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RESEARCH ARTICLE

SOFT UNION-THETA PRODUCT OF GROUPS

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ABSTRACT

Soft set theory provides a mathematically robust and algebraically versatile framework for modeling systems characterized by epistemic indeterminacy, vagueness, and parameter-dependent variability-features that pervade foundational inquiries in decision theory, engineering, economics, and the information sciences. At the heart of this formalism lies an extensive suite of algebraic operations and binary product constructs that collectively confer a rich internal structure upon the universe of soft sets, capable of faithfully representing intricate parametric interrelations. Within this conceptual setting, we introduce and rigorously investigate a novel soft product, referred to as the soft union-theta product, defined over soft sets whose parameter spaces are endowed with an intrinsic group-theoretic structure. The operation is meticulously axiomatized to ensure compatibility with generalized soft subsethood and equality relations, thereby preserving the formal $algebraic\ integrity\ of\ the\ resulting\ system.\ A\ comprehensive\ algebraic\ analysis\ is\ undertaken\ to\ characterize$ the operation's fundamental properties—including closure, associativity, commutativity, idempotency, and interactions with identity and absorbing elements—as well as its behavior in relation to the null and absolute soft sets. In parallel, the proposed product is analytically juxtaposed with existing soft binary operations within the stratified hierarchy of soft subset classifications, offering deeper insights into their relative expressive capacities and mutual structural coherence. Our results affirm that the soft union-theta product respects the algebraic constraints imposed by group-parameterized domains while generating a coherent and structurally consistent algebraic system over the space of soft sets. Two core algebraic contributions emerge from this study: (i) the integration of this product fortifies the internal operational harmony of soft set theory by embedding it into an axiomatic framework that preserves and extends fundamental algebraic behaviors; and (ii) the operation lays the groundwork for a generalized soft group theory, wherein soft sets over group-based parameter domains replicate the axiomatic signatures of classical group structures under suitably defined soft operations. By addressing the critical need for algebraic operations grounded in semantically meaningful and structurally sound axioms, this work significantly advances the algebraic unification and generalization of soft set theory. Beyond its theoretical depth, the proposed operation enables the construction of abstract algebra-driven soft computational models, with direct implications for multicriteria decision-making, algebraic classification mechanisms, and uncertainty-sensitive data analysis over group-structured semantic spaces. Consequently, the algebraic apparatus formulated herein not only extends the theoretical boundaries of soft algebra but also solidifies its foundational relevance in both abstract mathematical logic and applied analytical domains.

KEYWORDS

Soft sets; Soft subsets; Soft equalities; Soft union-theta product.

1. Introduction

A wide spectrum of mathematically sophisticated frameworks has been developed to model and analyze phenomena governed by uncertainty, vagueness, and indeterminacy—conditions that routinely manifest in domains such as engineering, economics, the social sciences, and medical diagnostics. Notwithstanding their theoretical depth, classical paradigms such as fuzzy set theory and probabilistic models continue to exhibit intrinsic epistemological and algebraic constraints. For instance, fuzzy set theory, as introduced, hinges on the subjectivity inherent in the assignment of membership functions, whereas probabilistic approaches presuppose the availability of repeatable events and known distributional structures—assumptions that are frequently violated in epistemically ambiguous or non-replicable real-world environments by (Zadeh, 1965).

In a landmark contribution, proposed soft set theory as a formally

minimalistic yet structurally adaptable framework that circumvents the limitations of conventional models by encoding uncertainty through parameter dependence rather than probabilistic likelihoods or fuzzy memberships (Molodtsov, 1999). Since its inception, the algebraic underpinnings of soft set theory have undergone substantial refinement. Foundational operations such as union, intersection, and AND/OR products were first introduced, while reinterpreted these constructions within an information-theoretic paradigm, thereby facilitating their applications to multivalued and relational systems (Maji et al., 2003; Pei and Miao, 2005). It further enriched the theoretical apparatus by defining restricted and extended variants of classical operations, which improved the expressive granularity and operational scope of soft systems (Ali et al., 2009). A sequence of subsequent investigations—including thosesystematically addressed conceptual ambiguities and introduced new binary products and generalized equalities, significantly advancing the algebraic landscape of soft set theory by (Yang, 2008; Feng et al., 2010;

