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RESEARCH ARTICLE

ANALYTICAL APPROXIMATE SOLUTION OF HEAT CONDUCTION EQUATION USING NEW HOMOTOPY PERTURBATION METHOD

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ABSTRACT

In this paper, the analytic solution of one-dimensional heat conduction equation is obtained by means of new homotopy perturbation method. This method is effectively applied to obtain the exact solution for the problems on hand. Some problems related to one dimensional heat equation have been discussed, which reveals the effectiveness and simplicity of the method. Numerical results have also been analysed graphically to show the rapid convergence of infinite series expansion.

KEYWORDS

New homotopy perturbation method, Heat conduction equation, Specific heat, Diffusivity, Density

1. INTRODUCTION

Various problems are modelled by linear and non-linear partial differential equations problems in the fields of physics, engineering etc. To solve such kind of partial differential equations (PDE), many methods are used to find the numerical or exact solutions. Homotopy perturbation method (HPM) is one of the methods used in recent years to solve various linear and non-linear PDE [1-3]. Initial and boundary value problems can be solved using HPM extensively. Many researchers and scientists show great interest in homotopy perturbation method. Huan was the first who described homotopy perturbation method.He showed that this method is a one of the powerful toolsused to investigate various problems which are arising nowadays. HPM is used for solving linear and non-linear ordinary and partial differential equations [4-10].

In HPM, complex linear or non-linear problem can be continuously distortedinto simpler ones. Perturbation theory and homotopy theory in topology is combined to develop homotopy perturbation method [4,11]. HPM is applicable to linear and non-linear boundary value problems [12]. The solution obtained by HPM gives the solution approximately near to the universally accepted method of separation of variable [1,13]. Recently, Biazar and Eslami proposed the new homotopy perturbation method (NHPM) [14,15]. Construction of an appropriate homotopy equation and selection of appropriate initial approximation guess are two important steps of NHPM [16]. The study reveals that with less computational work, we can construct proper homotopy by decomposition of source function in a correct way. New homotopy perturbation method is the most powerful tool which can be used to obtain analytical solution of various kinds of linear and nonlinear PDE's. This method is widely used by researchers to obtain solution of various functional equations [2].

To develop this new technique, HPM is combined with the decomposition of source function. The decomposition of a source function is the basis of homotopy used in this method because convergence of a solution is affected by the decomposition of source functions [14]. Different kind of homotopy can be formed using various decomposition of a source functions. This study is aimed at constructing suitable homotopy by decomposition of a source function which requires less computational efforts and made calculations in simpler form unlike other perturbation methods [14-16]. The obtained results directly imply the fact that NHPM is very influential as compared to HPM or any other perturbation technique. To establish exact solution of linear and non-linear problem with boundary and initial condition, new homotopy method is most appropriate method to apply [14].

This modified version is used to obtain analytic solution for one dimensional heat conduction equation with boundary and initial conditions which is same as the universally accepted exact solution. Some solved problems tell us about the capability and reliability of this method. Solutions with greater accuracy are given by this method for solving linear and non-linear different differential equations. The solution obtained using NHPMis considered in the form of an infinite series. The convergenceof solution to the exact solution is very rapid.

2. APPLICATION OF HEAT TRANSFER MODEL

One of the most important phenomena is diffusion which is employed in many industries to perform various processes used such as polymer, paper, leather, printing and textile [4-10,17]. Jean Baptiste Joseph Fourier was first who described heat conduction process. Solution of heat conduction process is obtained by many analytical and numerical methods [1,18]. In this paper, one dimensional heat conduction equation is solved using new perturbation method.

