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REVIEW ARTICLE

REAL PROJECTIVE SPACE

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ABSTRACT

This article will begin with the claim that Hamilton spent a great deal of time trying to figure out the three-dimensional complex numbers. He was never able to accomplish that.

Complex numbers are in the form a+bi where 'a' is a real part and 'bi' an imaginary with $i = \sqrt{-1}$ The motive behind the claim is that both i_2 and $j_2 = -1$; The failure may also have been caused by the lack of a proper definition for the field of complex numbers. To address this issue, the author of this article offers his own definition of the field of complex numbers with key vectots i,j,k taking values (-1,0,1) respectively Of course the field of complex numbers remains unchanged with $\vec{X} = x + yi + zj$ under transformation becoming $\vec{X} = xi + yj + zk$ vector but it has it's correspondence in the field of real numbers and it's a vector (-1,0,1) The entire process of transition between fields, in author's opinion, is possible thanks to the matrix of transformation It's form has already been explored earlier on by the same author in his 'Ternary Mathematics and 3D Placement of Logical Elements Justification'.

Professor Juan Weisz (Doctor of Philosophy, Northeastern University, Argentina) generously proposed the concept of field transition, which allows for conversion between imaginary and real numbers without altering the field's structure or the relationships between its constituent parts. In essense it should work for all entries just the same way it works for the entries of real numbers The fact that serves as the proof is in $A'A = 1$ and it works well for both Real and Imaginary number fields which is what we are aiming to prove.

KEYWORDS

complex numbers, matrices, combinations and integers

1. General Information

To solve the problem outlined in the introduction, one must select a variety of methods and resources. For our purposes, these will include complex numbers, matrices, combinations, and integers. These essential elements must be sufficient enough to solve the problem; the only thing lacking is adequate evidence that it will work for both real and imaginary numbers.

We will start with the 'reals' An intentionally chosen vector with components of -1,0,1 is used as a reference. The idea behind is that we can replace one component for another in 6 different ways making total equal to 3! combinations in our search we are going to use just some of the aforementioned combinations which should be sufficient enough to demonstrate the principle:

$$\Delta \begin{vmatrix} -1 & 0 & 1 \\ 5 & 7 & 9 \\ 3 & 6 & 7 \end{vmatrix} = 14$$

Another matrix will have identical entries except in the first row:

$$\Delta \begin{vmatrix} -1 & 1 & 0 \\ 5 & 7 & 9 \\ 3 & 6 & 7 \end{vmatrix} = -3$$

What has changed and why does we have different outcome? The answer is simple It's the reference plane

In the first example it was a plane which is expressed by (-1,0,1) while in the second the plane is different it's (-1,1,0) Change the order of entries in the first row and you will see a determinate change now what seems to be the problem? Well, we cannot do the same as far as the fields are concerned for example, we cannot build an imaginary plane in a 3 D space we only can have a projection in other words an imaginary plane does not exist in a real space or does it? Well according to our research, it does just like the one in a real space We can then speak of a reference plane in a Complex Space and a Real Space Let's take a closer look.

2. Materials And Methods

Let's use an analogy and see if our method works for the vectors with imaginary components

$$\Delta \begin{vmatrix} -i^2 & i^2 & i\pi \\ 1 & 2 & -3 \\ 1 & 2 & 0 \end{vmatrix} = 9$$

For the reference here A determinant of the diagonal matrix with the respective imaginary number entries $(-i^2, i^2, i\pi)$ is $-i\pi$

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$$\Delta \begin{vmatrix} -i^2 & & \\ & i^2 & \\ & & i\pi \end{vmatrix} = -i\pi$$

What does it possibly tell? Well, it can mean several things Firstly, that a three-component matrix with imaginary numbers as entries, can have a complex determinant and real determinant It can have both positive and negative value which also depends on the place of its entries (x, y, z) making it a unique vector in space that determines a reference plane. This is a rather bold statement and it needs further analysis but we can speak of matrices as a means of translating from one Space to another in our case from Complex to Real: $C \to R$. The fact that 0 number can be presented as real imaginary and complex by definition and vice versa makes it possible to convert between various groups of numbers Binary decimal Ternary Quaternary etc. Just like an isomorphism in various fields Our task though is to find a specific group of numbers to satisfy that condition.

