

## Matriks Sains Matematik (MSMK)

DOI : <http://doi.org/10.26480/msmk.02.2018.11.17>

Print ISSN : 2792-2792

Online ISSN : 2792-2792



I IO O SO S IO IOMS I SO I- O O O I S S

ri Mehmood Khattak <sup>1</sup> ar i Khan <sup>2</sup> o nis Khan <sup>2</sup> M hammad Ish a Khattak <sup>2</sup> ahad ama <sup>2</sup>

This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited

## ARTICLE DETAILS

## ABSTRACT

**Article History:**

Received 12 November 2017

Accepted 12 December 2017

Available online 1 January 2018

The main aim of this article is to introduce Soft b separation axioms in soft bi topological spaces. We discuss soft b separation axioms in soft bi topological spaces with respect to ordinary point and soft points. Further study the behavior of soft regular  $bT_3$  and soft normal  $bT_4$  spaces at different angles with respect to ordinary points as well as with respect to soft points. Hereditary properties are also discussed.

## KEYWORDS

Soft sets, soft points, soft bi-topological space, soft  $bT_i$  spaces ( $i=1,2,3,4$ ), soft b regular and soft b normal spaces.

## 1. INTRODUCTION

In real life condition the problems in economics, engineering, social sciences, medical science etc. We cannot beautifully use the traditional classical methods because of different types of uncertainties presented in these problems. To overcome these difficulties, some kinds of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, in which we can safely use a mathematical technique for business with uncertainties. But, all these theories have their inherent difficulties. To overcome these difficulties in the year 1999, Russian scientist, initiated the notion of soft set as a new mathematical technique for uncertainties, which is free from the above complications [1]. A group researchers, successfully applied the soft set theory in different directions, such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, probability, theory of measurement and so on [2,3]. After presentation of the operations of soft sets, the properties and applications of the soft set theory have been studied increasingly [4-6]. Others group researchers discussed the linkage between soft sets and information systems [7,8]. They showed that soft sets are class of special information system. In the recent year, many interesting applications of soft sets theory have been extended by embedding the ideas of fuzzy sets industrialized soft set theory, the operations of the soft sets are redefined and in indecision making method was constructed by using their new operations [9-20].

Recently, in 2011, there are some researcher launched the study of soft Topological spaces, they beautiful defined soft Topology as a collection of  $\tau$  of soft sets over  $X$  [20]. They also defined the basic conception of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axiom, soft regular and soft normal spaces and published their several behaviors. Min in scrutinized some belongings of these soft separation axioms [21]. A group researcher introduced some soft

operations such as semi open soft, pre-open soft,  $\alpha$ -open soft and  $\beta$ -open soft and examined their properties in detail [22]. Some of researcher introduced the concept of soft semi - separation axioms, in particular soft semi- regular spaces [23]. The concept of soft ideal was discussed for the first time [24]. They also introduced the concept of Soft local function. There concepts are discussed with a view to find new soft topological from the original one, called soft topological spaces with soft ideal  $(X, \tau, E, I)$ .

Application to different zone were further discussed [24-30]. The notion of super soft topological spaces was initiated for the first time [31]. They also introduced new different types of sub-sets of supra soft topological spaces and study the dealings between them in great detail. A researcher introduced the concept of semi open soft sets and studied their related properties, discussed soft separation axioms [32,33]. Mahanta introduced semi open and semi closed soft sets. [34,35]. On Soft  $\beta$ -Separation Axioms, Arockiarani, I, Arokialancy in generalized soft  $g\beta$  closed and soft  $gs\beta$  closed sets in soft topology are exposed [36].

In this present paper the concept of soft  $bT_0$ , soft  $bT_1$ , soft  $bT_2$  and soft  $bT_3$  spaces in Soft topological spaces is introduced with respect to soft points. The concept of soft  $bT_0$ ,  $bT_1$ ,  $bT_2$  and soft  $bT_3$  and soft  $bT_4$  spaces are introduced in soft bi topological spaces with respect to ordinary as well as soft points. Many mathematicians discussed soft separation axioms in soft topological spaces at full length with respect to soft open set, soft  $b$ -open set, soft semi-open set, soft  $\alpha$ -open set and soft  $\beta$ -open set [37-40]. They also worked over the hereditary properties of different soft topological structures in soft topology.

In this present work hand is tried and work is encouraged over the gap that exists in soft bi-topology. Related to Soft  $bT_3$  and soft  $bT_4$  spaces, some Proposition in soft bi topological spaces are discussed with respect

to ordinary points as well as with respect to soft points. When we talk about the distances between the points in soft topology then the concept of soft separation axioms will automatically come in play [41]. That is why these structures are catching our attentions. We hope that these results will be valuable for the future study on soft bi topological spaces to accomplish general framework for the practical applications and to solve the most intricate problems containing scruple in economics, engineering, medical, environment and in general mechanic systems of various kinds [41,42].

**1. PRELIMINARIES**

The following Definitions which are pre-requisites for present study.

**Definition 1** [1]. Let  $X$  be an initial universe of discourse and  $E$  be a set of parameters. Let  $P(X)$  denotes the power set of  $X$  and  $A$  be a non-empty sub-set of  $E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(X)$ . In other words, a set over  $X$  is a parameterized family of sub set of universe of discourse  $X$ . For  $e \in A, F(e)$  may be considered as the set of e-approximate elements of the soft set  $(F, A)$  and if  $e \notin A$  then  $F(e) = \phi$ , that is  $F_A = \{F(e): e \in A \subseteq E, F: A \rightarrow P(X)\}$  the family of all these soft sets over  $X$  denoted by  $SS(X)_A$ .

**Definition 2** [1]. Let  $F_A, G_B \in SS(X)_E$  then  $F_A$  is a soft subset of  $G_B$  denoted by  $F_A \subseteq G_B$ , if

- $A \subseteq B$  &
- $F(e) \subseteq G(e), \forall e \in A$

In this case  $F_A$  is said to be a soft subset of  $G_B$  and  $G_B$  is said to be a soft super set  $F_A, G_B \supseteq F_A$ .

**Definition 3** [5]. Two soft subsets  $F_A$  and  $G_B$  over a common universe of discourse set  $X$  are said to be equal if  $F_A$  is a soft subset of  $G_B$  and  $G_B$  is a soft subset of  $F_A$ .

**Definition 4** [5]. The complement of soft subset  $(F, A)$  denoted by  $(F, A)^c$  is defined by  $(F, A)^c = (F^c, A)$   $F^c \rightarrow P(X)$  is a mapping given by  $F^c(e) = U - F(e) \forall e \in A$  and  $F^c$  is called the soft complement function of  $F$ . Clearly  $(F^c)^c$  is the same as  $F$  and  $((F, A)^c)^c = (F, A)$ .

**Definition 5** [4]. The difference between two soft subset  $(G, E)$  and  $(F, E)$  over common of universe discourse  $X$  denoted by  $(F, E) - (G, E)$  is the soft set  $(H, E)$  where for all  $e \in E$ .

**Definition 6** [4]. Let  $(G, E)$  be a soft set over  $X$  and  $x \in X$  We say that  $x \in (F, E)$  and read as  $x$  belong to the soft set  $(F, E)$  whenever  $x \in F(e) \forall e \in E$  The soft set  $(F, E)$  over  $X$  such that  $F(e) = \{x\} \forall e \in E$  is called singleton soft point and denoted by  $x_E$ , or  $(x, E)$ .

**Definition 7** [3]. A soft set  $(F, A)$  over  $X$  is said to be Null soft set denoted by  $\emptyset$  or  $\emptyset_A$  if  $\forall e \in A, F(e) = \emptyset$ .

**Definition 8** [3]. A soft set  $(F, A)$  over  $X$  is said to be an absolute soft denoted by  $\bar{A}$  or  $X_A$  if  $\forall e \in A, F(e) = X$ . Clearly, we have.  $X_A^c = \emptyset_A$  and  $\emptyset_A^c = X_A$ .

