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RESEARCH ARTICLE

CLASSICAL APPROACH TO THE DEVELOPMENT OF RAYLEIGH WEIBULL DISTRIBUTION AND IT'S APPLICATION TO COVID 19 DATA.

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ABSTRACT

The goal of this work is to use the T-R(y) family technique to create the Raleigh Weibull Distribution (RWD), a new probability distribution that may be used to fit real-world data. A Covid-19 data set that was acquired from Mexico was used to develop and fit some of the newly suggested distribution's features. Different estimate methodologies were discussed, and the properties of the probability density function (pdf) and cumulative distribution function (CDF) were developed. The analysis shows that the extra form elements in the suggested model make the new distribution more flexible than the present distribution, which makes it more flexible than the current models. According to the fitted findings, the suggested RWD performed better than the existing distributions. Compared to the current Rayleigh Distribution, the recommended distribution (RWD) better fits real-world data. Consequently, a distribution (RWD) that can handle asymmetric datasets for both left- and right-skewed data was developed and different methods for estimating were explained. The analysis shows that the extra form elements in the suggested model make the new distribution more flexible than the present distribution, which makes it more flexible than the current models.

KEYWORDS

Raleigh, Covid-19, T-R(y), Properties, Raleigh Weibul.

1. INTRODUCTION

The Rayleigh distribution, which was first proposed by Lord Rayleigh in the fields of optics and acoustics in 1880, is one of the most widely used distributions for analyzing skewed data sets. Since then, it has gained widespread use in oceanography and communication theory to describe the instantaneous peak power of received radio signals. Engineers and scientists have given it a lot of attention to model radiation, wave propagation, synthetic aperture radar images, and other related phenomena. There is a cumulative distribution function (PDF) for a Rayleigh random variable X. $F(x;\beta) = 1 - e^{-\frac{x^2}{2\beta}}, \quad x \geq 0, \beta > 0$ (1)
with probability density function (pdf)

$$f(x:\lambda) = \frac{x}{\beta} e^{-\frac{x^2}{2\beta}}, \; x \geq 0, \beta > 0 \quad (2)$$

A continuous probability distribution is the Weibull distribution. Although it was first identified and used to characterize a unit size of distribution it was named after Swedish mathematician Waloddi Weibull, who detailed it in detail in 1951 by (Frechet, 1927; Rosin and Rammler, 1933). There are parameters for the scale and shape of the Weibull distribution. When analyzing lifespan data and for many other applications where a skewed distribution is needed, this distribution has grown in popularity.

$$F_R(x) = 1 - e^{-\lambda x^{\alpha}}, \quad 0 < x < \infty, \quad \lambda \text{ and } \alpha > 0 \quad (3)$$

$$f(x) = \lambda e^{-\lambda x^{\alpha}}, \quad 0 < x < \infty, \quad \lambda \text{ and } \alpha > 0 \quad (4)$$

A model can be introduced into a wider range of distributions by inducing a new shape parameter or parameters. This can result in considerably skewed and heavy-tailed distributions as well as additional flexibility in the form of new distributions. The goal of this study is to create the Rayleigh-Weibul Distribution (RWD), a novel probability density function, utilizing the T-R{Y} framework that suggested (Alzaatreh et al., 2014).

2. PROPOSED RAYLEIGH-WEIBULL DISTRIBUTION

A new Rayleigh-Weibull distribution is formed

2.1 Methodology

Raleigh cumulative distribution function CDF

$$F_T(x:\lambda) = 1 - e^{-\frac{x^2}{2\beta^2}} \qquad x > 0, \; \beta > 0 \quad (5)$$

Raleigh Probability distribution function PDF

$$f(x:\lambda) = \frac{x}{\beta} e^{-\frac{x^2}{2\beta^2}} \qquad x > 0, \; \beta > 0 \quad (6)$$