3. MATHEMATICAL MODEL OF HEAT CONDUCTION EQUATION

The one-dimensional heat equation

$$\frac{\partial U}{\partial \theta} = \beta \frac{\partial^2 U}{\partial z^2} \tag{1}$$

With B.C. and I.C.

$$U(0,\theta) = 0,\ U(1,\theta) = 0, \tag{2}$$

$$U(z,0) = h(z), \qquad 0 \le z \le 1. \tag{3}$$

4. BASIC IDEA OF NEW HOMOTOPY PERTURBATION METHOD

First of all, following homotopyis constructed for solving heat conduction equation using NHPM

$$(1-\mathrm{p})\left(\frac{\partial T}{\partial \theta} - U_0\right) + \mathrm{p}\left(\frac{\partial T}{\partial \theta} - \beta\frac{\partial^2 T}{\partial z^2}\right) = 0$$

or

$$\frac{\partial T(z,\theta)}{\partial \theta} = U_0(z,\theta) - \mathrm{p}\left(U_0 - \beta\frac{\partial^2 T}{\partial z^2}\right). \tag{4}$$

Taking $L^{-1} = \int_{\theta_0}^{\theta} (.)d\theta$ i.e. inverse operator on the both sides of equation (4), then

$$T(z,\theta) = \int_{\theta_0}^{\theta} U_0(z,\theta)d\theta - \mathrm{p}\int_{\theta_0}^{\theta}\left(U_0 - \beta\frac{\partial^2 T}{\partial z^2}\right)d\theta + T(z,\theta_0), \tag{5}$$

where, $T(z,\theta_0) = U(z,\theta_0)$.

Let the solution of equation (5) is given by

$$T = T_0 + \mathrm{p}T_1 + \mathrm{p}^2 T_2 + \mathrm{p}^3 T_3 + \dots, \tag{6}$$

where, $T_0, T_1, T_2, T_3, \dots$ are to be determined.

Suppose solution given by equation (6) is the solution of equation (5).On comparing the coefficients of powers of p and equating to zero and using equation (6) in equation (5), following are obtained:

$$\mathrm{p}^0: \ T_0(z,\theta) = \int_{\theta_0}^{\theta} U_0(z,\theta)d\theta + T(z,\theta_0)$$

$$\mathrm{p}^1: \ T_1(z,\theta) = -\int_{\theta_0}^{\theta}\left(U_0(z,\theta) - \beta\frac{\partial^2 T_0}{\partial z^2}\right)d\theta$$

$$\mathrm{p}^2: \ T_2(z,\theta) = \int_{\theta_0}^{\theta}\left(\beta\frac{\partial^2 T_1}{\partial z^2}\right)d\theta$$

$$\mathrm{p}^3: \ T_3(z,\theta) = \int_{\theta_0}^{\theta}\left(\beta\frac{\partial^2 T_2}{\partial z^2}\right)d\theta$$

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and so on... (7)

Consider the initial approximation of equation (1) as

$$U_0(z,\theta) = \sum_{n=0}^{\infty} c_n(z)P_n(\theta), \ T(z,0) = U(z,0), \ \ P_k(\theta) = \theta^k , \tag{8}$$

where, $P_1(\theta), P_2(\theta), P_3(\theta), \dots$ and $c_0(z), c_1(z), c_2(z), \dots$

arespecified functions andunknown coefficients respectively, depending on the problem.

Using equation (8) in (7), following are obtained :

$$T_0(z,\theta) = \left(c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \dots\right) + U(z,0)$$

$$T_1(z,\theta) = \left(-c_0(z) - \beta\pi^2 \sin \pi z\right)\theta + \left(-\frac{1}{2}c_1(z) + \frac{1}{2}\beta c_0''(z)\right)\theta^2 + \left(-\frac{1}{3}c_2(z) + \frac{1}{3}c_1''(z)\right)\theta^3 + \dots$$

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and so on... (9)

Now solving the above equations in such a manner that, $T_1(z,\theta) = 0$. Thereforeequation (9) reduces to

$$T_1(z,\theta) = T_2(z,\theta) = \dots = 0.$$

So $U(z,\theta) = T_0(z,\theta) = \sum_{n=0}^{\infty} c_n(z)P_n(\theta)$ is obtained solution which is found to be exactly same as the exact solution obtained through method of separation of variable.