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Jiang et al., 2010; Ali et al., 2011; Neog and Sut, 2011; Fu, 2011; Ge and Yang, 2011; Singh and Onyeozili, 2012a-d; Zhu and Wen, 2013; Onyeozili and Gwary, 2014; Sen, 2014). In recent years, the discipline has witnessed a marked expansion of its algebraic framework through the introduction of rigorously defined novel operations. Distinguished among these are the works, whose contributions have collectively established a robust, extensible, and internally coherent algebraic infrastructure for soft set theory (Eren and Çalışıcı, 2019; Stojanović, 2021; Sezgin et al., 2023a, 2023b; Sezgin and Dağtoros, 2023; Sezgin and Demirci, 2023; Sezgin and Çalışıcı, 2024; Sezgin and Yavuz, 2023a, 2023b; 2024; Sezgin and Çağman, 2024, 2025; Sezgin and Sarıalioğlu, 2024a, 2024b; Sezgin and Şenyiğit, 2025).

A pivotal area within this evolving body of work concerns the formalization and generalization of soft equality and soft inclusion. The original conception of soft subsets was generalized by (Maji et al., 2003; Pei and Miao, 2005; Feng et al., 2010). They advanced the theoretical framework through the introduction of soft congruences, embedding equivalence classes into the soft set universe (Qin and Hong, 2010). They further extended the algebraic semantics by developing the notion of J-soft equality, together with associated distributive principles (Jun and Yang, 2011). Subsequently proposed the concepts of L-soft subsets and Lequality, uncovering foundational deviations from classical algebraic norms—most notably, the failure of distributive identities in generalized soft settings (Liu et al., 2012). Formalized a typology of soft subsets under L-equality and established the validity of associativity, commutativity, and distributivity within certain quotient structures, which were shown to form commutative semigroups (Feng and Yongming, 2013). Broader generalizations—including g-soft, gf-soft, and T-soft equalities—have also been proposed, facilitating a lattice-theoretic and congruence-based reinterpretation of soft algebraic systems (Abbas et al., 2014, 2017; Alshami, 2019; Alshami and El-Shafei, 2020).

The definitional foundation of soft set theory was significantly restructured, who eliminated internal inconsistencies and provided an operationally coherent axiomatic basis, enabling a more rigorous algebraic treatment (Çağman and Enginoğlu, 2010). Concurrently, efforts to develop binary soft products over algebraic structures have flourished. The soft intersection–union product has been extended to rings, semigroups, and groups, producing soft algebraic entities such as soft union rings, semigroups, and groups (Sezer, 2012; Sezgin, 2016; Muştuoğlu et al., 2016). Its dual, the soft union–intersection product, has likewise been formulated within group-theoretic, semigroup-theoretic, and ring-theoretic contexts, with their structural behaviors critically determined by the presence or absence of identity and inverse elements in the parameter domain (Kaygisiz, 2012; Sezer et al., 2015; Sezgin et al., 2017).

Building upon this corpus, the present study introduces a new soft product—the soft union-theta product—defined over soft sets whose parameter sets are equipped with group-theoretic structure. This operation is rigorously formalized and subjected to comprehensive algebraic scrutiny. We examine its fundamental properties, including closure, associativity, commutativity, idempotency, and distributivity, and its interactions with identity and absorbing elements. The compatibility of the proposed operation with generalized soft inclusion and equality is established, ensuring its integration into the broader algebraic architecture of soft set theory. Moreover, a comparative analysis is conducted with preexisting soft products to evaluate its relative expressive power and algebraic coherence within soft subset hierarchies. The product's behavior with respect to null and absolute soft sets is also formally characterized. Our theoretical findings confirm that the soft union-theta product satisfies desirable axiomatic criteria and facilitates a coherent algebraic mechanism for aggregating soft information across group-structured parameter domains. In doing so, it extends classical group-theoretic concepts into the soft set framework and lays the conceptual groundwork for the development of a generalized soft group theory defined via rigorously constructed binary operations. The remainder of this manuscript is structured as follows: Section 2 presents the foundational preliminaries, including definitions and basic algebraic structures relevant to soft sets. Section 3 introduces the soft union-theta product and systematically develops its algebraic theory. Section 4 synthesizes the primary theoretical results and outlines directions for future inquiry, particularly in relation to the expansion of soft algebra and its applications in abstract algebraic systems and uncertainty modeling.