β is the scale of the parameters

Weibull cumulative distribution function CDF

$$F_R(x) = 1 - e^{-\lambda x^{\theta}} \qquad 0 < x < \infty \qquad \theta, \lambda \text{ and } x > 0 \quad (7)$$

$$f(x) = \theta\lambda e^{-\lambda x^{\theta}} \qquad 0 < x < \infty \qquad \theta, \lambda \text{ and } x > 0 \quad (8)$$

formulation of new distribution from T-R(y) family

The quantile from the exponential distribution is $p = 1 - e^{-\lambda x}$ (9)

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$x = \left[-\frac{1}{\lambda}\log(1-p)\right]$ (10)

By standardization $\lambda = 1$

Therefore $x = [-\log(1-p)]$ this is the quantile function

By substitution $F_R(x) = p$

$x = -\log(1 - F_R(x))$

$G(x) = \int_0^{W(F_R(x))} ar(t)dt = R\{W(F(x))\}$ and

$g(x) = \{ddxW(F(x))\}r\{W(F(x))\}.$

$G(x) = \int_0^{-\log(1-F_R(x))} r(t)dt = F_T(W(F_R(x)))$

$G(x) = F_T(-\log(1 - F_R(x)))$

The Propose RW cumulative distribution is given as

$G(x) = 1 - e^{-\frac{\lambda(x)^{2\theta}}{2\beta^2}} \quad 0 < x < \infty$ (11)

Let $2\theta = \alpha$

The quantile of the Propose RW Distribution

$x = \frac{\left(-2\beta^2 log(1-p)\right)^{\frac{1}{\alpha}}}{\lambda}$ (12)

The probability distribution is given by differentiating the CDF

i.e $g(x) = \frac{d}{dx}W(F(x)).r\{W(F(x))\}$

$g(x) = \frac{\alpha\lambda^2}{2\beta^2}x^{\alpha-1}e^{-\frac{\lambda^2 x^\alpha}{2\beta^2}} \quad 0 < x < \infty \quad \alpha, \beta and\ \lambda > 0$ (13)

$Meadian = \frac{(-2\beta ln2)^\alpha}{\lambda^2}$ (14)

2.2 Moment and Properties of Proposed Raleigh weibull distribution (GRWD)

$E(x^r) = \int_{-\infty}^{\infty} x^r f(x)\, dx$

$E(x^r) = \int_0^\infty x^r \frac{\alpha\lambda^2}{2\beta^2}x^{\alpha-1}e^{-\frac{\lambda^2 x^\alpha}{2\beta^2}}\, dx, \quad \alpha, \beta and\ \lambda > 0$ (15)

r^{th} Moment

$E(x^r) = \left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{r}{\alpha}} \Gamma_{\frac{r}{\alpha}+1}$, where $\Gamma_{\frac{r}{\alpha}+1} = \frac{r}{\alpha}\Gamma_{\frac{r}{\alpha}}$

1^{st} Moment i.e. Meam

$E(x) = \mu = \left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{1}{\alpha}} \frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}}$ (16)

2^{nd} Moment

$E(x^2) = \mu_2' = \left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{2}{\alpha}} \frac{2}{\alpha}\Gamma_{\frac{2}{\alpha}}$ (17)

$Var(x) = E(x-\mu)^2$

$Var(x) = E(x^2) - \mu^2$

$Var(x) = \mu_2' - \mu^2$

The expression for variance, standard deviation, Skewness, and Kurtosis can be obtained from equations 16, and 17 as:

$Var(x) = \left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{2}{\alpha}} \left[\frac{2}{\alpha}\Gamma_{\frac{2}{\alpha}} - \left(\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}}\right)^2\right]$ (18)

$S.D = \left(\left(\frac{2\beta^2}{\lambda^1}\right)^{\frac{1}{\alpha}} \left[\frac{2}{\alpha}\Gamma_{\frac{2}{\alpha}} - \left(\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}}\right)^2\right]\right)^{\frac{1}{2}}$ (19)