If $U_0(z,\theta)$ is analytic at $\theta = \theta_0$,

$U_0(z,\theta) = \sum_{n=0}^{\infty} c_n(z)(\theta - \theta_0)^n$ is the taylor series expansion whichcan be used in equation (9).

5. APPLICATIONS OF NEW HOMOTOPY PERTURBATION METHOD

Problem 5.1

Consider one dimensional heat conduction equation

$$\frac{\partial U}{\partial \theta} = \beta\frac{\partial^2 U}{\partial z^2} \tag{10}$$

withB.C. and I.C.

$$U(0,\theta) = 0, \ U(1,\theta) = 0, \tag{11}$$

$$U(z,0) = \sin \pi z, \qquad 0 \le z \le 1. \tag{12}$$

First of all, following homotopy is constructed for solving heat conduction equation using NHPM

$$(1-\mathrm{p})\left(\frac{\partial T}{\partial \theta} - U_0\right) + \mathrm{p}\left(\frac{\partial T}{\partial \theta} - \beta\frac{\partial^2 T}{\partial z^2}\right) = 0$$

or

$$\frac{\partial T(z,\theta)}{\partial \theta} = U_0(z,\theta) - \mathrm{p}\left(U_0 - \beta\frac{\partial^2 T}{\partial z^2}\right). \tag{13}$$

Taking $L^{-1} = \int_{\theta_0}^{\theta} (.)d\theta$ i.e.inverse operator on the both sides of equation (13), then

$$T(z,\theta) = \int_{0}^{\theta} U_0(z,\theta)d\theta - \mathrm{p}\int_{0}^{\theta}\left(U_0 - \beta\frac{\partial^2 T}{\partial z^2}\right)d\theta + T(z,0). \tag{14}$$

Let the solution of the equation (14) is

$$T = T_0 + \mathrm{p}T_1 + \mathrm{p}^2 T_2 + \mathrm{p}^3 T_3 + \dots, \tag{15}$$

where, $T_0, T_1, T_2, \ldots$ are to be determined.

Let us suppose equation (15) is the solution of equation (14).Comparing the coefficients of powers of p and equating to zero and using equation (15) in equation (14), following are obtained:

$$p^0: \; T_0(z,\theta) = \int_0^\theta U_0(z,\theta)d\theta + T(z,0)$$

$$p^1: \; T_1(z,\theta) = -\int_0^\theta \left(U_0(z,\theta) - \beta \frac{\partial^2 T_0}{\partial z^2} \right) d\theta$$

$$p^2: \; T_2(z,\theta) = \int_0^\theta \left(\beta \frac{\partial^2 T_1}{\partial z^2} \right) d\theta$$

$$p^3: \; T_3(z,\theta) = \int_0^\theta \left(\beta \frac{\partial^2 T_2}{\partial z^2} \right) d\theta$$

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and so on... (16)

Consider initial approximation of equation (10) as

$$U_0(z,\theta) = \sum_{n=0}^{\infty} c_n(z) P_n(\theta), \;\; T(z,0) = U(z,0), \;\; P_k(\theta) = \theta^k \;, \tag{17}$$

where, $P_1(\theta), P_2(\theta), P_3(\theta), \ldots$ and $c_0(z), c_1(z), c_2(z), \ldots$ are specified functions and unknown coefficients respectively, depending on the problem.

Using equation (17) in equation (16), following are obtained:

$$T_0(z,\theta) = \left(c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \ldots \right) + \sin \pi z$$

$$T_1(z,\theta) = \left(-c_0(z) - \beta\pi^2 \sin \pi z \right)\theta + \left(-\frac{1}{2}c_1(z) + \frac{1}{2}\beta c_0''(z) \right)\theta^2 + \left(-\frac{1}{3}c_2(z) + \frac{1}{3}c_1''(z) \right)\theta^3 + \ldots$$

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and so on... (18)