2. Preliminaries

This section undertakes a rigorous and systematic re-examination of the foundational definitions and algebraic axioms that underpin the theoretical architecture developed in the subsequent discourse. While soft set theory was originally formulated as a parameter-dependent formalism

for modeling uncertainty, its structural and operational framework underwent a significant axiomatic refinement in the seminal reformulation by (Çağman and Enginoğlu, 2010; Molodtsov, 1999). This revision not only rectified formal inconsistencies in the original model but also substantially enhanced the theory's internal logical coherence and broadened its applicability across a diverse spectrum of algebraic, computational, and decision-theoretic contexts. The present investigation adopts this refined formalism as its foundational axiomatic substrate. Accordingly, all algebraic constructs, operational definitions, and theoretical generalizations introduced in this study are rigorously formulated within this enhanced framework, thereby ensuring maximal internal consistency, structural soundness, and adherence to contemporary standards governing the algebraic theory of soft systems.

Definition 2.1. (Çağman and Enginoğlu, 2010) Let E be a parameter set, U be a universal set, P(U) be the power set of U, and $\mathcal{H} \subseteq E$. Then, the soft set \int_Y over U is a function such that $f_{\mathcal{H}} \colon E \to P(U)$, where for all $w \notin \mathcal{H}$, $f_{\mathcal{H}}(w) = \emptyset$. That is,

$$f_{\mathcal{H}} = \{(w, f_{\mathcal{H}}(w)) : w \in E\}$$

From now on, the soft set over U is abbreviated by SS.

Definition 2.2. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an \mathcal{SS} . If $f_{\mathcal{H}}(w) = \emptyset$ for all $w \in E$, then $f_{\mathcal{H}}$ is called a null SS and indicated by \emptyset_E , and if $f_{\mathcal{H}}(w) = U$, for all $w \in E$, then $f_{\mathcal{H}}$ is called an absolute SS and indicated by U_E .

Definition 2.3. (Çağman and Enginoğlu, 2010) Let $\mathfrak{f}_{\mathcal{H}}$ and \mathcal{G}_{\aleph} be two \mathcal{SS} s. If $\mathfrak{f}_{\mathcal{H}}(w) \subseteq \mathcal{G}_{\aleph}(w)$, for all $w \in E$, then $\mathfrak{f}_{\mathcal{H}}$ is a soft subset of \mathcal{G}_{\aleph} and indicated by $\mathfrak{f}_{\mathcal{H}} \cong \mathcal{G}_{\aleph}$. If $\mathfrak{f}_{\mathcal{H}}(w) = \mathcal{G}_{\aleph}(w)$, for all $w \in E$, then $\mathfrak{f}_{\mathcal{H}}$ is called soft equal to \mathcal{G}_{\aleph} , and denoted by $\mathfrak{f}_{\mathcal{H}} = \mathcal{G}_{\aleph}$.

Definition 2.4. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an \mathcal{SS} . Then, the complement of $f_{\mathcal{H}}$, denoted by $f_{\mathcal{H}}^c$, is defined by the soft set $f_{\mathcal{H}}^c$: $E \to P(U)$ such that $f_{\mathcal{H}}^c(e) = U \setminus f_{\mathcal{H}}(e) = (f_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 2.5. (Sezgin et al., 2025b) Let f_K and g_N be two \mathcal{SS} s. Then, f_K is called a soft S-subset of g_N , denoted by $f_K \subseteq_S g_N$, if for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_N(w) = \mathcal{D}$, where \mathcal{M} and \mathcal{D} are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two SSs f_K and g_N are said to be soft S-equal, denoted by $f_K =_S g_N$, if $f_K \subseteq_S g_N$ and $g_N \subseteq_S f_K$.

It is obvious that if $f_K =_S \mathcal{G}_{\mathcal{R}_V}$, then f_K and $\mathcal{G}_{\mathcal{R}}$ are the same constant functions, that is, for all $w \in E$, $f_K(w) = \mathcal{G}_{\mathcal{R}}(w) = \mathcal{M}$, where \mathcal{M} is a fixed set.