**Definition 9** [32]. Let  $(G, E)$  be a soft set over  $X$  and  $e_G \in X_A$ , we say that  $e_G \in (F, E)$  and read as  $e_G$  belong to the soft set  $(F, E)$  whenever  $e_G \in F(e) \forall e \in E$ . the soft set  $(F, E)$  over  $X$  such that  $F(e) = \{e_G\}, \forall e \in E$  is called singleton soft point and denoted by  $e_G$ , or  $(e_G, E)$ .

**Definition 10** [32]. The soft set  $(F, A) \in SSX_A$  is called a soft point in  $X_A$ , denoted by  $e_F$ , if for the element  $e \in A, F(e) \neq \emptyset$  and  $F(e') = \emptyset$  if for all  $e' \in A - \{e\}$

**Definition 11** [32]. The soft point  $e_F$  is said to be in the soft set  $(G, A)$ , denoted by  $e_F \in (G, A)$  if for the element  $e \in A, F(e) \subseteq G(e)$ .

**Definition 12** [32]. Two soft sets  $(G, A), (H, A)$  in  $SSX_A$  are said to be soft disjoint, written  $(G, A) \cap (H, A) = \emptyset_A$  If  $G(e) \cap H(e) = \emptyset \forall e \in A$ .

**Definition 13** [32]. The soft point  $e_G, e_H \in X_A$  are disjoint, written  $e_G \neq e_H$ , if their corresponding soft sets  $(G, A)$  and  $(H, A)$  are disjoint.

**Definition 14** [5]. The union of two soft sets  $(F, A)$  and  $(G, B)$  over the common universe of discourse  $X$  is the soft set  $(H, C)$ , where,  $C = A \cup B \forall e \in C$

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in (B - A) \\ F(e), & \text{if } e \in A \cap B \end{cases}$$

Written as  $(F, A) \cup (G, B) = (H, C)$

**Definition 15** [3]. The intersection  $(H, C)$  of two soft sets  $(F, A)$  and  $(G, B)$  over common universe  $X$ , denoted  $(F, A) \cap (G, B)$  is defined as  $C = A \cap B$  and  $H(e) = F(e) \cap G(e), \forall e \in C$ .

**Definition 16** [19]. Let  $(F, E)$  be a soft set over  $X$  and  $Y$  be a non-empty sub set of  $X$ . Then the sub soft set of  $(F, E)$  over  $Y$  denoted by  $(Y_F, E)$ , is defined as follow  $Y_{F(\alpha)} = Y \cap F(\alpha), \forall \alpha \in E$  in other words  $(Y_F, E) = Y \cap (F, E)$ .

**Definition 17** [19]. Let  $\tau$  be the collection of soft sets over  $X$ , then  $\tau$  is said to be a soft topology on  $X$ , if

- $\emptyset, X \in \tau$
- The union of any number of soft sets in  $\tau$  belongs to  $\tau$
- The intersection of any two soft sets in  $\tau$  belong to  $\tau$

The triplet  $(X, \tau, E)$  is called a soft topological space.

**Definition 18** [36]. Let  $(X, \tau, E)$  be a soft topological space over  $X$  then the member of  $\tau$  are said to be soft open sets in  $X$ .

**Definition 19** [36]. Let  $(X, \tau, E)$  be a soft topological space over  $X$ . A soft set  $(F, A)$  over  $X$  is said to be a soft closed set in  $X$ , if its relative complement  $(F, E)^c$  belong to  $\tau$ .

**Definition 20** [37] Let  $(X, \tau, E)$  be a soft topological space and  $(F, E) \subseteq SS(X)_A$  then  $(F, E)$  is called a b open soft set.  $(F, E) \subseteq Cl(int(F, E) \cup int(Cl(F, E)))$  The set of all b open soft set is denoted by  $BOS(X, \tau, E)$  or  $BOS(X)$  and the set of all b closed soft set is denoted by  $BOS(X, \tau, E)$  or  $BOS(X)$

**Proposition 1** [37]. Let  $(X, \tau, E)$  be a soft topological space over  $X$ . If  $(X, \tau, E)$  is soft b  $T_3$ -space, then for all  $x \in X, x_E = (x, E)$  is b-closed soft set.

**2. SOFT B-SEPARATION AXIOMS OF SOFT TOPOLOGICAL SPACES.**

**Definition 21** [36]. Let  $(X, \tau, A)$  be a soft Topological space over  $X$  and  $x, y \in X$  such that  $x \neq y$  if there exist at least one soft open set  $(F_1, A)$  or  $(F_2, A)$  such that  $x \in (F_1, A), y \notin (F_1, A)$  or  $y \in (F_2, A), x \notin ((F_2, A))$  then  $(X, \tau, A)$  is called a soft  $T_0$  space.

**Definition 22** [36]. Let  $(X, \tau, A)$  be a soft Topological spaces over  $X$  and  $x, y \in X$  such that  $x \neq y$  if there exist soft open sets  $(F_1, A)$  and  $(F_2, A)$  such that  $x \in (F_1, A), y \notin (F_1, A)$  and  $y \in (F_2, A), x \notin ((F_2, A))$  then  $(X, \tau, A)$  is called a soft  $T_1$  space.

**Definition 23** [36]. Let  $(X, \tau, A)$  be a soft Topological space over  $X$  and  $x, y \in X$  such that  $x \neq y$  if there exist soft open set  $(F_1, A)$  and  $(F_2, A)$  such that  $x \in (F_1, A)$ , and  $y \in (F_2, A)$ , and  $F_1 \cap F_2 = \emptyset$  Then  $(X, \tau, A)$  is called soft  $T_2$  spaces.

**Definition 24** [32]. Let  $(X, \tau, A)$  be a soft Topological space over  $X$  and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if we can search at least one soft open set  $(F_1, A)$  or  $(F_2, A)$  such that  $e_G \in (F_1, A), e_H \notin (F_1, A)$  or  $e_H \in (F_2, A), e_G \notin ((F_2, A))$  then  $(X, \tau, A)$  is called a soft  $T_0$  space.

**Definition 25** [32]. Let  $(X, \tau, A)$  be a soft Topological spaces over  $X$  and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if we can search soft open sets  $(F_1, A)$  and  $(F_2, A)$  such that  $e_G \in (F_1, A), e_H \notin (F_1, A)$  and  $e_H \in (F_2, A), e_G \notin ((F_2, A))$  then  $(X, \tau, A)$  is called a soft  $T_1$  space.

**Definition 26** [32]. Let  $(X, \tau, A)$  be a soft Topological space over  $X$  and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if we can search soft open set  $(F_1, A)$  and  $(F_2, A)$  such that  $e_G \in (F_1, A)$ , and  $e_H \in (F_2, A)$   $(F_1, A) \cap (F_2, A) = \emptyset_A$  Then  $(X, \tau, A)$  is called soft  $T_2$  space.

**Definition 27** [37]. Let  $(X, \tau, E)$  be a soft topological space over  $X$  and  $x, y \in X$  such that  $x \neq y$ . If we can find soft b open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  Then  $(X, \tau, E)$  is called soft b  $T_0$  space.

**Definition 28** [37]. Let  $(X, \tau, E)$  be a soft topological space over  $X$

and  $x, y \in X$  such that  $x \neq y$ . If we can find two soft b open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  and  $y \in (G, E)$  and  $x \notin (G, E)$ . Then  $(X, \tau, E)$  is called soft b  $T_1$  space.

**Definition 29** [37]. Let  $(X, \tau, E)$  be a soft topological space over  $X$  and  $x, y \in X$  such that  $x \neq y$ . If we can find two soft b open soft sets such that  $x \in (F, E)$  and  $y \in (G, E)$  moreover  $(F, E) \cap (G, E) = \phi$ . Then  $(X, \tau, E)$  is called a soft b  $T_2$  space.