Now solving the above equations in such a manner that, $T_1(z,\theta) = 0$. Then fromequation (18) we get

$$c_0(z) = -\beta\pi^2 \sin \pi z \,,$$
$$c_1(z) = \beta^2\pi^4 \sin \pi z \,,$$
$$c_2(z) = -\beta^3\pi^6 \sin \pi z \,.$$

$$\begin{aligned} U(z,\theta) = T_0(z,\theta) &= \sin \pi z + c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \ldots \\ &= \sin \pi z \left[1 - \beta\pi^2\theta + \beta^2\pi^4 \frac{\theta^2}{2} - \beta^3\pi^6 \frac{\theta^3}{3} + \ldots \right] \\ &= \sin \pi z e^{-\beta\pi^2\theta} \,, \end{aligned} \tag{19}$$

whichissame as the universally acceptedexact solution for the problem which is demonstrated by Table 1 and Figure 1.

Table 1 and Figure 1 represents the solution $U(z,\theta)$ for $0 \le z \le 1, \; 0 \le \theta \le 0.5$ and $\beta = 0.05$.

Table 1: $U(z,\theta)$ *for* $0 \le z \le 1, 0 \le \theta \le 0.5$ *for* $\beta = 0.05$

z	$\theta = 0$	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$
0	0	0	0	0	0	0
0.1	0.309017	0.279975	0.253662	0.229823	0.208224	0.188654
0.2	0.587785	0.532544	0.482495	0.437149	0.396065	0.358842
0.3	0.809017	0.732984	0.664097	0.601684	0.545136	0.493903
0.4	0.951057	0.861674	0.780693	0.707322	0.640846	0.580618
0.5	1	0.906018	0.820869	0.743722	0.673825	0.610498
0.6	0.951057	0.861674	0.780693	0.707322	0.640846	0.580618
0.7	0.809017	0.732984	0.664097	0.601684	0.545136	0.493903
0.8	0.587785	0.532544	0.482495	0.437149	0.396065	0.358842
0.9	0.309017	0.279975	0.253662	0.229823	0.208224	0.188654
1	0	0	0	0	0	0

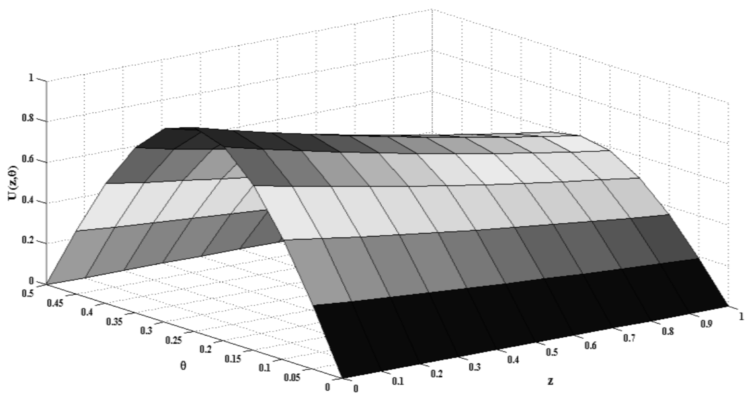

Figure 1: $U(z,\theta)$ *for* $0 \le z \le 1, 0 \le \theta \le 0.5$ *for* $\beta = 0.05$

Problem 5.2

Consider heat conduction equation

$$\frac{\partial U}{\partial \theta} = \beta \frac{\partial^2 U}{\partial z^2} \tag{20}$$

withB.C. and I.C.

$$U(0,\theta) = 0, \quad U(1,\theta) = 0\ , \tag{21}$$

$$U(z,0) = \sin 3\pi z, \qquad 0 \le z \le 1. \tag{22}$$

First of all, following homotopy is constructed for solving heat conduction equation using NHPM

$$(1-\mathrm{p})\left(\frac{\partial T}{\partial \theta} - U_0\right) + \mathrm{p}\left(\frac{\partial T}{\partial \theta} - \beta \frac{\partial^2 T}{\partial z^2}\right) = 0$$

Or

$$\frac{\partial T(z,\theta)}{\partial \theta} = U_0(z,\theta) - \mathrm{p}\left(U_0 - \beta \frac{\partial^2 T}{\partial z^2}\right). \tag{23}$$

