shown in Figs. 1, 2, and 3. We take two parabolic kind W fibers with the same V- value 6.0 and C- value (1.5) but with dissimilar (1.4975 ,1.5000) ρ -values (Ray et al., 2020; Mishra et al., 1984). Next, by the same procedure stated above, the field of LP_{11}mode for these two categories of graded W fibers are assessed. In the disappearance and appearance of nonlinearity, Figs. 4 and 5 delineate the changes of the LP_{11} field with R in respect of the above mentioned graded W fiber-samples. Likewise, we continue our analysis on two standard fibers having step W profile with the same V- number 4.0 and C -value 2.0 but dissimilar ρ-values as 1.3333 and 1.2500 (Ray et al., 2020; Monerie, 1982). Proceeding as before, we predict the LP_{11} fields of these standard fibers having step- W profile corresponding to the chosen types of nonlinearity and also in linear state. Variation of ψ(R) against R for these step-W fibers are presented in Figs.6 and 7. Solid lines like——— in each of above stated graphs represent the exact values of the field ψ(R) computed using the finite element approach (Hayata et al., 1987). It can be seen that our findings clearly equal with the exact ones in every instance. In addition, the condition $n_{NL}P = 0$correlates to linearity, and the findings are also quite close with the available exact data (Hayata et al., 1987).By using this iterative Chebyshev approach to obtain the solution of a (3x3) determinant, any interested person can correctly determine the fields of the LP_{11} mode in dual- mode dispersion -controlled Kerr category nonlinear fibres. Thus, the findings are significant in terms of prudent selection of dual-mode fibers in the field of communication and sensors to reduce modal noise. Moreover, it offers wide scope for implementation in various sectors of nonlinear photonic-technology.

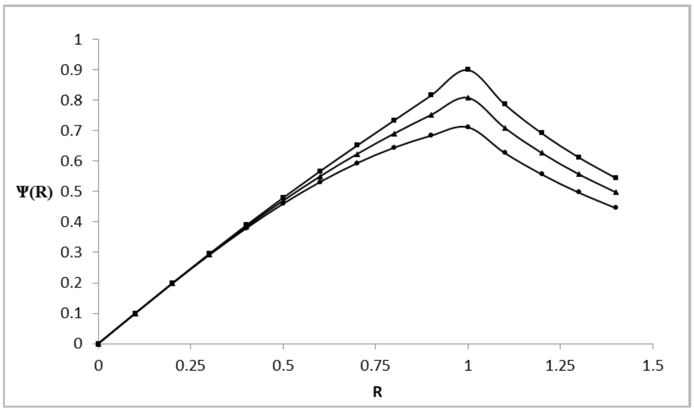

Figure 1: Ψ (R) versus R for LP_{11}mode of Trapezoidal -Index fiber with " *V = 4.0 and S=0.25* at different$n_{NL}P$" " (Predicted results: ■ for $n_{NL}P = +1.5X10^{-14}m^2$,▲ for $n_{NL}P = 0$ and ● for $n_{NL}P = -1.5X10^{-14}m^2$; ——— exact results) "

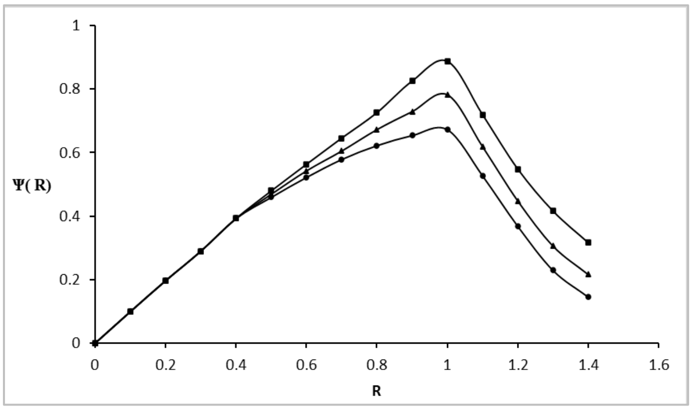

Figure 2: Ψ (R) versus R for LP_{11} mode of Trapezoidal- Index fiber with "*V = 4.0 and S=0.50* at different $n_{NL}P$" " (Predicted results:■ for $n_{NL}P = +1.5X10^{-14}m^2$, ▲ for $n_{NL}P = 0$ and ● for $n_{NL}P = -1.5X10^{-14}m^2$; ——— exact results) "