**Definition 30** [37]. Let  $(X, \tau, E)$  be a soft topological space  $(G, E)$  and b closed soft set in  $X$  and  $x \in X$  such that  $x \notin (G, E)$ . If there occurs soft b open sets  $(F_1, E)$  and  $(F_2, E)$  such that  $x \in (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Then  $(X, \tau, E)$  is called soft b regular spaces. A soft b regular  $T_1$  Space is called soft b  $T_3$  space.

**Definition 31** [37]. Let  $(X, \tau, E)$  be a soft topological space and  $(F, E), (G, E)$  be soft b closed sets in  $X$  such that  $(F, E) \cap (G, E) = \phi$ . If there occurs b open soft sets  $(F_1, E)$  and  $(F_2, E)$  such that  $(F, E) \subseteq (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$  then  $(X, \tau, E)$  is called a soft b normal space. A soft b normal  $T_1$  space is called soft b  $T_4$  space.

### 3. SOFT B-SEPARATION AXIOMS OF SOFT BI-TOPOLOGICAL SPACES

Let  $X$  is an initial set and  $E$  be the non-empty set of parameters. In soft bi topological space over the soft set  $X$  is introduced [36]. Soft separation axioms in soft bi topological spaces were introduced by Basavaraj and Ittanagi [36]. In this section we introduced the concept of soft b  $T_3$  and b  $T_4$  spaces in soft bi topological spaces with respect to ordinary as well as soft points and some of its basic properties are studied and applied to different results in this section.

**Definition 32** [36]. Let  $(X, \tau_1, E)$  and  $(X, \tau_2, E)$  be two different soft topologies on  $X$ . Then  $(X, \tau_1, \tau_2, E)$  is called a soft bi topological space. The two soft topologies  $(X, \tau_1, E)$  and  $(X, \tau_2, E)$  are independently satisfy the axioms of soft topology. The members of  $\tau_1$  are called  $\tau_1$  soft open set. And complement of  $\tau_1$ . Soft open set is called  $\tau_1$  soft closed set. Similarly, the member of  $\tau_2$  are called  $\tau_2$  soft open sets and the complement of  $\tau_2$  soft open sets are called  $\tau_2$  soft closed set.

**Definition 33** [36]. Let  $(X, \tau_1, \tau_2, E)$  be a soft topological space over  $X$  and  $Y$  be a non-empty subset of  $X$ . Then  $\tau_{1Y} = \{(Y_E, E) : (F, E) \in \tau_1\}$  and  $\tau_{2Y} = \{(G_E, E) : (G, E) \in \tau_2\}$  are said to be the relative topological on  $Y$ . Then  $(Y, \tau_{1Y}, \tau_{2Y}, E)$  is called relative soft bi-topological space of  $(X, \tau_1, \tau_2, E)$ .

#### 3.1 Soft b-Separation axioms with respect to ordinary points

In this section we introduced soft b separation axioms in soft bi topological space with respect to ordinary points and discussed some results with respect to these points in detail.

**Definition 34.** In a soft bi topological space  $(X, \tau_1, \tau_2, E)$

1)  $\tau_1$  said to be soft b  $T_0$  space with respect to  $\tau_2$  if for each pair of distinct points  $x, y \in X$  there exists  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  similarly,  $\tau_2$  is said to be soft b  $T_0$  space with respect to  $\tau_1$  if for each pair of distinct points  $x, y \in X$  there exists  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$ . Soft bi topological spaces  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_0$  space if  $\tau_1$  is soft b  $T_0$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_0$  space with respect to  $\tau_1$ .

2)  $\tau_1$  is said to be soft b  $T_1$  space with respect to  $\tau_2$  if for each pair of distinct points  $x, y \in X$  there exists a  $\tau_1$  soft b open set  $(F, E)$  and  $\tau_2$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  and  $y \in (G, E)$  and  $x \notin (G, E)$ . Similarly,  $\tau_2$  is said to be soft b  $T_1$  space with respect to  $\tau_1$  if for each pair of distinct points  $x, y \in X$  there exist a  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  and  $y \in (G, E)$  and  $x \notin (G, E)$ . Soft bi topological spaces  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_1$  space if  $\tau_1$  is soft b  $T_1$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_1$  space with respect to  $\tau_1$ .

3)  $\tau_1$  is said to be soft b  $T_2$  space with respect to  $\tau_2$  if for each pair of distinct points  $x, y \in X$  there exists a  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \in (G, E)$ ,  $(F, E) \cap (G, E) = \phi$ . Similarly,  $\tau_2$  is said to be soft b  $T_2$  space with respect to  $\tau_1$  if for each pair of distinct points  $x, y \in X$  there exist a  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \in (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . The soft bi topological space  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_2$  space if  $\tau_1$  is soft b  $T_2$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_2$  space with respect to  $\tau_1$ .

**Definition 35.** In a soft bi topological space  $(X, \tau_1, \tau_2, E)$

1)  $\tau_1$  is said to be soft b  $T_3$  space with respect to  $\tau_2$  if  $\tau_1$  is soft b  $T_1$  space with respect to  $\tau_2$  and for each pair of distinct points  $x, y \in X$ , there exists a  $\tau_1$  soft b closed soft set  $(G, E)$  such that  $x \notin (G, E)$ ,  $\tau_1$  soft b open set  $(F_1, E)$  and  $\tau_2$  soft b open set  $(F_2, E)$  such that  $x \in (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Similarly,  $\tau_2$  is said to be soft b  $T_3$  space with respect to  $\tau_1$  if  $\tau_2$  is soft b  $T_1$  space with respect to  $\tau_1$  and for each pair of distinct points  $x, y \in X$  there exists a  $\tau_2$  soft b closed set  $(G, E)$  such that  $x \notin (G, E)$ ,  $\tau_2$  soft b open set  $(F_1, E)$  and  $\tau_1$  soft b open set  $(F_2, E)$  such that  $x \in (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ .  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_3$  space if  $\tau_1$  is soft b  $T_3$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_3$  space with respect to  $\tau_1$ .

2)  $\tau_1$  is said to be soft b  $T_4$  space with respect to  $\tau_2$  if  $\tau_1$  is soft b  $T_1$  space with respect to  $\tau_2$ , there exists a  $\tau_1$  soft b closed set  $(F_1, E)$  and  $\tau_2$  soft b closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$  also there exists  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_1$  b open set,  $(G_1, E)$  is soft  $\tau_2$  b open set such that  $(F_1, E) \subseteq (F_3, E)$ ,  $(F_2, E) \subseteq (G_1, E)$ . Similarly,  $\tau_2$  is said to be soft b  $T_4$  space with respect to  $\tau_1$  if  $\tau_2$  is soft b  $T_1$  space with respect to  $\tau_1$ , there exists  $\tau_2$  soft b closed set  $(F_1, E)$  and  $\tau_1$  soft b closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$  also there exists  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_2$  b open set,  $(G_1, E)$  is soft  $\tau_1$  b open set such that  $(F_1, E) \subseteq (F_3, E)$ ,  $(F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$ . Thus,  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_4$  space if  $\tau_1$  is soft b  $T_4$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_4$  space with respect to  $\tau_1$ .

**Proposition 2.** Let  $(Y, \tau_Y, E)$  be a soft sub space of a soft topological space  $(X, \tau, E)$  and  $(F, E) \in SS(X)$  then

1. If  $(F, E)$  is b open soft set in  $Y$  and  $Y \in \tau$ , then  $(F, E) \in \tau$
2.  $(F, E)$  is b open soft set in  $Y$  if and only if  $(F, E) = Y \cap (G, E)$  for some  $(G, E) \in \tau$ .
3.  $(F, E)$  is b closed soft set in  $Y$  if and only if  $(F, E) = Y \cap (H, E)$  for some  $(H, E) \in \tau$  soft b close set.