Taking $L^{-1} = \int_{\theta_0}^{\theta} (.) d\theta$ i.e.inverse operator on the both sides of (23), then

$$T(z,\theta) = \int_0^{\theta} U_0(z,\theta) d\theta - \mathrm{p}\int_0^{\theta}\left(U_0 - \beta \frac{\partial^2 T}{\partial z^2}\right) d\theta + T(z,0)\,. \tag{24}$$

Let the solution of the (24) is

$$T = T_0 + \mathrm{p}T_1 + \mathrm{p}^2 T_2 + \mathrm{p}^3 T_3 + \ldots, \tag{25}$$

where, $T_0, T_1, T_2, \ldots$ are to be determined.

Suppose equation (25) is the solution of equation (24).Comparing the coefficients of powers of p and equating to zero and using equation (25) in equation (24), following are obtained:

$$\mathrm{p}^0: \quad T_0(z,\theta) = \int_0^{\theta} U_0(z,\theta) d\theta + T(z,0)$$

$$\mathrm{p}^1: \quad T_1(z,\theta) = -\int_0^{\theta}\left(U_0(z,\theta) - \beta \frac{\partial^2 T_0}{\partial z^2}\right) d\theta$$

$$\mathrm{p}^2: \quad T_2(z,\theta) = \int_0^{\theta}\left(\beta \frac{\partial^2 T_1}{\partial z^2}\right) d\theta$$

$$\mathrm{p}^3: \quad T_3(z,\theta) = \int_0^{\theta}\left(\beta \frac{\partial^2 T_2}{\partial z^2}\right) d\theta$$

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and so on. (26)

Consider initial approximation of equation (20) as

$$U_0(z,\theta) = \sum_{n=0}^{\infty} c_n(z) P_n(\theta), \quad T(z,0) = U(z,0), \;\; P_k(\theta) = \theta^k\ , \tag{27}$$

where, $P_1(\theta), P_2(\theta), P_3(\theta), \ldots$ and $c_0(z), c_1(z), c_2(z), \ldots$ are specified functions and unknown coefficients respectively, depending on the problem.

Using equation (27) in (26), following are obtained:

$$T_0(z,\theta) = \left(c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \ldots\right) + \sin 3\pi z$$

$$T_1(z,\theta) = \left(-c_0(z) - 9\beta\pi^2 \sin 3\pi z\right)\theta + \left(-\frac{1}{2}c_1(z) + \frac{1}{2}\beta c_0''(z)\right)\theta^2 + \left(-\frac{1}{3}c_2(z) + \frac{1}{3}c_1''(z)\right)\theta^3 + \ldots$$

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and so on... (28)

Now solving the above equations in such a manner that, $T_1(z,\theta) = 0$. Thereforeequation (28) reduces to

$$c_0(z) = -3^2 \beta\pi^2 \sin \pi z\,,$$

$$c_1(z) = 3^4 \beta^2 \pi^4 \sin \pi z\,,$$

$$c_2(z) = -3^6 \beta^3 \pi^6 \sin \pi z.$$

$$\begin{aligned} U(z,\theta) = T_0(z,\theta) &= \sin \pi z + c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \ldots \\ &= \sin \pi z\left[1 - 3^2\beta\pi^2\theta + 3^4\beta^2\pi^4\frac{\theta^2}{2} - 3^6\beta^3\pi^6\frac{\theta^3}{3} + \ldots\right] \\ &= \sin \pi z e^{-9\beta\pi^2\theta}\ , \end{aligned} \tag{29}$$

whichissame as the universally acceptedexact solution for the problemwhich is demonstrated by Table 2 and Figure 2.

Table 2 and Figure 2 represents the solution $U(z,\theta)$ for $0 \le z \le 1,\ 0 \le \theta \le 0.5$ and $\beta = 0.05$.