3^{rd} Moment

$E(x^3) = \mu_3' = \left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{3}{\alpha}} \frac{3}{\alpha}\Gamma_{\frac{3}{\alpha}}$ (20)

4^{th} Moment

$E(x^4) = \mu_2' = \left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{4}{\alpha}} \frac{4}{\alpha}\Gamma_{\frac{4}{\alpha}}$ (21)

$Skewness = \frac{E(x-\mu)^3}{\sigma^3} = \frac{\mu_3' - 3\mu_2'\mu + 2\mu^3}{\sigma^3}$

$$S.K = \frac{\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{3}{\alpha}}\frac{3}{\alpha}\Gamma_{\frac{3}{\alpha}} - 3.\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{2}{\alpha}}\frac{2}{\alpha}\Gamma_{\frac{2}{\alpha}}.\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{1}{\alpha}}\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}} + 2.\left(\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{1}{\alpha}}\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}}\right)^3}{\left(\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{2}{\alpha}}\left[\frac{2}{\alpha}\Gamma_{\frac{2}{\alpha}} - \left(\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}}\right)^2\right]\right)^{\frac{3}{2}}} \quad (22)$$

$Kurtosis = \frac{E(x-\mu)^4}{\sigma^4} = \frac{\mu_4' - 4\mu_3'\mu + 6\mu_2'\mu^2 - 3\mu^3}{\sigma^4}$

$$K.T = \frac{\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{4}{\alpha}}\frac{4}{\alpha}\Gamma_{\frac{4}{\alpha}} - 4.\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{3}{\alpha}}\frac{3}{\alpha}\Gamma_{\frac{3}{\alpha}}.\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{1}{\alpha}}\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}} + 6.\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{2}{\alpha}}\frac{2}{\alpha}\Gamma_{\frac{2}{\alpha}}.\left(\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{1}{\alpha}}\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}}\right)^2 - 3.\left(\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{1}{\alpha}}\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}}\right)^3}{\left[\left(\left(\frac{2\beta^2}{\lambda^2}\right)^{\frac{2}{\alpha}}\left[\frac{2}{\alpha}\Gamma_{\frac{2}{\alpha}} - \left(\frac{1}{\alpha}\Gamma_{\frac{1}{\alpha}}\right)^2\right]\right)\right]^2}$$

(23)

Maximum Likelihood Estimation

In this section, we determine the maximum likelihood estimates (MLEs) of the parameters of the GRED distribution.

Maximum Likelihood Estimation

In this section, we determine the maximum likelihood estimates (MLEs) of the parameters of the RWD distribution.

$\widehat{\alpha} = \frac{n}{\alpha} - \frac{n\lambda^2}{2\beta}\sum x_i{}^\alpha lnx_i$ (24)

$\widehat{\beta} = \frac{\lambda^2 x^\alpha}{\beta} - \frac{2n}{b}$ (25)

$\widehat{\lambda} = \frac{2n}{\lambda} - \frac{\lambda \sum x^2}{\beta}$ (26)

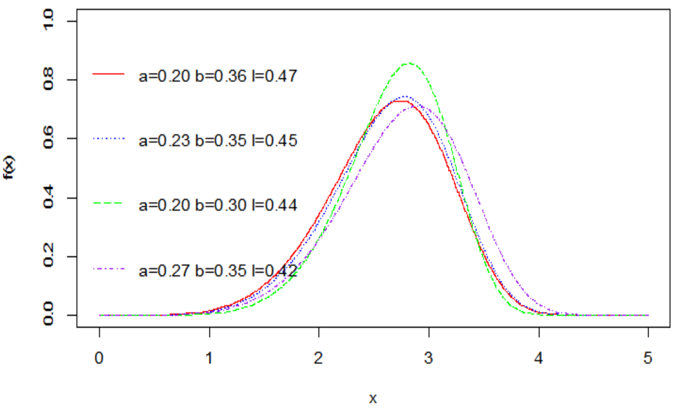

Figure 1: Plot of Probability density function of RW

Figure 1 above shows the graph of probability density function of RW distribution.

