

## REVIEW ARTICLE

## FIBONACCI SEQUENCES AND IT'S COMPLETENESS

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## ARTICLE DETAILS

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## ABSTRACT

This paper attempts to describe the Fibonacci numbers, their qualities, completeness, and mathematical completeness. The Fibonacci sequence may be seen in a number of stunning natural events. As a result, this paper begins by introducing the Fibonacci sequence and then goes on to discuss some of its properties. The Fibonacci sequence is mathematically complete, as well as having natural and artistic manifestations. The series itself is infinite, and any positive integer may be expressed as the sum of individual Fibonacci numbers. The Fibonacci sequence and golden ratio act as testimonies. The Fibonacci sequence, which has ancient mathematical foundations, continues to fascinate our minds and show subtle patterns in a variety of fields. As we interpret the complexities of these mathematical masterworks, we develop a better understanding of how mathematics and the world around us.

## KEYWORDS

Fibonacci number and sequence, Pascal triangle, Golden number, Natural Phenomenon.

### 1. INTRODUCTION

Mathematicians have long been devoted to the explaining nature, dating back to the early actions of the ancient Greeks. After all, mathematics is really an investigation for forms of all types, and where improved to discover such misdeeds than in wildlife? A nearer look at wildlife discloses some charming truths about the basic loveliness of world (Grigas, 2003).

More than just a hobby, pattern discovery in nature aids us make mathematical tools that permit us to make estimates. The amazing thing is that many biological species display mathematical designs as well. It is an example of a common and fascinating sequence in nature (Bicknell, 1973). It has several uses in nature. It may be found in a variety of biological situations, including tree branching and floral petal patterns.

Fibonacci was among "greatest European mathematicians of the Middle Ages." His grave is located at the base of the cemetery Tower in Pisa Cathedral. He was born in 1170 and passed away in 1240. Fibonacci aging among the Moors and had a North African education before traveling extensively throughout the Mediterranean coast. He subsequently paid visits to traders, teaching them various mathematical skills while also assisting them with their business. He rapidly demonstrated that the "Hindu-Arab" technique had a significant advantage over the others (Grigas, 2003).

Fibonacci was first to transfer the Hindu-Arab system to Europe. The following 10 base numerals with decimal points and a zero sign are used: 1 2 3 4 5 6 7 8 9 and 0. It is related to whole and decimal numbers in terms of finance and business.

The initial Fibonacci number is always zero, while the second is always one. The Fibonacci sequence may be seen in a wide range of wonderful natural events. Leonardo Fibonacci, an Italian mathematician, discovered Fibonacci numbers in the 13th century (Dasdan.,2018). The study richly defined the history and application of Fibonacci numbers (Bortner &

Peterson, 2016)

#### 1.1 Novelty

While Fibonacci sequences themselves are ancient and well-known, the novelty often arises from innovative applications, discoveries, or interpretations. Over time, researchers and enthusiasts have found new ways to apply Fibonacci sequences in diverse areas such as mathematics, computer science, art, and even finance. For example, recent studies might explore novel applications of Fibonacci sequences in data compression, cryptography, or algorithmic design.

#### 1.2 Innovation

Innovation in Fibonacci sequences can manifest in several ways. It could involve developing new algorithms or computational methods to efficiently generate or analyze large Fibonacci sequences. Innovations might also come from integrating Fibonacci sequences with other mathematical concepts or theories to solve previously unsolved problems. Additionally, innovations in visualization techniques allow for a deeper understanding of Fibonacci patterns and structures.

#### 1.3 Significance

The significance of Fibonacci sequences lies in their ubiquity and relevance across various domains. In mathematics, Fibonacci sequences have connections to number theory, combinatorics, and algebra. They also appear in the arrangement of leaves on a stem, the branching of trees. Understanding Fibonacci sequences can lead to insights into the underlying mathematical principles governing these phenomena, contributing to both theoretical knowledge and practical applications.

### 2. PRELIMINARY DEFINITIONS

#### 2.1 Fibonacci Sequence

Fibonacci sequences are a numbers or sequence of numbers, each

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## 5. COMPLETENESS OF FIBINACCI SEQUENCES

### 5.1 Nature's Completeness

One of the amazing elements in Fibonacci sequence is its widespread use throughout nature. Fibonacci numbers and the golden ratio appear to influence the development patterns of many living species, ranging from the prearrangement of leaves on a stalk to spirals of pinecone, sunflower seeds, and flower florets.

A pinecone's spirals, for example, frequently correlate to Fibonacci numbers, as does the number of spirals in each direction. Nature's intrinsic order has been ascribed to the efficiency of packing and optimizing space, which results in optimal growth and resource usage (Falcon and Plaza, 2007; Falcon, S. 2011).