**Proof.** 1) Let  $(F, E)$  be a soft open set in  $Y$ , then there does exists a soft b open set  $(G, E)$  in  $X$  such that  $(F, E) = Y \cap (G, E)$ . Now, if  $Y \in \tau$  then  $Y \cap (G, E) \in \tau$  by the third condition of the definition of a soft topological space and hence  $(F, E) \in \tau$ .

2) Follows from the definition of a soft subspace.

3) If  $(F, E)$  is soft b closed in  $Y$  then we have  $(F, E) = Y \setminus (G, E)$ , for some  $(G, E) \in \tau_Y$ . Now,  $(G, E) = Y \cap (H, E)$  for some soft b open set  $(H, E) \in \tau$ . for any  $\alpha \in E$ .  $F(\alpha) = Y(\alpha) \setminus G(\alpha) = Y \setminus G(\alpha) = Y \setminus (Y(\alpha) \cap H(\alpha)) = Y \setminus (Y \cap H(\alpha)) = Y \setminus H(\alpha) = Y \cap (X \setminus H(\alpha)) = Y \cap (H(\alpha))^c = Y \cap (H(\alpha))^c$ . Thus  $(F, E) = Y \cap (H, E)$  is soft b closed in  $X$  as  $(H, E) \in \tau$ . Conversely, suppose that  $(F, E) = Y \cap (G, E)$  for some soft b closed set  $(G, E)$  in  $X$ . This qualifies us to say that  $(G, E)' \in \tau$ . Now, if  $(G, E) = (X, E) \setminus (H, E)$  where  $(H, E)$  is soft b open set in  $\tau$  then for any  $\alpha \in E$   $F(\alpha) = Y(\alpha) \cap G(\alpha) = Y \cap G(\alpha) = Y \cap (X(\alpha) \setminus H(\alpha)) = Y \cap (X(\alpha) \setminus H(\alpha)) = Y \cap (X \setminus H(\alpha)) = Y \setminus H(\alpha) = Y \setminus (Y \cap H(\alpha)) = Y \setminus (Y(\alpha) \cap H(\alpha))$ . Thus  $(F, E) = Y \setminus (Y \cap (H, E))$ . Since  $(H, E) \in \tau$ ,  $So(Y \cap (H, E)) \in \tau$ . So  $(Y \cap (H, E)) \in \tau_Y$  and hence  $(F, E)$  is soft b closed in  $Y$ .

**Proposition 3.** Let  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over  $X$ . Then, if  $(X, \tau_1, E)$  and  $(X, \tau_2, E)$  are soft b  $T_3$  space then  $(X, \tau_1, \tau_2, E)$  is a pair wise soft b  $T_3$  space.

**Proof:** Suppose  $(X, \tau_1, E)$  is a soft b  $T_3$  space with respect to  $(X, \tau_2, E)$  then according to definition for  $x, y \in X, x \neq y$ , by using Theorem 1,  $(y, E)$  is soft b closed set in  $\tau_2$  and  $x \notin (y, E)$  there exists a  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $x \in (F, E)$ ,  $y \in (y, E) \subseteq (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . Hence  $\tau_1$  is soft b  $T_2$  space with respect to  $\tau_2$ . Similarly, if  $(X, \tau_2, E)$  is a soft b  $T_3$  space with respect to  $(X, \tau_1, E)$  then according to definition for  $x, y \in X, x \neq y$ , by using Theorem 1,  $(x, E)$  is b closed soft set in  $\tau_1$  and  $y \notin (x, E)$  there exists a  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $y \in (F, E)$ ,  $x \in (x, E) \subseteq (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . Hence  $\tau_2$  is soft b  $T_2$  space. this implies that  $(X, \tau_1, \tau_2, E)$  is a pair wise soft b  $T_2$  space.

**Proposition 4.** Let  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over  $X$ . if  $(X, \tau_1, E)$  and  $(X, \tau_2, E)$  are soft b  $T_3$  space then  $(X, \tau_1, \tau_2, E)$  is a pair wise soft b  $T_3$  space.

**Proof:** Suppose  $(X, \tau_1, E)$  is a soft b  $T_3$  space with respect to  $(X, \tau_2, E)$  then according to definition for  $x, y \in X, x \neq y$  there exists a  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  and for each point  $x \in X$  and each  $\tau_1$  b closed soft set  $(G_1, E)$  such that  $x \notin (G_1, E)$  there exists a  $\tau_1$  soft b open set  $(F_1, E)$  and  $\tau_2$  soft b open set  $(F_2, E)$  such that  $x \in (F_1, E)$ ,  $(G_1, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Similarly,  $(X, \tau_2, E)$  is a soft b  $T_3$  space with respect to  $(X, \tau_1, E)$ . So according to definition for  $x, y \in X, x \neq y$  there exists  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  and for each point  $x \in X$  and each  $\tau_2$  b closed soft set  $(G_1, E)$  such that  $x \notin (G_1, E)$  there exists a  $\tau_2$  soft

b open set  $(F_1, E)$  and a  $\tau_1$  soft b open set  $(F_2, E)$  such that  $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Hence  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_3$  space.

**Proposition 5.** If  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over X if  $(X, \tau_1, E)$  and  $(X, \tau_2, E)$  are soft b  $T_4$  space then  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_4$  space.

**Proof:** Suppose  $(X, \tau_1, E)$  is soft b  $T_4$  space with respect to  $(X, \tau_2, E)$ . So according to definition for  $x, y \in X, x \neq y$  there exist a  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  each  $\tau_1$  soft b closed set  $(F_1, E)$  and a  $\tau_2$  soft b closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . There exist  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_2$  b open set  $(G_1, E)$  is soft  $\tau_1$  b open set  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$ . Similarly,  $\tau_2$  is soft b  $T_4$  space with respect to  $\tau_1$  so according to definition for  $x, y \in X, x \neq y$  there exists a  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  and for each  $\tau_2$  soft b closed set  $(F_1, E)$  and  $\tau_1$  soft b closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . there exists  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_2$  b open set  $(G_1, E)$  is soft  $\tau_1$  b open set such that  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$  hence  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_4$  space.

**Proposition 6.** Let  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over X and Y be a non-empty subset of X. if  $(X, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_3$  space. Then  $(Y, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_3$  space.

**Proof:** First we prove that  $(Y, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_1$  space. Let  $x, y \in X, x \neq y$  if  $(X, \tau_1, \tau_2, E)$  is pair wise space then this implies that  $(X, \tau_1, \tau_2, E)$  is pair wise soft  $\tau_1$  space. So there exists  $\tau_1$  soft b open  $(F, E)$  and  $\tau_2$  soft b open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  now  $x \in Y$  and  $x \notin (G, E)$ . Hence  $x \in Y \cap (F, E) = (Y_F, E)$  then  $y \notin Y \cap (F, E)$  for some  $\alpha \in E$ . this means that  $\alpha \in E$  then  $y \notin Y \cap F(\alpha)$  for some  $\alpha \in E$ .

Therefore,  $y \notin Y \cap (F, E) = (Y_F, E)$ . Now  $y \in Y$  and  $y \in (G, E)$  hence  $y \in Y \cap (G, E) = (Y_G, E)$  where  $(G, E) \in \tau_2$ . Consider  $x \notin (G, E)$  this means that  $\alpha \in E$  then  $x \notin Y \cap G(\alpha)$  for some  $\alpha \in E$ . There fore  $x \notin Y \cap (G, E) = (Y_G, E)$  thus  $(Y, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_1$  space.

Now we prove that  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_3$  space then  $(X, \tau_1, \tau_2, E)$  is pair wise soft b regular space.