Probability density function plot of RW distribution exhibit various shape depending on the combination of scales and shapes parameters. It shows that it could be high peaked depending on the values of the parameters. As we increase the shape of the parameter the tail becomes heavier.

The graph is a bell shape and the origin of the RW distribution are the same, the parameters start from the same origin.

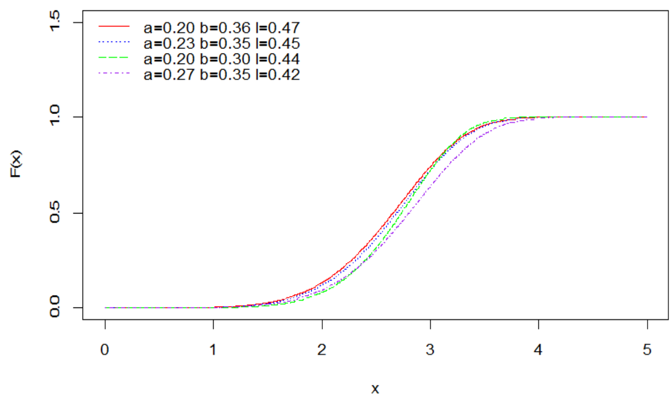

Figure 2: Plot of Cumulative distribution of RW

Figure 2 shows the graph of cumulative distribution function of RW distribution. Cumulative distribution plot of RW distribution shows a certificatory level of cumulative plot since the plot is not exceeding 1, which shows that our distribution is a true probability distribution function.

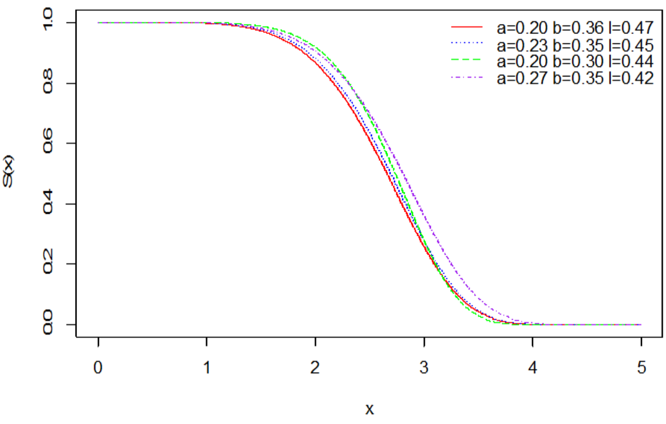

Figure 3: Plot of Survival Function of RW Distribution

RW delayed its drop. The survival function of RW dropping across the x-axis as the shape and scale's parameter increases.

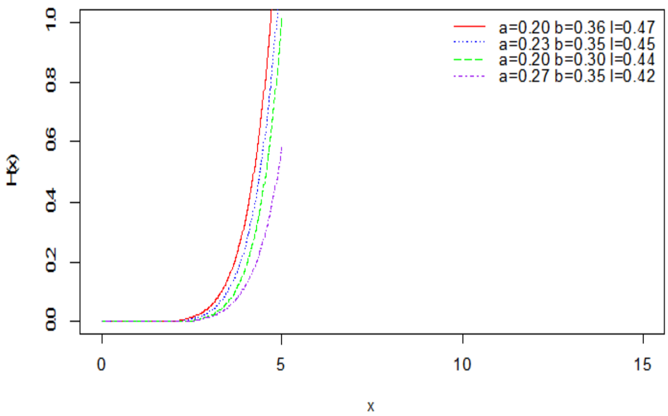

Figure 4: Plot of Hazard function of RW

Figure 4 above show the graph of hazard function of RW distribution. Hazard function of RW is an increase function of the shape and scale parameters.

