ZIBELINE INTERNATIONAL™ PUBLISHING

ISSN: 2521-0831 (Print)
ISSN: 2521-084X(Online)
CODEN: MSMAD

Matrix Science Mathematic (MSMK)

DOI: http://doi.org/10.26480/msmk.02.2023.69.72

CrossMark

RESEARCH ARTICLE

A COMPARATIVE STUDY OF TWO METHODS FOR SOLVING QUADRATIC EQUATIONS

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ARTICLE DETAILS

Article History:

Received 09 April 2023
Revised 12 May 2023
Accepted 19 June 2023
Available online 23 June 2023

ABSTRACT

For the past millennia, various methods had been developed to solve quadratic equations with one unknown. Among high school mathematics curriculum worldwide, quadratic equations teaching and learning intensively utilizes the completing squares method. The present study compared Loh's method to completing squares to examine the difference in students performance in solving quadratic equations. A random sample of 21 students were put into control (n=11) and experimental groups (n=10). The experimental group solved two essay type questions using Loh's method whiles the control group used completed squares. Students test scores were analysed non-parametrically by Mann-Whitney U test. The results revealed statistically significant difference in test scores between control group $(\text{median} = 2.00, \text{n} = 11)$ and experimental group $(\text{median} = 5.00, \text{n} = 10)$. The median for the experimental group was significantly higher than that of the control group. The study recommended that teachers, curriculum developers and policy makers should be highly interested in intuitive approaches, particularly Loh's method, that discourage rote memorization.

KEYWORDS

Algebra, Completing Squares, High Schools, Loh's Method, Mathematics, Quadratic Equations

1. INTRODUCTION

The etymology of algebra is traced back to ninth century Arabic mathematics texts (Oaks, 2018). Algebra is a translation of the word al-jabrin (Guner & Uygun, 2016). It is a broad term that houses various symbolic concepts including quadratic functions and equations. Quadratic equation is a corkscrew to mathematics study at higher levels (Alhassan & Agyei, 2020). Among the important mathematics concepts, quadratic equations cannot be ruled out. Previous studies demonstrated that much attention has been paid due to its numerous applications in the natural and applied sciences (Alhassan & Agyei, 2020; Didiş, 2018; Didis & Erbas, 2015; Hong & Choi, 2014; Vaiyavutjamai & Clements, 2006; Wirts, 2020). For example, in physics, quadratic equation is used to model projectile motion. Similarly, in chemistry, engineering, economics, and so on. In ancient times, quadratic equation was conceptualized as the technique to solve the area or either side of a rectangle of length and breadth (Guner & Uygun, 2016). The equation is a second-degree polynomial of the form $ax^2 + bx + c = 0$, where a, b and c are constants.

In solving quadratic equations, the nature of the roots is determined by the value of the discriminant. The discriminant is a constant expressed by $b^2 - 4ac$. The conditions are; (i) if $b^2 - 4ac < 0$, the equation has complex roots (ii) if $b^2 - 4ac = 0$, the equation has repeated roots, and (ii) if $b^2 - 4ac > 0$, the equation has two distinct real roots; x_1, x_2. The evolution of mathematics has caused the emergence of several techniques to solve problems. However, survivability of a technique depends on greatest power and generality (Alhassan & Agyei, 2020). On similar grounds, numerous methods (graphical, algebraic, geometrical and numerical) have been useful in solving quadratic equations. Including factorization method, the easiest symbolic technique, which works by expanding quadratic expressions into their respective factors. However, if factorization fails, the method of completing squares becomes hopeful. The method of completing squares has been dominant among high school mathematics curriculum worldwide. It is trustworthy method that takes the form of perfect square quadratic expressions to release students' tensions of factoring cannot-be-factored terms (Alhassan & Agyei, 2020). The Ghanaian high school curriculum recommends the method of completing squares to solve quadratic equations (NaCCA, 2020).