Definition 2.6. (Sezgin et al., 2025b) Let f_K and g_N be two SSs. Then, f_K is called a soft A-subset of g_N , denoted by $f_K \cong_A g_N$, if, for each $a, b \in E$, $f_K(a) \subseteq g_N(b)$.

Definition 2.7. (Sezgin et al., 2025b) Let f_K and g_N be two SSs. Then, f_K is called a soft S-complement of g_N , denoted by $f_K =_S (g_N)^c$, if, for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_N(w) = \mathcal{D}$, where \mathcal{M} and \mathcal{D} are two fixed sets and $\mathcal{M} = \mathcal{D}'$. Here, $\mathcal{D}' = U \setminus \mathcal{D}$.

From now on, let G be a group, and $S_G(U)$ denotes the collection of all SSs over U, whose parameter sets are G; that is, each element of $S_G(U)$ is an SS parameterized by G.

Definition 2.8. (Muştuoğlu et al., 2016) Let f_G and g_G be two SSs. Then, the soft intersection-union product $f_G \otimes_{i/u} g_G$ is defined by

$$\left(\mathfrak{f}_{G} \otimes_{i/u} \mathfrak{g}_{G}\right)(x) = \bigcap_{x=y_{Z}} \left(\mathfrak{f}_{G}\left(y\right) \cup \mathfrak{g}_{G}(z)\right), \qquad y, z \in G$$

for all $x \in G$.

For additional information on SSs, we refer to (Aktas and Çağman, 2007; Alcantud et al., 2024; Ali et al., 2015; Ali et al., 2022; Atagün et al., 2019; Atagün and Sezgin, 2015; Atagün and Sezgin, 2017; Atagün and Sezgin, 2018; Atagün and Sezgin, 2022; Feng et al., 2008; Gulistan and Shahzad, 2014; Gulistan et al., 2018; Jana et al., 2019; Karaaslan, 2019; Khan et al., 2017; Mahmood et al., 2015; Mahmood et al., 2018; Manikantan et al., 2023; Memiş, 2022; Özlü and Sezgin, 2020; Riaz et al., 2023; Sezer and Atagün, 2016; Sezer et al., 2017; Sezer et al., 2013; Sezer et al., 2014; Sezgin and Ilgin, 2024; Sezgin et al., 2022; Sezgin and Onur, 2024; Sezgin et al., 2024; Sezgin and Orbay, 2022; Sezgin et al., 2019; Sun et al., 2008; Tunçay and Sezgin, 2016; Ullah et al., 2018; Sezgin et al., 2024a, 2024b.).

3. SOFT UNION-THETA PRODUCT OF GROUPS

In this section, we formally introduce a novel binary operation on soft sets, designated as the soft union-theta product, defined over parameter domains equipped with group-theoretic structure. A rigorous algebraic investigation is conducted to systematically delineate the foundational structural properties of this operation, including its compatibility with generalized soft equalities and its behavior under various soft inclusion hierarchies. Special attention is devoted to analyzing the interplay between the proposed product and established soft subset taxonomies,

thereby situating it within the broader algebraic landscape of soft set operations. To concretize the theoretical exposition, a carefully selected suite of illustrative examples is provided, elucidating the operational dynamics and subtle algebraic features of the construction. In addition, the product's interaction with preexisting soft binary operations is examined within the context of soft subset classifications, offering refined insight into its algebraic coherence and integrability. Collectively, these results underscore the structural consistency of the soft union—theta product and demonstrate its potential as a foundational component in the ongoing algebraic enrichment of soft set theory.

Definition 3.1. Let f_G and g_G be two SSs. Then, the soft union-theta product $f_G \otimes_{u/t} g_G$ is defined by

$$\left(\oint_{G} \bigotimes_{u/t} \mathcal{G}_{G} \right) (x) = \bigcup_{x = v_{Z}} \left(\oint_{G} (y) \theta \mathcal{G}_{G}(z) \right) = \bigcup_{x = v_{Z}} \left(\left(\oint_{G} (y) \right)' \cap \left(\mathcal{G}_{G}(z) \right)' \right),$$

 $y,z \in G$

for all $x \in G$.