3. Real Life Application

Table 1: data represents a COVID-19 mortality rate data belongs to Mexico of 108 days, which is recorded from 4 March to 20 July 2020.

8.826,6.105,10.383,7.267,13.220,6.015,10.855,6.122,10.685,10.035,5.242,7.630,14.604,7.903,6.327,9.391,14.962,4.730,3.215,16.498,11.665,9.284,12.878,6.656,3.440,5.854,8.813,10.043,7.260,5.985,4.424,4.344,5.143, 9.935,7.840,9.550,6.968,6.370, 3.537,3.286, 10.158, 8.108, 6.697, 7.151, 6.560,2.988,3.336,6.814,8.325,7.854,8.551,3.228,3.499,3.751,7.486,6.625,6.140,4.909,4.661,1.867,2.838,5.392,12.042,8.696,6.412,3.395,1.815,3.327,5.406,6.182,4.949, 4.089, 3.359, 2.070, 3.298, 5.317, 5.442, 4.557, 4.292, 2.500, 6.535, 4.648, 4.697, 5.459, 4.120,3.922,3.219, 1.402, 2.438, 3.257, 3.632, 3.233, 3.027,2.352, 1.205, 2.077, 3.778,3.218,2.926, 2.601, 2.065, 1.041, 1.800, 3.029, 2.058, 2.326, 2.506, 1.923

Source: Department of Statistics Auchi Polytechnic, 2018/2019 first semester result

In this section, we fit the RW distribution to real life data sets and compare it fitted values with that of its sub-model. The data represents a COVID-19 mortality rate data belongs to Mexico of 108 days, which is recorded from 4 March to 20 July 2020.

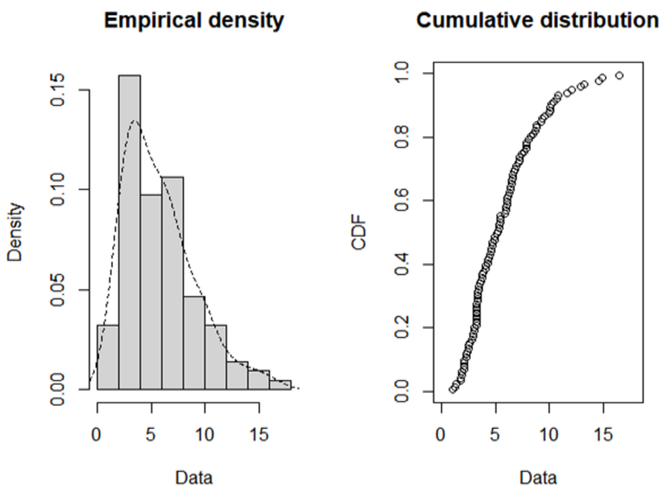

Figure 5: Histogram and CDF Plots of an Empirical Distribution for The data represents a COVID-19 mortality rate data belongs to Mexico of 108 days, which is recorded from 4 March to 20 July 2020.

Table2: Summary Statistics of Covid 19 Data

Min.	Max.	Mean	Median	1st Quart.	3rd Quart.	S.D.	Skewness	Kurtosis
1.041	3.251	5.758	5.192	2.57	3.03	3.254165	0.972951	0.6134055

The statistical summary table from Table 7 shows that the Covid 19 data is under disperse, non-symmetry and right-skewed.

Table 3: Log-likelihood, Information Criterion Goodness-of-fit statistics and P-values of Covid Data.