Let  $y \in Y$  and  $(G, E)$  be a soft b closed set in Y such that  $y \notin (G, E)$  where  $(G, E) \in \tau_1$  then  $(G, E) = (Y, E) \cap (F, E)$  for some soft b closed set in  $\tau_1$ . hence  $y \notin (Y, E) \cap (F, E)$  but  $y \in (Y, E)$ , so  $y \notin (F, E)$  since  $(X, \tau_1, \tau_2, E)$  is soft b  $T_3$  space

$(X, \tau_1, \tau_2, E)$  is soft b regular space so there exists  $\tau_1$  soft b open set  $(F_1, E)$  and  $\tau_2$  soft b open set  $(F_2, E)$  such that

$$y \in (F_1, E), (G, E) \subseteq (F_2, E) \\ (F_1, E) \cap (F_2, E) = \phi$$

Take  $(G_1, E) = (Y, E) \cap (F_2, E)$  then  $(G_1, E), (G_2, E)$  are soft b open set in Y such that

$$y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) \\ = (G_2, E) \\ (G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi \\ (G_1, E) \cap (G_2, E) = \phi$$

There fore  $\tau_{1Y}$  is soft b regular space with respect to  $\tau_{2Y}$ . similarly, Let  $y \in Y$  and  $(G, E)$  be a soft b closed sub set in Y. such that  $y \notin (G, E)$ , where  $(G, E) \in \tau_2$  then  $(G, E) = (Y, E) \cap (F, E)$  where  $(F, E)$  is some soft b closed set in  $\tau_2$ .  $y \notin (Y, E) \cap (F, E)$  But  $y \in (Y, E)$  so  $y \notin (F, E)$  since  $(X, \tau_1, \tau_2, E)$  is soft b regular space so there exists  $\tau_2$  soft b open set  $(F_1, E)$  and  $\tau_1$  soft b open set  $(F_2, E)$ . Such that

$$y \in (F_1, E), (G, E) \subseteq (F_2, E) \\ (F_1, E) \cap (F_2, E) = \phi$$

Take

$$(G_1, E) = (Y, E) \cap (F_1, E) \\ (G_1, E) = (Y, E) \cap (F_1, E)$$

Then  $(G_1, E)$  and  $(G_2, E)$  are soft b open set in Y such that

$$y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = \\ (G_2, E)$$

$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$$

There fore  $\tau_{2Y}$  is soft b regular space with respect to  $\tau_{1Y}$ .

$\Rightarrow (X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_3$  space.

**Proposition 7.** Let  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over X and Y be a soft b closed sub space of X. if  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_4$  space then  $(Y, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_4$  space.

**Proof:** Since  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_4$  space so this implies that  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_1$  space as proved above.

We prove  $(X, \tau_1, \tau_2, E)$  is pair wise soft b normal space.

Let  $(G_1, E), (G_2, E)$  be soft b closed sets in Y such that

$$(G_1, E) \cap (G_2, E) = \phi \\ (G_1, E) = (Y, E) \cap (F_1, E) \\ (G_2, E) = (Y, E) \cap (F_2, E)$$

Then

And

For some soft b closed sets such that  $(F_1, E)$  is soft b closed set in  $\tau_1$   $(F_2, E)$

is soft b closed set in  $\tau_2$ .

And  $(F_1, E) \cap (F_2, E) = \phi$

From **Proposition 2.** Since, Y is soft b closed sub set of X then  $(G_1, E), (G_2, E)$  are soft b closed sets in X such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since  $(X, \tau_1, \tau_2, E)$  is pair wise soft b normal space. So there exists soft b open sets  $(H_1, E)$  and  $(H_2, E)$  such that

$$(H_1, E) \text{ is soft b open set in } \tau_1 \text{ and } (H_2, E) \text{ is soft b open set in } \tau_2 \text{ such that}$$

$$(G_1, E) \subseteq (H_1, E)$$

$$(G_2, E) \subseteq (H_2, E)$$

$$(H_1, E) \cap (H_2, E) = \phi$$

Since

$$(G_1, E), (G_2, E) \subseteq (Y, E)$$

Then

$$(G_1, E) \subseteq (Y, E) \cap (H_1, E)$$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And

$$[(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$$

Where  $(Y, E) \cap (H_1, E)$  and  $(Y, E) \cap (H_2, E)$  are soft b open sets in Y there fore  $\tau_{1Y}$  is soft b normal space with respect to  $\tau_{2Y}$ . Similarly, let

$$(G_1, E), (G_2, E) \text{ be soft b closed sub set in } Y \text{ such that}$$

$$(G_1, E) \cap (G_2, E) = \phi$$

Then

$$(G_1, E) = (Y, E) \cap (F_1, E)$$

And

$$(G_2, E) = (Y, E) \cap (F_2, E)$$

For some soft b closed sets such that  $(F_1, E)$  is soft b closed set in  $\tau_2$   $(F_2, E)$  soft b closed set in  $\tau_1$  and

$$(F_1, E) \cap (F_2, E) = \phi$$

From **Proposition 2.** Since, Y is soft b closed sub set in X then  $(G_1, E), (G_2, E)$  are soft b closed sets in X such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since  $(X, \tau_1, \tau_2, E)$  is pair wise soft b normal space so there exists soft b open sets  $(H_1, E)$  and  $(H_2, E)$

Such that  $(H_1, E)$  is soft b open set is  $\tau_2$  and  $(H_2, E)$  is soft b open set in  $\tau_1$  such that

$$(G_1, E) \subseteq (H_1, E)$$

$$(G_2, E) \subseteq (H_2, E)$$

$$(H_1, E) \cap (H_2, E) = \phi$$

Since

$$(G_1, E), (G_2, E) \subseteq (Y, E)$$

Then

$$(G_1, E) \subseteq (Y, E) \cap (H_1, E)$$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And

$$[(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$$

Where  $(Y, E) \cap (H_1, E)$  and  $(Y, E) \cap (H_2, E)$  are soft b open sets in Y there fore  $\tau_{2Y}$  is soft b normal space with respect to  $\tau_{1Y}$ .

$\Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_4$  space.

### 3.2 Soft b-Separation axioms with respect to ordinary points

In this section, we introduced soft b separation axioms in soft topology and in soft bi topology with respect to soft points. With the application of these soft b separation axioms different result are discussed.

**Definition 36.** Let  $(X, \tau, A)$  be a soft Topological space over X and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if there can happen at least one soft b open set  $(F_1, A)$  OR  $(F_2, A)$  such that  $e_G \in (F_1, A), e_H \notin (F_1, A)$  or  $e_H \in (F_2, A), e_G \notin ((F_2, A))$  then  $(X, \tau, A)$  is called a soft b  $T_0$  space.

**Definition 37.** Let  $(X, \tau, A)$  be a soft Topological spaces over X and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if there can happen soft b open sets  $(F_1, A)$  and  $(F_2, A)$  such that  $e_G \in (F_1, A), e_H \notin (F_1, A)$  and  $e_H \in (F_2, A), e_G \notin ((F_2, A))$  then  $(X, \tau, A)$  is called soft b  $T_1$  space.

**Definition 38.** Let  $(X, \tau, A)$  be a soft Topological space over X and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if there can happen soft b open sets  $(F_1, A)$  and  $(F_2, A)$  such that  $e_G \in (F_1, A)$ , and  $e_H \in (F_2, A)$   $(F_1, A) \cap (F_2, A) = \phi_A$  Then  $(X, \tau, A)$  is called soft b  $T_2$  space.

**Definition 39.** Let  $(X, \tau, E)$  be a soft topological space  $(G, E)$  be b closed soft set in X and  $e_G \in X_A$  such that  $e_G \notin (G, E)$ . If there occurs soft b open sets  $(F_1, E)$  and  $(F_2, E)$  such that  $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Then  $(X, \tau, E)$  is called soft b regular spaces. A soft b regular  $T_1$  Space is called soft b  $T_3$  space.

**Definition 40.** In a soft bi topological space  $(X, \tau_1, \tau_2, E)$

1)  $\tau_1$  said to be soft b  $T_0$  space with respect to  $\tau_2$  if for each pair of distinct points  $e_G, e_H \in X_A$  there happens  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (G, E)$ , Similarly,  $\tau_2$  is said to be soft b  $T_0$  space with respect to  $\tau_1$  if for each pair of distinct points  $e_G, e_H \in X_A$  there happens  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$ . Soft bi topological spaces  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_0$  space if  $\tau_1$  is soft b  $T_0$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_0$  spaces with respect to  $\tau_1$ .