Note here that since G is a group, there always exist $y,z\in G$ such that =yz, for all $x\in G$. Let the order of the group G be n, that is, |G|=n. Then, it is obvious that there exist n different combinations of writing styles for each $x\in G$ such that x=yz, where $y,z\in G$. Besides, for more on the theta (θ) operation of sets, we refer to Sezgin et al. (2023c).

Note 3.2. The soft union-theta product is well-defined in $S_G(U)$. In fact, let \mathfrak{f}_G , \mathfrak{g}_G , σ_G , $\mathfrak{k}_G \in S_G(U)$ such that $(\mathfrak{f}_G,\mathfrak{g}_G) = (\sigma_G,\mathfrak{k}_G)$. Then, $\mathfrak{f}_G = \sigma_G$ and $\mathfrak{g}_G = \mathfrak{k}_G$, implying that $\mathfrak{f}_G(x) = \sigma_G(x)$ and $\mathfrak{g}_G(x) = \mathfrak{k}_G(x)$ for all $x \in G$. Thereby, for all $x \in G$,

$$(\mathfrak{f}_{G} \otimes_{u/t} \mathfrak{g}_{G})(x) = \bigcup_{x=yz} (\mathfrak{f}_{G}^{c}(y) \cap \mathfrak{g}_{G}^{c}(z))$$
$$= \bigcup_{x=yz} (\sigma_{G}^{c}(y) \cap \mathfrak{k}_{G}^{c}(z))$$
$$= (\sigma_{G} \otimes_{u/t} \mathfrak{k}_{G})(x)$$

Hence, $f_G \otimes_{u/t} g_G = \sigma_G \otimes_{u/t} k_G$.

Example 3.3. Consider the group $G = \{2, 6\}$ with the following operation:

	ລ	б
ລ	ລ	б
б	б	2

Let f_G and g_G be two SSs over $U=D_2=\{< x,y>: x^2=y^2=e, xy=yx\}=\{e,x,y,yx\}$ as follows:

$$f_G = \{(2, \{e, x, y\}), (6, \{e, yx\})\} \text{ and } g_G = \{(2, \{x, yx\}), (6, \{e, y\})\}$$

Since Q = QQ = bb, $(f_G \otimes_{u/t} g_G)(Q) = (f_G^c(Q) \cap g_G^c(Q)) \cup (f_G^c(b) \cap g_G^c(b)) = \{x\}$ and since b = Qb = bQ, $(f_G \otimes_{u/t} g_G)(b) = (f_G^c(Q) \cap g_G^c(b)) \cup (f_G^c(b) \cap g_G^c(Q)) = \{y, yx\}$ is obtained. Hence,

$$f_G \otimes_{u/t} g_G = \{(Q, \{x\}), (b, \{y, yx\})\}$$

Proposition 3.4. The set $S_G(U)$ is closed under the soft union-theta product. That is, if f_G and g_G are two SSs, then so is $f_G \otimes_{u/t} g_G$.

PROOF. It is obvious that the soft union-theta product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft union-theta product.

Proposition 3.5. The soft union-theta product is not associative in $S_G(U)$

PROOF. Consider the group G and the SSs f_G and g_G over $U = \{e, x, y, yx\}$ in Example 3.3. Let $h_G = \{(\mathfrak{Q}, \{e, y\}), (\mathfrak{b}, \{x, yx\})\}$ be an SS over U. Since $f_G \otimes_{u/t} g_G = \{(\mathfrak{Q}, \{x\}), (\mathfrak{b}, \{y, yx\})\}$, then

$$\left(\mathscr{f}_G \otimes_{u/t} \mathscr{g}_G \right) \otimes_{u/t} \hbar_G = \left\{ (\mathfrak{Q}, \{e, yx\}), (\mathfrak{b}, \{e, x, y\}) \right\}$$

Moreover, since $g_G \otimes_{u/t} h_G = \{(2, \emptyset), (6, U)\}$, then

$$f_G \bigotimes_{u/t} (g_G \bigotimes_{u/t} h_G) = \{ (\mathfrak{Q}, \{yx\}), (\mathfrak{b}, \{x,y\}) \}$$

Thereby,
$$(f_G \otimes_{u/t} g_G) \otimes_{u/t} h_G \neq f_G \otimes_{u/t} (g_G \otimes_{u/t} h_G)$$
.