DISTRIBUTION	LL	AIC	BIC	K-S	C-M	AD	P-value
RW	-258.9824	523.9647	532.0111	0.8480602	25.2108289	147.5051918	0.7077
Rayleigh	-269.2302	540.4605	543.1426	0.09334079	0.16297970	29.2168	1.332e-15

Table 6 shows the estimated values for all the shapes and scale parameters for the proposed RW distribution and its related sub-model Raleigh distribution. The estimates of log-likelihood function (LL), information criterion (Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)) shows that RW distribution gives a better result, from all the values than Raleigh distribution. The estimated values of the goodness of fit, for the proposed RW distribution and its sub-model Raleigh distribution. The estimates of Kolmogorov-Smirnov statistic, Cramer-von Mises statistic, and Anderson-Darling statistic shows that RW distribution gives a better result, from all the values than Raleigh distribution. The table above shows the P-value of the proposed distribution RW and Raleigh distribution, and it can be seen that the p-value of the new distribution is 0.7077 which signifies that the data follow the new distribution (RW)because the more the p-value tends to 1 the better the distribution. The p-value for Raleigh distribution is 1.332e-15 which is very far from 1, this is an indication that the data did not follow Raleigh distribution.

Fitting of the distribution ' Rweibull ' by maximum likelihood

Table 4: Estimated Unknown Parameters

Estimate	Std. Error
α = 2.0798275	0.1315763
β = 1.7951190	8388.6080009
λ = l 0.1389194	0.0207910

Correlation matrix (A) =

$$\begin{pmatrix} 1.000000e+00 & -1.451166e-05 & -9.469148e-01 \\ -1.451166e-05 & 1.000000e+00 & 1.361322e-05 \\ -9.469148e-01 & 1.361322e-05 & 1.000000e+00 \end{pmatrix}$$

The asymptotic variance covariance matrix for the estimated parameters is

$$A_{ij}^{-1} = \begin{pmatrix} 0.017312310 & -1.601712e-02 & -0.0025903819 \\ -0.016017119 & 7.036874e+07 & 0.0023742490 \\ -0.002590382 & 2.374249e-03 & 0.0004322657 \end{pmatrix}$$

Correlation matrix above shows that the parameters exhibit both positive and negative correlation coefficients depending on the combination of the parameters. This allows us to see which pairs have a negative and positive correlation.

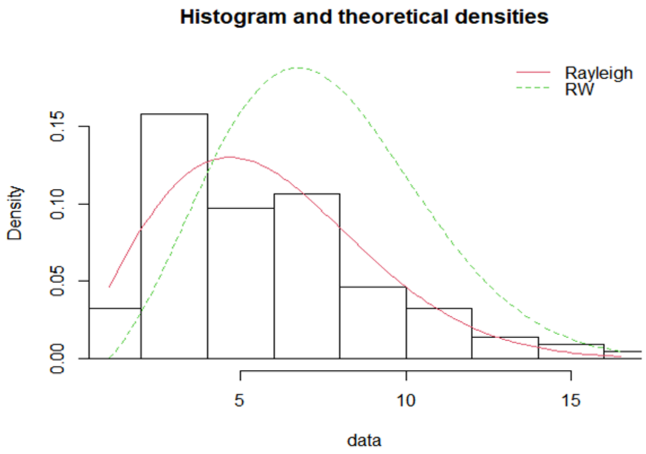

Figure 6: Fitted Plot of RW and Raleigh distribution Model on Covid 19 Data

Figure 6 shows the data represent a COVID-19 mortality rate data belongs to Mexico of 108 days, which is recorded from 4 March to 20 July 2020. The theoretical density of RW distribution has a better spread than Raleigh distribution model on the data set.

4. Conclusions

The RW distribution can be used to analyze lifetime data. It is observed in certain situations that the RW distribution provides a better fit in terms of the lower Kolmogorov Smirnov distance, Anderson-Darling and Cramer-von, BIC, AIC statistic or higher log-likelihood value than the Rayleigh distributions, see details in table 3 and 4. It shows from the analysis that RW distribution is more flexible compared to Rayleigh distribution as it is said in the literature that introduction of a new scale parameter makes the new distribution to be more flexible than the existing distribution. It is shown that these newly generated distributions are very flexible and are capable of fitting various types of data (Alzaatreh et al., 2014). The results studied here will help to decide which distribution to be used, based on the aging properties of the newly generated distributions, and also to find the better one in terms of various stochastic orders.

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