2)  $\tau_1$  is said to be soft b  $T_1$  space with respect to  $\tau_2$  if for each pair of

distinct points  $e_G, e_H \in X_A$  there happens a  $\tau_1$  soft b open set  $(F, E)$  and  $\tau_2$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (G, E)$  and  $e_H \in (G, E)$  and  $e_G \notin (G, E)$ . Similarly,  $\tau_1$  is said to be soft b  $T_1$  space with respect to  $\tau_1$  if for each pair of distinct points  $e_G, e_H \in X_A$  there exist a  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (G, E)$  and  $e_H \in (G, E)$  and  $e_G \notin (G, E)$ . Soft bi topological spaces  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_1$  space if  $\tau_1$  is soft b  $T_1$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_1$  spaces with respect to  $\tau_1$ .

3)  $\tau_1$  is said to be soft b  $T_2$  space with respect to  $\tau_2$ , if for each pair of distinct points  $e_G, e_H \in X_A$  there happens a  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (G, E)$  and  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . Similarly,  $\tau_2$  is said to be soft b  $T_2$  space with respect to  $\tau_1$  if for each pair of distinct points  $e_G, e_H \in X_A$  there happens a  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \in (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . The soft bi topological space  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T$  space if  $\tau_1$  is soft b  $T_2$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_2$  space with respect to  $\tau_1$ .

**Definition 41.** In a soft bi topological space  $(X, \tau_1, \tau_2, E)$

1)  $\tau_1$  is said to be soft b  $T_3$  space with respect to  $\tau_2$  if  $\tau_1$  is soft b  $T_1$  space with respect to  $\tau_2$  and for each pair of distinct points  $e_G, e_H \in X_A$ , there exists a  $\tau_1$  b closed soft set  $(G, E)$  such that  $e_G \notin (G, E)$ ,  $\tau_1$  soft b open set  $(F_1, E)$  and  $\tau_2$  soft b open set  $(F_2, E)$  such that  $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Similarly,  $\tau_2$  is said to be soft b  $T_3$  space with respect to  $\tau_1$  if  $\tau_2$  is soft b  $T_1$  space with respect to  $\tau_1$  and for each pair of distinct points  $e_G, e_H \in X_A$  there exists a  $\tau_2$  soft b closed set  $(G, E)$  such that  $e_G \notin (G, E)$ ,  $\tau_2$  soft b open set  $(F_1, E)$  and  $\tau_1$  soft b open set  $(F_2, E)$  such that  $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ .  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_3$  space if  $\tau_1$  is soft b  $T_3$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_3$  space with respect to  $\tau_1$ .

2)  $\tau_1$  is said to be soft b  $T_4$  space with respect to  $\tau_2$  if  $\tau_1$  is soft b  $T_1$  space with respect to  $\tau_2$ , there exists a  $\tau_1$  soft b closed set  $(F_1, E)$  and  $\tau_2$  soft b closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$  also there exists  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_1$  b open set,  $(G_1, E)$  is soft  $\tau_2$  b open set such that  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ . Similarly,  $\tau_2$  is said to be soft b  $T_4$  space with respect to  $\tau_1$  if  $\tau_2$  is soft b  $T_1$  space with respect to  $\tau_1$ , there exists  $\tau_2$  soft b closed set  $(F_1, E)$  and  $\tau_1$  soft b closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . Also there exists  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_2$  b open set,  $(G_1, E)$  is soft  $\tau_1$  b soft set such that  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$ . Thus,  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft b  $T_4$  space if  $\tau_1$  is soft b  $T_4$  space with respect to  $\tau_2$  and  $\tau_2$  is soft b  $T_4$  space with respect to  $\tau_1$ .

**Proposition 8.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ . If  $(X, \tau, E)$  is soft b  $T_3$ -space, then for all  $e_G \in X_E$   $e_G = (e_G, E)$  is b-closed soft set.

**Proof:** We want to prove that  $e_G$  is b-closed soft set, which is sufficient to prove that  $e_G^c$  is b-open soft set for all  $e_H \in \{e_G\}^c$ . Since  $(X, \tau, E)$  is soft b  $T_3$ -space, then there exists soft b set sets  $(F, E)_{e_H}$  and  $(G, E)$  such that  $e_{H_E} \subseteq (F, E)_{e_H}$  and  $e_{G_E} \cap (F, E)_{e_H} = \phi$  and  $e_{G_E} \subseteq (G, E)$  and  $e_{H_E} \cap (G, E) = \phi$ . It follows that,  $\cup_{e_H \in \{e_G\}^c} (F, E)_{e_H} = e_{G_E}^c$ . Now, we want to prove that  $e_{G_E}^c \subseteq \cup_{e_H \in \{e_G\}^c} (F, E)_{e_H}$ . Let  $\cup_{e_H \in \{e_G\}^c} (F, E)_{e_H} = (H, E)$ . Where  $H(e) = \cup_{e_H \in \{e_G\}^c} (F, E)_{e_H}$  for all  $e \in E$ . Since  $e_{G_E}^c(e) = (e_G)^c$  for all  $e \in E$  from Definition 9, so, for all  $e_H \in \{e_G\}^c$  and  $e \in E$   $e_{G_E}^c(e) = \{e_G\}^c = \cup_{e_H \in \{e_G\}^c} \{e_H\} = \cup_{e_H \in \{e_G\}^c} (F, E)_{e_H} = H(e)$ . Thus,  $e_{G_E}^c \subseteq \cup_{e_H \in \{e_G\}^c} (F, E)_{e_H}$  from Definition 2, and so,  $e_{G_E}^c = \cup_{e_H \in \{e_G\}^c} (F, E)_{e_H}$ .

This means that,  $e_{G_E}^c$  is soft b-open set for all  $e_H \in \{e_G\}^c$ . Therefore,  $e_{G_E}$  is b-closed soft set.

**Proposition 9.** Let  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over  $X$ . Then, if  $(X, \tau_1, E)$  and  $(X, \tau_2, E)$  are soft b  $T_3$  space, then  $(X, \tau_1, \tau_2, E)$  is a pair wise soft b  $T_2$  space.

**Proof:** Suppose  $(X, \tau_1, E)$  is a soft b  $T_3$  space with respect to  $(X, \tau_2, E)$ , then according to definition for,  $e_G \neq e_H, e_G, e_H \in X_A$ , by using Theorem 8,  $(e_H, E)$  is soft b closed set in  $\tau_2$  and  $e_G \notin (e_H, E)$  there exists a  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $e_G \in (F, E), e_H \in (Y, E) \subseteq (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . Hence,  $\tau_1$  is soft b  $T_2$  space with respect to  $\tau_2$ . Similarly, if  $(X, \tau_2, E)$  is a soft b  $T_3$  space with respect to  $(X, \tau_1, E)$ , then according to definition for,  $e_G \neq e_H, e_G, e_H \in X_A$ , by using Theorem 8,  $(e_G, E)$  is b closed soft set in  $\tau_1$  and  $y \notin (x, E)$  there exists a  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $e_H \in (F, E), e_G \in (x, E) \subseteq (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . Hence,  $\tau_2$  is a soft b  $T_2$  space. Thus  $(X, \tau_1, \tau_2, E)$  is a pair wise soft b  $T_2$  space.

**Proposition 10.** Let  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over  $X$ . if  $(X, \tau_1, E)$  and  $(X, \tau_2, E)$  are soft b  $T_3$  space then  $(X, \tau_1, \tau_2, E)$  is a pair wise soft b  $T_3$  space.