Though, for the past millennia, the method of completing squares had been convenient, studies (Alhassan & Agyei, 2020; Didis & Erbas, 2015) and teaching experiences revealed understudies face difficulties to understand and apply sequentially. This conceptually challenging notion could be due to improper understanding of the solution strategy. As a result, learners memorize the algorithm through rote learning techniques with no technical comprehension. Students main goal is to provide solutions ruling out conceptual understanding. Simply put, the method of completing squares encourages rote memorization disrupting meaningful mathematics learning. In 2019, Poh Shen Loh proposed a new method to solve quadratic equations which works by the ideas of Viète (Loh, 2019; Francois, 1579; Descartes, 1954). The method also revolves around ancient Babylonians techniques of working with squares (McMillan, 1984). Though, this method is closer to Savage (1989), however, it differs in procedural, and in algorithmic structure. However, in 2019, a new method was drawn from ancient Babylonians techniques by Professor Poh Shen Loh of Carnegie Mellon University. In spite of the importance of Loh's (2019) method, little is known concerning its effectiveness in mathematics teaching and learning circles in Ghanaian high schools. The present study compares Loh's method to completing squares to investigate the advantages if implemented in high schools in Ghana.

Quick Response Code	Access this article online	
	Website: www.matrixsmathematic.com	**DOI:** 10.26480/msmk.02.2023.69.72

Cite The Article: Samuel Kwaku Boadu, Gyasi Alfred Bannor (2023). A Comparative Study of Two Methods for Solving Quadratic Equations. *Matrix Science Mathematic, 7(2): 69-72.*

2. Objectives of The Study

The study sought to identify the difference in students test score between completing squares and Loh's method.

2.1 Study Hypotheses

The study sought to test the following hypotheses;

H₀: There is no statistically significant difference in students test score between experimental and control group.

Hₐ: There is statistically significant difference in students test score between experimental and control group.

2.2 The Method of Completing Squares

The historical background of what is known today as completing squares is attributed to the contributions of Muhammad ibn Musa al-Khwarizmi (ca. 780–850).

The results of his immense work in the ninth century, is an earlier Islamic algebra manuscript called "The Calculation of al-Jabr and al-Muqabala" (Guner & Uygun, 2016). Al-Khwarizmi presented an exhaustive symbolic procedure for solving quadratic equations using geometric approaches. The method of completing squares is an all-round technique to find the roots, graph quadratic equations and work with different characteristics of conic sections such as finding the center and radius of circles (Guner & Uygun, 2016). Completing the square is very useful in case of non-factorability. The procedure is applied if students encounter difficulties to split quadratic equation into its corresponding factors. According to completing squares, quadratic equation in standard form can be expressed in perfect square (Alhassan & Agyei, 2020). In order to complete squares, one should be cognizant the algorithm is better for monic equations (Tonapi, 2022). A brief solution of the general quadratic equation by the method of completing squares is presented below.

We complete squares for the equation;

$$ax^2 + bx + c = 0$$

We ensure the constant term, c is moved to the right-hand side of the equation

$$ax^2 + bx = -c$$

We ensure the equation is monic-there is a unit coefficient of x^2 by dividing through by a

$$\frac{a}{a}x^2 + \frac{b}{a}x = -\frac{c}{a}$$

We have the monic equation;

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

We find $\frac{1}{2}$ of $\frac{b}{a}$, square it and add to both sides of the equation

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

We expressed the terms as perfect square

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

We find square root of both sides to solve for x

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

We make x the subject

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Therefore, since quadratic equation has two roots;

$$x_1 = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \text{ or } x_2 = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

The method of completing squares presents more brief tactical procedures to solve quadratic equations, however, studies had shown students face problems. A study conducted by Didis and Erbas (2015) revealed that great number of students prefer to work quadratic equations by formula or factorization without regard to completing squares. Students who tried found it challenging to balance the equations by supplementing constants to both sides. Similarly, computations with fractions are inevitable, however, because of apprehensiveness from lack of the ability to perform fractional calculations, students are deterred from using the method (Guner & Uygun, 2016). As a matter of fact, different convenient approach will be significant. Therefore, Poh Shen Loh's method could become better replacement but intensive investigation is what it needs, which is the core objective of the present study.