Table 2: $U(z,\theta)$ *for* $0 \le z \le 1, 0 \le \theta \le 0.5$ *for* $\beta = 0.05$

z	$\theta = 0$	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$
0	0	0	0	0	0	0
0.1	0.809017	0.518888	0.332805	0.213454	0.136906	0.087809
0.2	0.951057	0.609989	0.391235	0.250931	0.160942	0.103225
0.3	0.309017	0.198198	0.12712	0.081532	0.052293	0.03354
0.4	-0.58779	-0.37699	-0.2418	-0.15508	-0.09947	-0.0638
0.5	-1	-0.64138	-0.41137	-0.26384	-0.16922	-0.10854
0.6	-0.58779	-0.37699	-0.2418	-0.15508	-0.09947	-0.0638
0.7	0.309017	0.198198	0.12712	0.081532	0.052293	0.03354
0.8	0.951057	0.609989	0.391235	0.250931	0.160942	0.103225
0.9	0.809017	0.518888	0.332805	0.213454	0.136906	0.087809
1	0	0	0	0	0	0

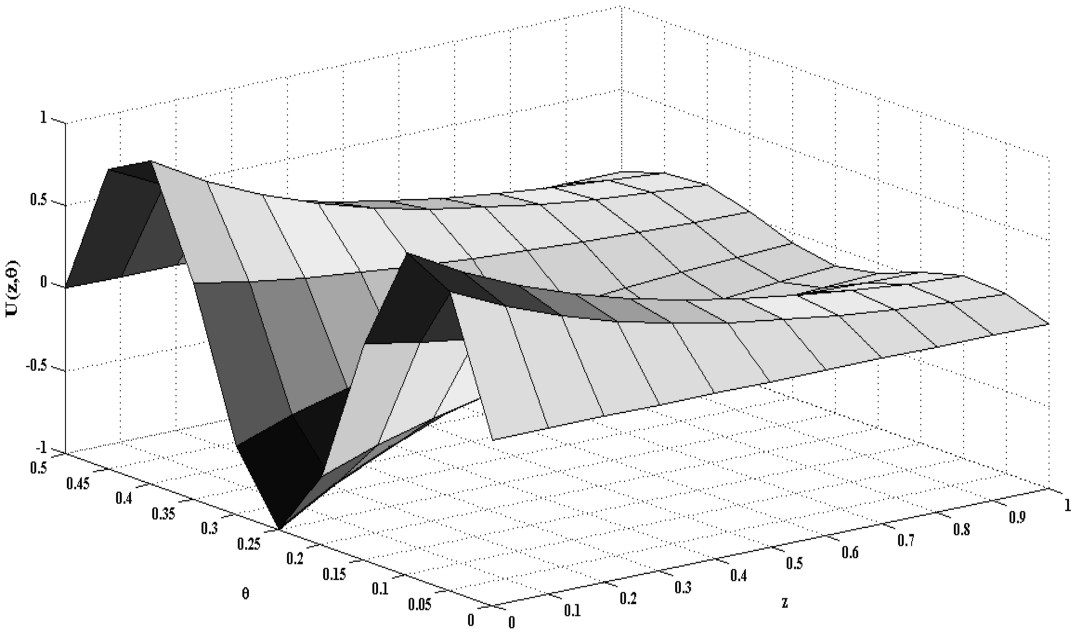

Figure 2: $U(z,\theta)$ for $0 \le z \le 1,\ 0 \le \theta \le 0.5$ for $\beta = 0.05$

Problem 5.3

Consider heat conduction equation

$$\frac{\partial U}{\partial \theta} = \frac{\partial^2 U}{\partial z^2} \tag{30}$$

withB.C. and I.C.

$$U(0,\theta) = 0,\ \ U(1,\theta) = 0\ , \tag{31}$$

$$U(z,0) = \sin z\ . \tag{32}$$

First of all, following homotopy is constructed for solving heat conduction equation using NHPM

$$(1-\mathrm{p})\left[\frac{\partial T}{\partial \theta} - U_0\right] + \mathrm{p}\left[\frac{\partial T}{\partial \theta} - \frac{\partial^2 T}{\partial z^2}\right] = 0$$

or

$$\frac{\partial T}{\partial \theta} = U_0 - \mathrm{p}\left(U_0 - \frac{\partial^2 T}{\partial z^2}\right). \tag{33}$$