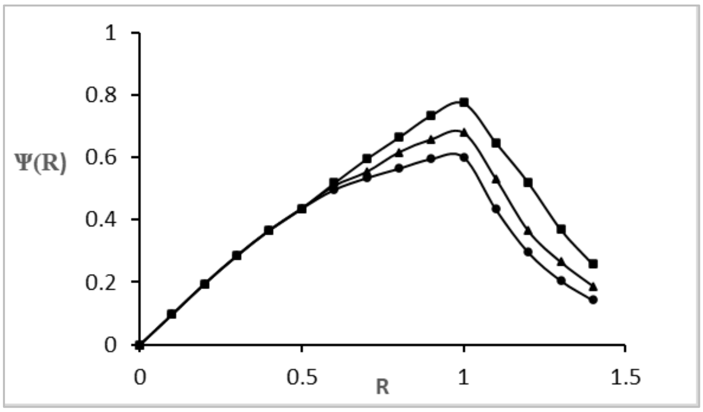

Figure 3: Ψ (R) versus R for LP_{11}mode of Trapezoidal- Index fiber with "*V = 4.0 and S=0.75* at different $n_{NL}P$ " " (Predicted results:■ for $n_{NL}P = +1.5X10^{-14}m^2$, ▲ for $n_{NL}P = 0$ and ● for $n_{NL}P = -1.5X10^{-14}m^2$; ——— exact results)"

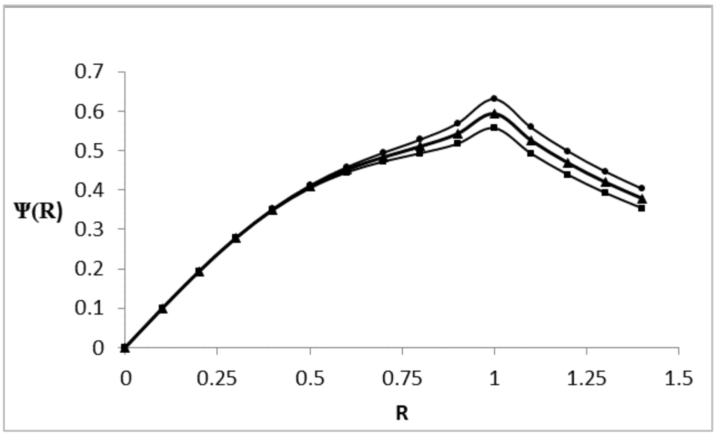

Figure 4: Ψ (R) versus R for LP_{11} mode of Parabolic-W fiber with " V = 6.0,1/C= 0.6666 and ρ =1.4975 at different $n_{NL}P$" " (Predicted results:■ for $n_{NL}P$ = +1.5X10^{-14}m^2 , ▲ for $n_{NL}P$ = 0 and ● for $n_{NL}P$ = -1.5X10^{-14}m^2; ——— exact results)"

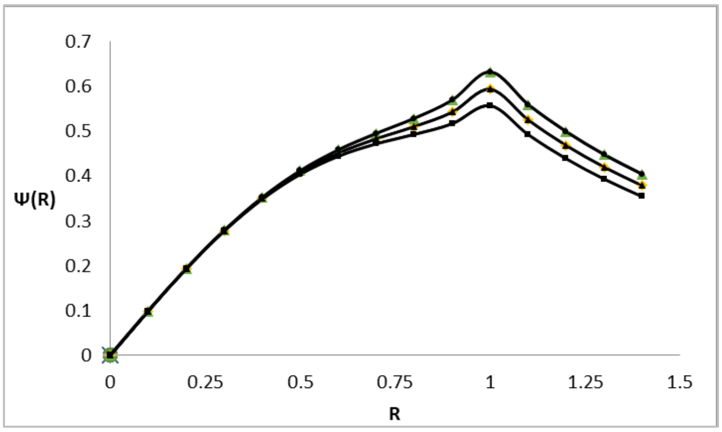

Figure 5: Ψ (R) versus R for LP_{11}mode of Parabolic-W fiber with " V = 6.0 ,1/C= 0.6666 and ρ =1.5000 with different $n_{NL}P$ " " (Predicted results:■ for $n_{NL}P$ = +1.5X10^{-14}m^2 , ▲ for $n_{NL}P$ = 0 and ● for $n_{NL}P$ = -1.5X10^{-14}m^2; ——— exact results) "