Proposition 3.6. The soft union-theta product is not commutative in $S_G(U)$. However, if G is an abelian group, then the soft union-theta product is commutative in $S_G(U)$.

PROOF. Let f_G and g_G be two SSs and G be an abelian group. Then, for all $x \in G$

$$\left(f_G \bigotimes_{u/t} g_G \right)(x) = \bigcup_{x=yz} \left(f_G^c(y) \cap g_G^c(z) \right)$$

$$= \bigcup_{x=zy} (g_G^c(z) \cap f_G^c(y))$$
$$= (g_G \otimes_{u/t} f_G)(x)$$

Example 3.7. Consider the $SSs f_G$ and g_G over $U = \{e, x, y, yx\}$ in Example 3.3. Then,

$$f_G \otimes_{u/t} f_G = \{(2, \{x\}), (6, \{y, yx\})\}, \text{ and } g_G \otimes_{u/t} f_G = \{(2, \{x\}), (6, \{y, yx\})\}\}$$

implying that $f_G \otimes_{u/t} g_G = g_G \otimes_{u/t} f_G$.

Proposition 3.8. The soft union-theta product is not idempotent in $S_G(U)$.

PROOF. Consider the $f_G SS$ in Example 3.3. Then,

$$f_G \bigotimes_{u/t} f_G = \{(Q, \{x, y, yx\}), (b, \emptyset)\}$$

implying that $f_G \bigotimes_{u/t} f_G \neq f_G$. \square

Proposition 3.9. Let f_G be a constant SS. Then, $f_G \otimes_{u/t} f_G = f_G^c$.

PROOF. Let f_G be a constant SS such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(\mathfrak{f}_G \otimes_{u/t} \mathfrak{f}_G)(x) = \bigcup_{x=yz} (\mathfrak{f}_G^c(y) \cap \mathfrak{f}_G^c(z))$$

= $\mathfrak{f}_G^c(x)$

Thereby, $f_G \bigotimes_{u/t} f_G = f_G^c$. \square

Remark 3.10. Let $S_G^*(U)$ be the collection of all constant SSs. Then, the soft union-theta product is not idempotent in $S_G^*(U)$ either.

Proposition 3.11. Let f_G be an SS. Then, $U_G \otimes_{u/t} f_G = f_G \otimes_{u/t} U_G = \emptyset_G$.

PROOF. Let f_G be an SS. Then, for all $x \in G$,

$$(U_G \otimes_{u/t} f_G)(x) = \bigcup_{x=yz} (U_G^c(y) \cap f_G^c(z))$$
$$= \bigcup_{x=yz} (\emptyset \cap f_G^c(z))$$
$$= \emptyset_G(x)$$

Similarly,

$$(\mathfrak{f}_G \otimes_{u/t} U_G)(x) = \bigcup_{x=yz} (\mathfrak{f}_G^c(y) \cap U_G^c(z))$$
$$= \bigcup_{x=yz} (\mathfrak{f}_G^c(y) \cap \emptyset)$$
$$= \emptyset_G(x)$$

Thereby, $U_G \otimes_{u/t} \mathscr{Y}_G = \mathscr{Y}_G \otimes_{u/t} U_G = \emptyset_G. \square$

Proposition 3.12. Let f_G be a constant SS. Then, $\emptyset_G \bigotimes_{u/t} f_G = f_G \bigotimes_{u/t} \emptyset_G = f_G^c$.