### 5.2 Completeness in Architecture

The Fibonacci sequence and its golden ratio have long inspired artists and builders, who incorporate these mathematical ideas into their work. The Parthenon in Athens, Egypt's Pyramids, and numerous pieces of art and architecture throughout history all mirror the Fibonacci sequence's harmonic proportions.

The aesthetic attraction of these mathematical links inspired the concept of completeness in art. The Fibonacci sequence and the golden ratio are frequently used to create a feeling of balance and proportion, whether in art composition, architectural design, or musical arrangement (Falcon, and Plaza, 2007; Falcon, S. 2011).

### 5.3 Mathematical Completeness

The Fibonacci sequence is mathematically complete, in addition to its expressions in nature and art. The sequence itself is endless. One may express every positive integer as the sum of two different Fibonacci sequences. This characteristic, known as Zeckendorf's theorem, emphasizes the Fibonacci sequence's basic completeness and flexibility in expressing whole numbers. Some theorems demonstrate their completeness (Zeckendorf, 1972).

Theorem 5: Fibonacci sequences of numbers are complete for  $a_n = F_n (n \geq 0)$ .

Proof: Now, constructing a Fibonacci Table 3 as

$F_1$	$F_2$	$F_3$	$F_4$	$F_5$	$F_6$	.....
1	1	2	3 4	5 6	7 8	.....

We can write

$$\begin{aligned}
 F_1 &= F_2 = 1, \\
 F_3 &= F_2 + F_1 = 2, \\
 F_4 &= F_4 + F_2 = 3 \\
 F_4 + F_2 &= 4 \\
 F_5 &= F_4 + F_3 = 5 \\
 F_4 + F_3 + F_2 &= 6, \\
 F_5 + F_3 &= 7, \\
 F_6 &= F_5 + F_4 = 8,
 \end{aligned}$$

Each positive integer from 1 to 8 can be characterized in two ways as a sum of Fibonacci numbers, with each number looking only once in each form. We know that,

$F_n - 1 = F_1 + F_2 + F_3, \dots + F_{n-2}$ . We also see in Table 3 that  $3 \leq n \leq 6$  and each integer

$m = 1, 2, 3, \dots, F_n - 1$  can denoted as sum of all Fibonacci numbers  $F_1, F_2, \dots, F_{n-2}$ . Now, for  $n = 3$ ,

$$F_n = F_3 = 2, F_n - 1 = 2 - 1 = 1$$

$$F_{n-2} = F_1 = 1.$$

We can also show for  $n = 4, 5, 6$ . Let the integer  $m = 1, 2, 3, \dots, F_k - 1, k \geq 3$  can be denoted with Fibonacci numbers  $F_1, F_2, \dots, F_{k-2}$ . If  $F_{k-1}$  is added to each of the given illustration,

$$\text{Then } F_k + F_{k-1} - 1 = F_{k-1} - 1.$$

$$\text{If } k = 3, \text{ then } F_k - 1 = F_3 - 1 = 1 \text{ and } 1 + F_{k-1} = 1 + F_2 = 2.$$

Similarly, we can show for  $k = 4, k = 5$  and  $F_k - F_{k-1} \geq 2$ , we have  $F_k - 1 \geq 1 + F_{k-1}$ . There is an overlap.

Theorem 6: The Fibonacci number sequence,  $a_n = F_n, n \geq 1$  is complete even when  $F_n$  is lacking.

Proof: If we remove  $F_4$  in the Theorem5, then every integer ranging from one to eight can be denoted by a sum of other Fibonacci numbers. Theorem 5 also states that any numbers  $m = 1, 2, \dots, F_{n+1} - 1$  is denoted by Fibonacci numbers

$$F_1, F_2, \dots, F_{n-1}, \text{ without } F_n.$$

Now,  $F_{n+1}$  denote itself, and when we add  $F_{n+1}$  to the  $m = 1, 2, \dots, F_{n+1} - 1$

gives  $m = 1, 2, \dots, 2F_{n+1} - 1$ . Since,  $2F_{n+1} - 1 < 2F_{n+2} - 1$  gives

$$a_n = F_n, n \geq 1 \text{ and is complete.}$$

## 6. ADVANCEMENT

Advances in Fibonacci sequences often come hand in hand with advances in computational power, mathematical theory, and interdisciplinary collaboration. With the advent of high-performance computing, researchers can explore larger Fibonacci sequences and analyze their properties more comprehensively. Advancements in mathematical techniques, such as number theory or algebraic geometry, provide new tools for studying Fibonacci sequences and their generalizations. Interdisciplinary collaboration raises innovation by bringing together experts from different fields to tackle complex problems involving Fibonacci sequences.

## 7. CONCLUSION

The Fibonacci sequence and the golden ratio serve as testaments to the intrinsic beauty and completeness entrenched in the fabric of our existence, from nature to creative endeavors and mathematical ideas.

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