3. Methodology And Research

Everyone remembers Complex number system and the ring rules that apply to every system of counting.

They are:

$(x1, y1) + (x2, y2) = (x1 + x2, y1 + y2);$

$(x1, y1)(x2, y2) = (x1x2 - y1y2, x1y2 + x2y1)$

The same set of rules applies to rings within different groups Complex Imaginary Real etc. In our case we are looking at a specific formula for a Ternary set such that the aforementioned principle would hold no matter what the entry would be a Ternary set is a specific class of numbers even more so is a Balanced set It requires that not only does the conformity holds but also the cardinality of a set here we must delve into a history and show how this problem was solved by various Mathematicians before and certainly we cannot forget Hamilton who set the rules $S = S(q) + v(q)$. A quick look at the form of a quaternion allows an onlooker to see a striking similarity with vectors with the left part being a scalar and the right a vector In other words a scalar or an integer is placed in correspondence with the vector The only issue is that, although this representation is a good two-component vector, it does not meet our first condition i.e a scalar is represented by a balanced set, in our case, a set of three numbers $(-1, 0, 1)$. Right quaternions are almost a perfect way to convert between decimal and Binary but what about if we want to operate within Binary and Ternary system of counting and we don't need much conversion Then the issue appears. How can we quickly find a vector with entries say $(-1, 0, 1)$ to convert between various types of numbers.

4. Results

Before answers are given let's see what the definition of set is and perhaps under this definition, we can operate to find satisfactory answer to our question. Here we are: Ternary notation is the technique of expressing numbers in base 3. That is, every number $x \epsilon R$ is expressed in the form:

$$x = \sum_{j \epsilon Z} r_j 3_j \qquad (1)$$

In our case, we will suggest a similar solution. But before we do a recursive look at what has been achieved thus far should've been taken our set is not a single unit but a triplet therefore every element should be expressed in the form

$x = \sum j \epsilon Z^{j,j,j}$

$\forall j \epsilon Z : j \epsilon (-1, 0, 1)$

All is left for us to do is to define j

Let's look for any additional identities that we can utilize. Ternary Balanced operator. Here is one:

$$\sum_{k=0}^{n-1} e^{2\pi i \frac{k}{n}} = 0 \qquad (2)$$

As a matter of fact, it’s a beautiful identity because it also corresponds to $q = ai + bj + ck : |q = 0$. It is not hard to find all components provided that $x = \sum j \epsilon Z^{j,j,j}$

(*Important! 'j' in the last expression is an operator and not a dimension)

The unique entries or an Eigenvector of this set will be numbers of the type $j\ e^k : |k = xi\pi$.

5. Proof

$\forall j \in Z : i \epsilon (-1, 0, 1)$

Here comes a long history list First in it stands a Wick rotation unit which helps to find a system at any given state. The point is that the states are quantified making it easy to calculate That same very principle is utilized in our research of all the states available although we are only interested in three, namely (-1,0,1). They are equally as important for calculation as they are for Physics Electronics Computer Science etc. In that view our formula becomes:

$$x = \sum_{j=-1}^{1} j e^{\pi i} \qquad (3)$$

It is only characterized by 3 states or vectors which are mutually orthogonal making the final result equal 0 the dot product that is cross product on the other hand makes the transition between Decimal and Ternary quite plausible. However, the final result requires further explanation, so without further ado, the following relation can be used to describe the sum of balanced trits.

$$\sum x e^{x 2\pi i} = j;\ \forall j \in Z : j \ is \ a \ triple \ set \qquad (4)$$

Our formula establishes the relationship between vector field $F(x, y, z)$ and real numbers x.

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