2.3 Poh Shen Loh's Method

Loh was highly interested in simple solution strategies for novice quadratic equation learners, so he drew an intuitive approach to clear off memorization. Poh Shen Loh's entirely new technique extends the relationship between the solutions x_1 and x_2. The procedure of Loh is based on Viète's (Milicic & Plangg, 2019) ideas of the relationship between roots, and ancient Babylonians' substitution method, that is, two numbers with known sum and product can be found (Katz, 1997). However, the method is not reliant on Vieta's formulas as well as the theorem that quadratic equation is always solved to two roots. Moreover, the method works backwards (Missa, 2022), by focusing on the factorization $x^2 + Px + Q = (x - m)(x - n)$ (Loh, 2019). Loh's ExpiiDaily Challenge videos reveals that the method is suitable under all circumstances even if the results would generate to complex roots (Tonapi, 2022).

A simple solution to the general quadratic equation, $ax^2 + bx + c = 0$ using Loh's approach is presented below.

We start by expressing the equation into monic form, thus

$$\frac{a}{a}x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

We let

$$P = \frac{b}{a}$$

And

$$Q = \frac{c}{a}$$

We have

$$x^2 + Px + Q = 0$$

We factorize to obtain

$$x^2 + Px + Q = (x - m)(x - n)$$

The factorization is zero when at least $x = m$ or $x = n$. We let m and n be two numbers which sum to $-P$ and multiply to Q such that the truth set of $ax^2 + bx + c$ is $\{x: x = m, n\}$. But then, two numbers add to $-P$ if they average to $-\frac{P}{2}$.

In this case, the numbers that multiply to Q are of the form $-\frac{P}{2} \pm k$, where k is not known. This means that

$$Q = \left(-\frac{P}{2} - k\right)\left(-\frac{P}{2} - k\right) = \left(-\frac{P}{2}\right)^2 - k^2$$

$$k = \sqrt{\frac{P^2}{4} - Q}$$

Since m and n are of the form $-\frac{P}{2} \pm k$, then $\{m, n\} = -\frac{P}{2} \pm \sqrt{\frac{P^2}{4} - Q}$. Is the solution to the general quadratic equation.

2.4 Solving Example by Loh's method

We solve $6x^2 - 5x + 1 = 0$ by the Loh's method

We divide through by 6 to obtain $x^2 - \frac{5}{6}x + \frac{1}{6} = 0$

We find $\sqrt{\frac{\left(-\frac{5}{6}\right)^2}{4} - \frac{1}{6}} = \frac{1}{12}$

The solution is

$$x = -\frac{\left(-\frac{5}{6}\right)}{2} \pm \frac{1}{12}$$

$$\therefore x = \frac{5}{12} \pm \frac{1}{12}$$

3. Method

3.1 Research Design

The present study employed non-equivalent control group design (Cohen et al., 2018). This is quasi-experimental technique that allows subjects exposure to a program, or treatment to observe its effect on an outcome (Phillips & Phillips, 2013). Thus, experimental technique in cases where sample selection is biased. In educational research, participants are subjected to learning treatments or programs, and the relative influence of such treatments on their performance is determined by measuring within subjects variation (Creswell, 2009). However, this study randomly assigned participants to two groups (experimental and control groups) of students. The experimental group was put under the tutelage of the researcher on Loh (2019) method for solving quadratic equations for a period of twenty minutes. At th e end, participants solved two essay type questions. The control group was assigned to use completing squares method while, on the other hand, the experimental group was assigned to use the alternative method proposed by (Loh, 2019).