Taking $L^{-1} = \int_{\theta_0}^{\theta} (.) d\theta$ i.e.inverse operator on the both sides of (33), then

$$T(z,\theta) = \int_0^{\theta} U_0(z,\theta) d\theta - \mathrm{p}\int_0^{\theta}\left(U_0 - \frac{\partial^2 T}{\partial z^2}\right) d\theta + T(z,0)\ , \tag{34}$$

where, $T(z,0) = U(z,0)$.

Let the solution of (34) is

$$T = T_0 + \mathrm{p}T_1 + \mathrm{p}^2 T_2 + \mathrm{p}^3 T_3 + \ldots, \tag{35}$$

where, $T_0, T_1, T_2, \ldots$ are to be determined.

Suppose equation (35) is the solution of equation (34).Comparing the coefficients of powers of p and equating to zero and using equation (35) in equation (34), following are obtained:

$$\mathrm{p}^0: \quad T_0(z,\theta) = T(z,0) + \int_0^{\theta} U_0(z,\theta) d\theta$$

$$\mathrm{p}^1: \quad T_1(z,\theta) = -\int_0^{\theta}\left(U_0(z,\theta) - \frac{\partial^2 T_0}{\partial z^2}\right) d\theta$$

$$\mathrm{p}^2: \quad T_2(z,\theta) = \int_0^{\theta}\left(\frac{\partial^2 T_1}{\partial z^2}\right) d\theta$$

$$\mathrm{p}^3: \quad T_3(z,\theta) = \int_0^{\theta}\left(\frac{\partial^2 T_2}{\partial z^2}\right) d\theta$$

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and so on... (36)

Consider initial approximation of equation (30) as

$$U_0(z,\theta) = \sum_{n=0}^{\infty} c_n(z) P_n(\theta),\ \ T(z,0) = U(z,0),\ \ P_k(\theta) = \theta^k\ , \tag{37}$$

where, $P_1(\theta), P_2(\theta), P_3(\theta), \ldots$ and $c_0(z), c_1(z), c_2(z), \ldots$ are specified functions and unknown coefficients respectively, depending on the problem.

Using equation (37) in (36), following are obtained:

$$T_0(z,\theta) = \left(c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \ldots\right) + \sin z$$

$$T_1(z,\theta) = \left(-c_0(z) - \sin z\right)\theta + \left(-\frac{c_1(z)}{2} + \frac{c_0''(z)}{2}\right)\theta^2 + \left(-\frac{c_2(z)}{3} + \frac{c_1''(z)}{3}\right)\theta^3 + \ldots$$

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and so on... (38)

Now solving the above equations in such a manner that, $T_1(z,\theta)=0$.
Therefore equation (38) reduces to

$$c_0(z)=-\sin z,$$
$$c_1(z)=\sin z,$$
$$c_2(z)=-\sin z.$$

Therefore

$$U(z,\theta)=T_0(z,\theta)=\sin z+c_0(z)\theta+c_1(z)\frac{\theta^2}{2}+c_2(z)\frac{\theta^3}{3}+c_3(z)\frac{\theta^4}{4}+\dots$$
$$=\sin z\left[1-\theta+\frac{\theta^2}{2}-\frac{\theta^3}{3}+\dots\right]$$
$$=\sin ze^{-t}\ , \quad (39)$$

Which his same as the universally accepted exact solution for the problem which is demonstrated by Table 3 and Figure 3.

Table 3 and Figure 3 represents the solution $U(z,\theta)$ for $-5\le z\le 5,\ -5\le\theta\le 5$ and $\beta=0.05$.