**Proof:** Suppose  $(X, \tau_1, E)$  is a soft b  $T_3$  space with respect to  $(X, \tau_2, E)$  then

according to definition for  $e_G, e_H \in X_A$   $e_G \neq e_H$  there happens a  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  and for each point  $e_G \in X_A$  and each  $\tau_1$  b closed soft set  $(G_1, E)$  such that  $e_G \notin (G_1, E)$  there happens a  $\tau_1$  soft b open set  $(F_1, E)$  and  $\tau_2$  soft b open set  $(F_2, E)$  such that  $e_G \in (F_1, E), (G_1, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Similarly,  $(X, \tau_2, E)$  is a soft  $\beta_3$  space with respect to  $(X, \tau_1, E)$ . So according to definition for  $e_G, e_H \in X_A$   $e_G \neq e_H$  there exists  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $e_H \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  and for each point  $e_G \in X_A$  and each  $\tau_2$  b closed soft set  $(G_1, E)$  such that  $e_G \notin (G_1, E)$  there exists a  $\tau_2$  soft b open set  $(F_1, E)$  and a  $\tau_1$  soft b open set  $(F_2, E)$  such that  $e_G \in (F_1, E), (G_1, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Hence  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_3$  space.

**Proposition 11.** If  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over  $X$  if  $(X, \tau_1, E)$  and  $(X, \tau_2, E)$  are soft b  $T_4$  space then  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_4$  space.

**Proof:** Suppose  $(X, \tau_1, E)$  is soft b  $T_4$  space with respect to  $(X, \tau_2, E)$ . So according to definition for  $e_G, e_H \in X_A$   $e_G \neq e_H$  there happens a  $\tau_1$  soft b open set  $(F, E)$  and a  $\tau_2$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  each  $\tau_1$  soft b closed set  $(F_1, E)$  and a  $\tau_2$  soft b closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . There occurs  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_2$  b open set  $(G_1, E)$  is soft  $\tau_1$  b open set  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$ . Similarly,  $\tau_2$  is soft b  $T_4$  space with respect to  $\tau_1$  so according to definition for  $e_G, e_H \in X_A$   $e_G \neq e_H$  there happens a  $\tau_2$  soft b open set  $(F, E)$  and a  $\tau_1$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  and for each  $\tau_2$  soft b closed set  $(F_1, E)$  and  $\tau_1$  soft b closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . there occurs  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_2$  b open set  $(G_1, E)$  is soft  $\tau_1$  b open set such that  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$  hence  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_4$  space.

**Proposition 12.** Let  $(X, \tau_1, \tau_2, E)$  be a soft bi topological space over  $X$  and  $Y$  be a non-empty subset of  $X$ . if  $(X, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_3$  space. Then  $(Y, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_3$  space.

**Proof:** first we prove that  $(Y, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_1$  space. Let  $e_G, e_H \in X_A$   $e_G \neq e_H$  if  $(X, \tau_1, \tau_2, E)$  is pair wise space then this implies that  $(X, \tau_1, \tau_2, E)$  is pair wise soft  $\tau_1$  space. So there exists  $\tau_1$  soft b open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  now  $e_G \in Y$  and  $e_G \notin (G, E)$ . Hence  $e_G \in Y \cap (F, E) = (Y, F, E)$  then  $e_H \notin Y \cap F(\alpha)$  for some  $\alpha \in E$ . this means that  $\alpha \in E$  then  $e_H \notin Y \cap F(\alpha)$  for some  $\alpha \in E$ .

There fore,  $e_H \notin Y \cap (F, E) = (Y, F, E)$ . Now  $e_H \in Y$  and  $e_H \in (G, E)$ . Hence,  $e_H \in Y \cap (G, E) = (G, Y, E)$  where  $(G, E) \in \tau_2$ . Consider  $x \notin (G, E)$ . this means that  $\alpha \in E$  then  $x \notin Y \cap G(\alpha)$  for some  $\alpha \in E$ . There fore  $e_G \notin Y \cap (G, E) = (G, Y, E)$  thus  $(Y, \tau_{1Y}, \tau_{2Y}, E)$  is pair wise soft b  $T_1$  space.

Now, we prove that  $(X, \tau_1, \tau_2, E)$  is pair wise soft b  $T_3$  space then  $(X, \tau_1, \tau_2, E)$  is pair wise soft b regular space.

Let  $e_H \in Y$  and  $(G, E)$  be soft b closed set in  $Y$  such that  $e_H \notin (G, E)$  where  $(G, E) \in \tau_1$  then  $(G, E) = (Y, E) \cap (F, E)$  for some soft b closed set in  $\tau_1$ . hence  $e_H \notin (Y, E) \cap (F, E)$  but  $e_H \in (Y, E)$ , so  $e_H \notin (F, E)$  since  $(X, \tau_1, \tau_2, E)$  is soft b  $T_3$  space  $(X, \tau_1, \tau_2, E)$  is soft b regular space so there happens  $\tau_1$  soft b open set  $(F_1, E)$  and  $\tau_2$  soft b open set  $(F_2, E)$  such that

$$\begin{aligned} e_H \in (F_1, E), (G, E) \subseteq (F_2, E) \\ (F_1, E) \cap (F_2, E) = \phi \end{aligned}$$

Take  $(G_1, E) = (Y, E) \cap (F_2, E)$  then  $(G_1, E), (G_2, E)$  are soft b open sets in  $Y$  such that

$$\begin{aligned} e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) \\ = (G_2, E) \\ (G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi \\ (G_1, E) \cap (G_2, E) = \phi \end{aligned}$$

Therefore,  $\tau_{1Y}$  is soft b regular space with respect to  $\tau_{2Y}$ . Similarly, Let  $e_H \in Y$  and  $(G, E)$  be a soft b closed sub set in  $Y$  such that  $e_H \notin (G, E)$ , where  $(G, E) \in \tau_2$  then  $(G, E) = (Y, E) \cap (F, E)$  where  $(F, E)$  is some soft b closed set in  $\tau_2$ .  $e_H \notin (Y, E) \cap (F, E)$  but  $e_H \in (Y, E)$  so  $e_H \notin (F, E)$  since  $(X, \tau_1, \tau_2, E)$  is soft b regular space so there happens  $\tau_2$  soft b open set  $(F_1, E)$  and  $\tau_1$  soft b open set  $(F_2, E)$ . Such that

$$\begin{aligned} e_H \in (F_1, E), (G, E) \subseteq (F_2, E) \\ (F_1, E) \cap (F_2, E) = \phi \end{aligned}$$

Take

$$\begin{aligned} (G_1, E) = (Y, E) \cap (F_1, E) \\ (G_1, E) = (Y, E) \cap (F_1, E) \end{aligned}$$

Then  $(G_1, E)$  and  $(G_2, E)$  are soft  $\beta$  open set in  $Y$  such that

$$e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) =$$

$(G_2, E)$

$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$$

Therefore  $\tau_{2Y}$  is soft b regular space.