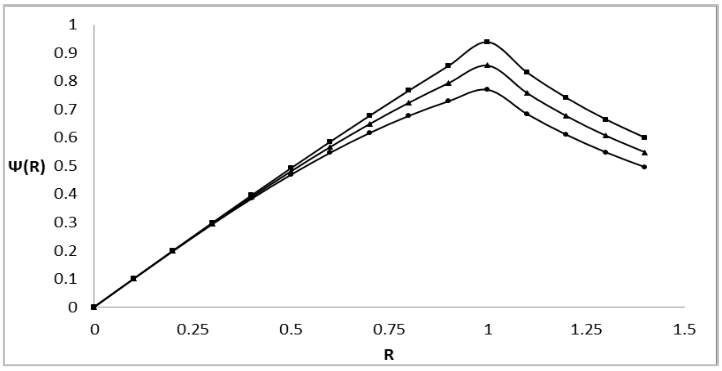

Figure 6: Ψ (R) versus R for LP_{11} mode of Step-W fiber with " *V = 4.0 ,1/C= 0.5 and ρ =5/4* at different $n_{NL}P$ " " (Predicted results:■ for $n_{NL}P$ = +1.5X10^{-14}m^2 , ▲ for $n_{NL}P$ = 0 and ● for $n_{NL}P$ = -1.5X10^{-14}m^2; ——— exact results) "

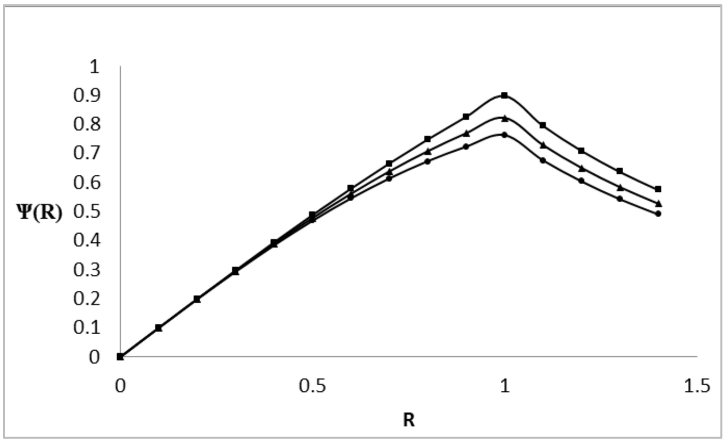

Figure 7: Ψ (R) R for LP_{11} mode of Step-W fiber with " *V = 4.0 ,1/C= 0.5 and ρ=4/3* at different $n_{NL}P$ " "(Predicted results:■ for $n_{NL}P$ = +1.5X10^{-14}m^2 , ▲ for $n_{NL}P$ = 0 and ● for $n_{NL}P$ = -1.5X10^{-14} m^2; ——— exact results)"

4. Conclusion

In this research work, we use an unique formalism incorporating the process of iteration to forecast modal- field patterns of the first higher mode (LP_{11}) of dispersion- controlled fibers in the appearance of third order nonlinearity. The formalism is straightforward to implement, but the results achieved are closely consistent with the available simulated correct results. In case of Trapezoidal -index fiber the normalized modal field Ψ (R) is maximum for positive nonlinearity and it is minimum for negative nonlinearity keeping the normalized modal field value Ψ (R) in between for linear condition for a particular normalised radial distance R exceeding 0.5. The same positions of field patterns are obtained for Step W dispersion-flattened fibers. But for Graded W fibers the opposite is found where the normalized modal field Ψ (R) is minimum for positive nonlinearity and it is maximum for negative nonlinearity keeping the normalized modal field value Ψ (R) in between for linear condition for a particular normalised radial distance R exceeding 0.75 . In the realm of nonlinear photonics, the results will be tremendously valuable in terms of minimizing modal noise caused by such nonlinearity.

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