Table 3: $U(z,\theta)$ *for* $-5\le\theta\le 0.5, -5\le z\le 5$ *for* $\beta=0.05$

z	$\theta=-5$	$\theta=-3$	$\theta=-1$	$\theta=0$	$\theta=1$	$\theta=3$	$\theta=5$
-5	142.317	19.26051	2.606626	0.958924	0.352769	0.047742	0.006461
-4	112.3194	15.20078	2.057202	0.756802	0.278412	0.037679	0.005099
-3	-20.9441	-2.83447	-0.3836	-0.14112	-0.05192	-0.00703	-0.00095
-2	-134.952	-18.2637	-2.47173	-0.9093	-0.33451	-0.04527	-0.00613
-1	-124.885	-16.9014	-2.28736	-0.84147	-0.30956	-0.2459	-0.00567
0	0	0	0	0	0	0	0
1	124.8854	16.9014	2.287355	0.841471	0.30956	0.041894	0.00567
2	134.9517	18.26373	2.471727	0.909297	0.334512	0.045271	0.006127
3	20.94407	2.834471	0.383604	0.14112	0.051915	0.007026	0.000951
4	-112.319	-15.2008	-2.0572	-0.7568	-0.27841	-0.03768	-0.0051
5	-142.317	-19.2605	-2.60663	-0.95892	-0.35277	-0.04774	-0.00646

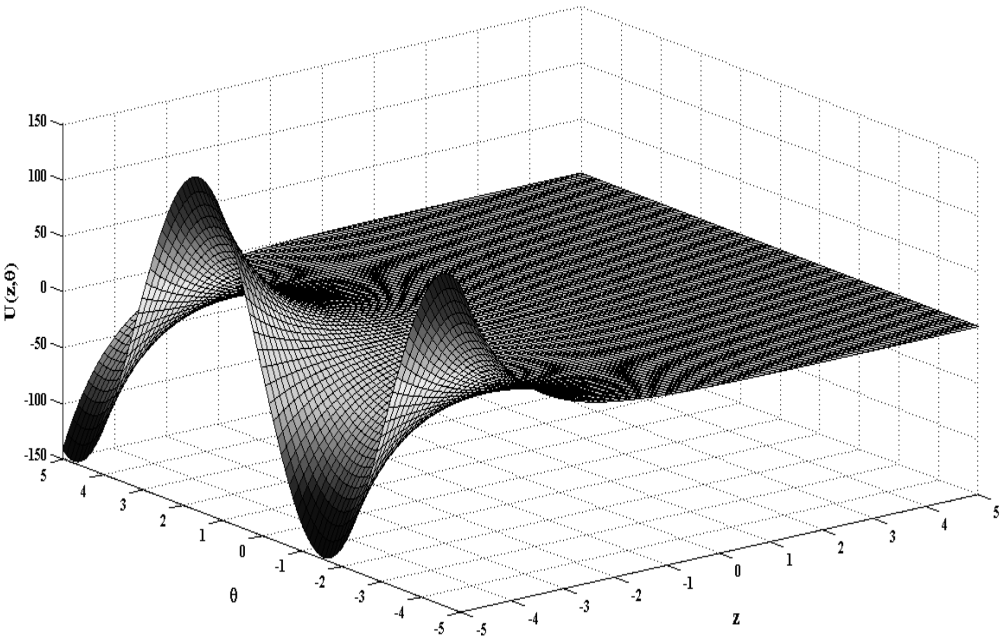

Figure 3: $U(z,\theta)$ *for* $-5\le z\le 5, -5\le\theta\le 5$ *for* $\beta=0.05$

6. CONCLUSION

The analytical approximate solutions of one-dimensional heat conduction equation are obtained by applying new homotopy perturbation method on three problems. It is found that new homotopy perturbation method (NHPM) converges very rapidly as compared to homotopy perturbation method (HPM) and other traditional methods. The exact solutions are obtained up to more accuracy using NHPM, while other methods including HPM leads to approximate solutions for the problems on hand. It is illustrated that NHPM is very prominent, when accuracy has a vital role to play. The numerical results also reflect the remarkable applicability of NHPM to linear and non-linear initial and boundary value problems. NHPM provides the rapid convergence of the series solution for linear as well as non-linear problems with less computational work.

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