3.2 Sample and Data Collection

The study setting was Asuoso Senior High School, a newly established community school in Offinso-north district of Ghana's Ashanti region. Including teachers, the school has maximum population of about nine hundred (900). The study applied purposive sampling to select the only two form three further/elective mathematics classes who have prior knowledge on quadratic equations as well as proficient in completing squares. These were 3 Arts 1 and 3 Science 1. By the fact that in this school, a few students are enrolled in further mathematics, on schedule, the two classes are combined for their elective mathematics lessons. The combined class comprised of thirty-two (32) students, but because of absenteeism, twenty-six (26) were randomly picked using Excel generated random numbers. Furthermore, timetable clashes did not allow five science students to partake in the testing, finally, reduc ing the sample size to twenty-one (21). The sample consisted of 10 students in the experimental group and 11 students in the control group. We sought consent from the school authority before proceeding, as well as ensured that anonymity and confidentiality are ethically conformed.

3.3 Tests and Measures

Test administration followed introduction of Loh (2019) method to the experimental group. Participants took a test that comprised two essay type questions. Both control and experimental groups solved similar quadratic equations. That is, control group used completing squares and experimental group applied Loh's method. The tests were scored according to the West African Examination Council further mathematics marking structure. The scheme has it that responses to essay type questions are marked relevant to the credentials M for method, B for bonus, and A for answer. In order to probe students cognitive potentials with fractions, we presented quadratic equations whose solutions would end in fraction arithmetic. In all, the total score of the test was 10.

4. Results and Discussion

The study sought to identify the difference in test scores among using the method of completing squares and Loh method for solving quadratic equations. Preliminary analyses were conducted by SPSS (version 23) to ascertain the properties of the distribution of test scores.

Normality was checked using kurtosis and skewness statistics. The results showed that the distribution of test scores was platykurtic and right skewed with kurtosis value of -1.173 and skewness of $.224$ respectively. Furthermore, Shapiro-Wilk test of normality was significant at 95% confidence level, that test scores were non-normally distributed (statistic $= .094$, df $= 21$, p $= .043$). According to Cohen et al. (2018), if distributions deviate from normality assumptions, then it may be statistically wrong to utilize parametric tests, instead non-parametric tests. We undertook a two-tailed Mann-Whitney two-samples rank-sum test to investigate the difference in test scores between the experimental and control groups. The two tailed Mann-Whitney U test is a non-parametric alternative to independent sample t-test; however, medians are compared instead of means. The results revealed statistically significant difference in test scores between control group (median $= 2.00$, n $= 11$) and experimental group (median $= 5.00$, n $= 10$). Hence, H_1 was supported. The median for the experimental group was significantly higher than that of the control group. As a matter of fact, the results show that Loh's method was substantially convenient for solving quadratic equations because the analysis produced sufficient effect size.

An effect size of $r = .446$ was fairly minimum to cause a change on students score. The results of the Mann-Whitney U test is presented on Table 1.

Table 1: Mann -Whitney U test

	Mean rank				
Variable	Experimental group	Control group	U	z	p
Test Score	13.85	8.41	26.50	-2.042	.043

Table1: *Mann-Whitney Test for Test Score by Group*

Source: (Field Survey, 2023)

5. Conclusion

The study concluded that students' performance in the test changed significantly by the Loh's method. Therefore, students found it more interesting and easy working with this method.

Recommendation

The study recommended that teachers should introduce students to Loh's alternative approach for solving quadratic equations, since, solving quadratic equations was seen easy with intuitive strategies. Additionally, policy makers and curriculum developers should include in the mathematics curriculum, more intuitive approaches, particularly Loh's method, that discourage rote memorization.

Limitations and Future Research Suggestions

The present study included smaller sample size ($n = 21$) that resulted to non-normally distributed data. In this case, sample selection biases rendered generalization impossible. Moreover, the study failed to control other factors that might alter results. The study suggests that future research should focus on larger sample size as well as include real-life questions that require critical thinking. Furthermore, we suggest that future analysis should consider control variables such as age, gender, attitude, perception etc.

Author Contributions

All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Acknowledgements

The authors would like to thank the anonymous reviewers for their comments and suggestions. We really appreciated their efforts to improve the quality and presentation of this work.

Ethical Statement

A letter was issued to the selected senior high school requesting permission to perform the study. The privacy and anonymity of the participants were respected.

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