PROOF. Let f_G be a constant SS such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{split} \left(\phi_{G} \otimes_{u/t} \mathfrak{f}_{G} \right) (x) &= \bigcup_{x = yz} \left(\phi_{G}^{\ c}(y) \cap \mathfrak{f}_{G}^{\ c}(z) \right) \\ &= \bigcup_{x = yz} \left(U \cap \mathfrak{f}_{G}^{\ c}(z) \right) \\ &= \mathfrak{f}_{G}^{\ c}(x) \end{split}$$

Similarly, for all $x \in G$,

$$(\mathfrak{f}_G \otimes_{u/t} \mathfrak{O}_G)(x) = \bigcup_{x=y_Z} (\mathfrak{f}_G^{\ c}(y) \cap \mathfrak{O}_G^{\ c}(z))$$
$$= \bigcup_{x=y_Z} (\mathfrak{f}_G^{\ c}(y) \cap U)$$
$$= \mathfrak{f}_G^{\ c}(x)$$

Thereby, $\emptyset_G \otimes_{u/t} \mathscr{F}_G = \mathscr{F}_G \otimes_{u/t} \mathscr{O}_G = \mathscr{F}_G^c$. \square

Proposition 3.13. Let f_G be a constant SS. Then, $f_G{}^c \otimes_{u/t} f_G = f_G \otimes_{u/t} f_G{}^c = g_G$.

PROOF. Let f_G be a constant SS such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$.

$$(\mathfrak{f}_G^c \otimes_{u/t} \mathfrak{f}_G)(x) = \bigcup_{x=yz} ((\mathfrak{f}_G^c)^c(y) \cap \mathfrak{f}_G^c(z))$$
$$= \bigcup_{x=yz} (\mathfrak{f}_G(y) \cap \mathfrak{f}_G^c(z))$$
$$= \emptyset_G(x)$$

Similarly, for all $x \in G$,

$$(\mathfrak{f}_G \otimes_{u/t} \mathfrak{f}_G^c)(x) = \bigcup_{x=yz} (\mathfrak{f}_G^c(y) \cap (\mathfrak{f}_G^c)^c(z))$$
$$= \bigcup_{x=yz} (\mathfrak{f}_G^c(y) \cap \mathfrak{f}_G(z))$$
$$= \emptyset_G(x)$$

Thereby, $f_G^c \otimes_{u/t} f_G = f_G \otimes_{u/t} f_G^c = \emptyset_G. \square$

Proposition 3.14. Let f_G and g_G be two SSs. If one of the following assertions is satisfied, then $f_G \otimes_{u/t} g_G = \emptyset_G$:

$$i. \, f_G =_S (g_G)^c$$

ii.
$$f_G = U_G$$
 or $g_G = U_G$

iii.
$$(f_G)^c \cong_A g_G$$

iv.
$$(f_G)^c \cong_S g_G$$

PROOF. Let f_G and g_G be two SSs.

i. Let $f_G = g(g_G)^c$. Thus, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$ where A and B are two fixed sets and A = B'. Then, for all $x \in G$,

$$(\mathfrak{f}_{G} \otimes_{u/t} \mathfrak{g}_{G})(x) = \bigcup_{x=yz} (\mathfrak{f}_{G}^{c}(y) \cap \mathfrak{g}_{G}^{c}(z))$$

$$= \bigcup_{x=yz} ((\mathfrak{g}_{G}^{c})^{c}(y) \cap \mathfrak{g}_{G}^{c}(z))$$

$$= \bigcup_{x=yz} (\mathfrak{g}_{G}(y) \cap \mathfrak{g}_{G}^{c}(z))$$

$$= \emptyset_{G}(x)$$

Thereby, $f_G \otimes_{u/t} g_G = \emptyset_G$. \square

ii. Without loss of generality, let $f_G=U_G$. Thus, for all $x\in G$, $f_G(x)=U_G(x)=U$. Then, for all $x\in G$,

$$\begin{split} \left(\oint_{G} \otimes_{u/t} g_{G} \right) (x) &= \bigcup_{x=yz} \left(\oint_{G} {}^{c}(y) \cap g_{G} {}^{c}(z) \right) \\ &= \bigcup_{x=yz} \left(U_{G} {}^{c}(y) \cap g_{G} {}^{c}(z) \right) \\ &= \bigcup_{x=yz} \left(\emptyset \cap g_{G} {}^{c}(z) \right) \\ &= \emptyset_{G}(x) \end{split}$$

Thereby, $f_G \otimes_{u/t} g_G = \emptyset_G$.