#### 4. CONCLUSION

Topology is the most important branch of mathematics which deals with mathematical structures. Recently, many researchers have studied the soft set theory which is initiated by Molodtsov and safely applied to many problems which contain uncertainties in our social life. Some researchers introduced and deeply studied the concept of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we have continued to study the properties of soft  $b$  separation axioms in soft bi topological spaces with respect to soft points as well as ordinary points of a soft topological spaces. We defined soft  $T_0$ ,  $T_1$ ,  $T_2$  and  $T_3$  spaces with respect to soft points and studied their behaviors in soft bi topological spaces. We also extended these axioms to different results. These soft separation axioms would be useful for the growth of the theory of soft topology to solve complex problems, comprising doubts in economics, engineering, medical etc. We also beautifully discussed some soft transmissible properties with respect to ordinary as well as soft points. We hope that these results in this paper will help the researchers for strengthening the toolbox of soft topology. In the next study, we extend the concept of soft semi open,  $\alpha$ -open, Pre-open and  $b^{**}$ -open soft sets in soft bi topological spaces with respect to ordinary as well as soft points

#### REFERENCES

- [1] Molodtsov, D.A. 1999. Soft set theory first results. Computers and Mathematics with Applications, 37, 19-31. [36] Ittanagi, B.M. 2014. Soft bi-topological spaces. International Journal of Computer Application, 107 (7), 1-4.
- [2] Ahmad, B., Kharal, A. 2011. Mapping of soft classes. New Mathematics and Natural Computation, 7 (3), 471-481.
- [3] Maji, P.K., Biswas, R., Roy, A.R. 2003. Soft set theory. Computers and Mathematics with Applications, 45, 555-562.
- [4] Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M. 2009. On some new operations in soft set theory. Computers and Mathematics with Applications, 57, 1547-1553.
- [5] Kovkov, D.V., Kolbanov, V.M., Moldtsov, D.A. 2007. Soft set theory-based optimization. Journal of Computer and System, 46 (6), 872-880.
- [6] Xiao, Z., Chen, L., Zhong, B., Ye, S. 2005. Recognition for soft information based on the theory of soft sets. J. Chen (Ed) Proceeding of ICSSM, 1404 -1106.
- [7] Pei, D., Miao, D. 2005. From soft sets to information systems, in: Hu, X., Liu, Q., Skowron, A., Lin, T.Y., Yager, R.R., Zhang, B. (Eds). Proceedings of Granular computing in: IEEE, 21, 617-621.
- [8] Ahmad, B., Kharal, B. 2009. On fuzzy soft sets. Advances in Fuzzy Systems, Avt, ID 586507, 6PP.
- [9] Aktas, H., Agman, W. 2007. Soft sets and soft groups. Information Sciences, 1(77), 2726-2735.
- [10] Cagman, N., Citak, F., Enginoglu, S. 2010. Fuzzy parameterized fuzzy soft sets theory and its application Turkish. Journal of Fuzzy systems, 1 (1), 21-35.
- [11] Kandil, A., Tantawy, O.A.E., El-Sheikh, S.A., Abd El-latif, A.M. 2015. Fuzzy soft semi connection properties in fuzzy soft topological spaces. Mathematical Sciences Letters, 4, 171-179.
- [12] Levine, N. 1963. Semi open sets and semi continuity in topological spaces. The American Mathematical Monthly, 70, 36-41.
- [13] Majk, P.K., Biswas, R., Roy, A.R. 2001. Fuzzy soft sets. Journal of Fuzzy Mathematics, 9 (3), 589-602.
- [14] Maji, P.K., Biswas, R., Roy, A.R. 2001. Intuitionistic fuzzy soft sets. Journal of fuzzy Mathematics, 9 (3), 677-691.
- [15] Majumdar, P., Samanta, S.K. 2010. Gernalized fuzzy soft sets. Computers and Mathematics with Applications, 59, 1425-1432.
- [16] Andrijevic, D. 1996. On b-open sets. Matematički Vesnik, 48, 24-32.
- [17] Zou, Y., Xiao, X. 2008. Data analysis approaches of soft sets under incomplete information. Knowledge Bond systems, 21, 941-945.
- [18] Chang, C.L. 1968. Fuzzy topological spaces. Journal of Mathematical Analysis and Applications, 24, 182-190.
- [19] Shabir, M., Naz, M. 2011. On soft topological spaces. Computers and Mathematics with Applications, 61, 1786 -1799.
- [20] Kandil, A., Tantawy, O.A.E., El-Sheikh, S.A., Abd-e-Latif, A.M. 2014. R-Operation and decomposition of some forms of soft continuity in soft topological spaces. FMI, 7, 181-196.
- [21] Kandil, A., Tantawy, O.A.E., El- Sheikh, S.A., Abd-e-Latif, A.M. 2014. Soft semi- separation axioms and irresolute soft function. FMI, 8 (2), 305-318.
- [22] Kandil, A., Tantawy, O.A.E., El-Sheikh, S.A., Abd-e-Latif, A.M. 2014. Soft ideal theory, soft local function and generated soft topological spaces. Applied Mathematics and Information Sciences, 8 (4), 1595-1603.
- [23] Kandil, A., Tantawy, O.A.E., El-Sheikh, S.A., Abd-e-Latif, A.M. 2015. Operation and decomposition of some forms of soft continuity of soft topological spaces via soft ideal. FMI, (3), 385-420.
- [24] Kandil, A., Tantawy, O.A.E., El-Sheikh, S.A., Abd-e-Latif, A.M. 2014. Soft connected ness via soft ideals. Journal of New results in science, 4, 90-108.
- [25] Kandil, A., Tantawy, O.A.E., El-Sheikh, S.A., Abd-e-Latif, A.M. 2015. Soft regularity and normally based on semi – open soft sets and soft ideal. Applied Mathematics and Information Sciences, 3, 47-55.
- [26] Kandkil, A., Tantawy, O.A.E., El-Sheikh, S.A., Abd-e-Latif, A.M. 2014. Soft semi soft ideal. South Asian Journal of Mathematics, 4 (6), 265-284.
- [27] Kandil, A., Tantawy, O.A.E., El- Sheikh, S.A., Abd El-e-Latif, A.M. 2014. Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces. Applied Mathematics & Information Sciences, (4), 1731-1740.
- [28] El -Sheikh, S.A., Abd-e-Latif, A.M. 2014. Decomposition of some type of supra soft sets and soft continuity. International journal of mathematics trends and technology, 9 (1), 37 -56.
- [29] Zorlutana, I., Akdog, N., Min, W.K., Atmaca, S. 2012. Remarks on soft topological spaces. FMI, 3 (2), 171-185.
- [30] Zadeh, L.A. 1965. Fuzzy sets. Information and control, 8, 338-353.
- [31] Chen, B. 2013. Soft Semi open sets and related properties in soft topological spaces. Applied Mathematics and Information Sciences, 7 (1), 287-294.
- [32] Hussain, S., Ahmad, B. 2015. Soft separation axioms in soft topological space submitted to Elsevier 28 September.
- [33] Mahanta, J., Das, P.K. 2012. On soft topological spaces via semi open and semi close soft sets are XIV: 1203, 1203.4133 VI P [Math, GN] 16 Mar.
- [34] Lancy, A., Arockiarani. 2013. On Soft  $\beta$ -Separation Axioms. IJMRS, 1 (5).
- [35] Arockiarani, Arokialancy. 2013. A Generalized soft  $g\beta$  closed sets and soft  $gs\beta$  closed sets in soft topological spaces. International Journal of Mathematical Archive, 4 (2), 1-7.
- [37] El-Sheikh, S.A., Hosny, R.A., Abd-e-Latif, A.M. 2015. Characterization of b soft separation Axioms in soft topological spaces. Information Sciences Letters, 4 (3), 125-133.
- [38] Naz, M., Shabir, M., Ali, M.I. 2015. Separation axiom in bi soft topological spaces. Arxiv. 1850 of .00866 v1 [Math, Gn] 29 August.
- [39] Buhaescu, T.T. 1996. Some observations Intuitionistic fuzzy relations. Itimerat Seminar on Functional Equations, 111-118.
- [40] Min, W.K. 2011. A note on soft topological spaces. Computers and Mathematics with Applications, 62, 3524-3528.
- [41] Kandil, A., Tantawy, O.A.E., El-Sheikh, S.A., Abd-e-Latif, A.M. 2014. Soft semi compactness via soft ideal. Applied Mathematics and Information

---

Information Sciences, 8 (5), 2297-2306.

[42] Sun, Q.M., Zhang, Z.L., Liu, J. 2008. Soft sets and soft modules. Proceeding of rough sets and Knowledge technology, third international conference, RSKT 2008, 17-19 May, Chengdu, China, 403-409.

[43] Park, C.H., Jun, Y.B., Ozturk, M.A. Soft W.S-Algebras. Communications of the Korean Mathematical society, 23(3), 313-324.