iii. Let $(f_G)^c \subseteq_A g_G$. Then, $(f_G)^c(y) \subseteq_A g_G(z)$, for each $y, z \in G$. Thus, for all $x \in G$,

$$\left(f_G \otimes_{u/t} g_G \right)(x) = \bigcup_{x = yz} \left(f_G^{\ c}(y) \cap g_G^{\ c}(z) \right) = \emptyset_G(x)$$

Thereby, $f_G \otimes_{u/t} g_G = g_G$. Here note that, in classical set theory, if $A' \subseteq B$, then $A' \cap B' = g$, where A and B are fixed sets.

iv. The proof of similar to Proposition 3.14 (iii). □

Proposition 3.15. Let f_G and g_G be two SSs. If $f_G = g_G = g_G$, then $f_G \otimes_{u/t} g_G = U_G$.

PROOF. Let $f_G = g_G = \emptyset_G$. Then, for all $x \in G$, $f_G(x) = g_G(x) = \emptyset_G(x) = \emptyset$. Thus, for all $x \in G$,

$$\left(\oint_{G} \bigotimes_{u/t} \mathcal{G}_{G} \right) (x) = \bigcup_{x = yz} \left(\oint_{G} {}^{c}(y) \cap \mathcal{G}_{G} {}^{c}(z) \right)$$

$$= \bigcup_{x=yz} (\emptyset_G^c(y) \cap \emptyset_G^c(z))$$

$$= \bigcup_{x=yz} (U \cap U)$$

$$= U_G(x)$$

Thereby, $f_G \otimes_{u/t} g_G = U_G$.

Proposition 3.16. Let f_G and g_G be two SSs. If $f_G \subseteq_S g_G$, then $f_G \otimes_{u/t} g_G = g_G^c$.

PROOF. Let f_G and g_G be two SSs and $f_G \subseteq_S g_G$. Hence, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$, where A and B are two fixed sets and $A \subseteq B$. Moreover, since $g_G^{\ c}(x) \subseteq f_G^{\ c}(x)$, for all $x \in G$,

$$\left(f_G \otimes_{u/t} g_G \right)(x) = \bigcup_{x = y_Z} \left(f_G^c(y) \cap g_G^c(z) \right) = g_G^c(x)$$

Thereby, $f_G \bigotimes_{u/t} g_G = g_G^c$. \square

Proposition 3.17. Let f_G and g_G be two SSs. If $g_G \cong_S f_G$, then $f_G \otimes_{u/t} g_G = f_G^c$.

PROOF. Let f_G and g_G be two SSs and $g_G \subseteq_S f_G$. Hence, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$, where A and B are two fixed sets and $B \subseteq A$. Moreover, since $f_G^{c}(x) \subseteq g_G^{c}(x)$, for all $x \in G$,

$$\left(\oint_G \bigotimes_{u/t} \mathcal{G}_G \right)(x) = \bigcup_{x = \forall z} \left(\oint_G {}^c(y) \cap \mathcal{G}_G{}^c(z) \right) = \oint_G {}^c(x)$$

Thereby, $f_G \otimes_{u/t} g_G = f_G^c$. \square

Proposition 3.18. Let f_G and g_G be two SSs. Then, $(f_G \otimes_{u/t} g_G)^c = f_G \otimes_{i/u} g_G$.

PROOF. Let f_G and g_G be two SSs. Then, for all $x \in G$,

$$\begin{split} \left(\oint_{G} \otimes_{u/t} \mathcal{G}_{G} \right)^{c}(x) &= \left(\bigcup_{x=yz} \left(\oint_{G} {}^{c}(y) \cap \mathcal{G}_{G} {}^{c}(z) \right) \right)' \\ &= \bigcap_{x=yz} \left(\oint_{G} {}^{c}(y) \cup \mathcal{G}_{G} {}^{c}(z) \right)' \\ &= \bigcap_{x=yz} \left(\oint_{G} (y) \cup \mathcal{G}_{G} (z) \right) \\ &= \oint_{G} \otimes_{i/u} \mathcal{G}_{G}(x) \end{split}$$

Thereby, $(\mathscr{F}_G \otimes_{u/t} \mathscr{G}_G)^c = \mathscr{F}_G \otimes_{i/u} \mathscr{G}_G$.

Proposition 3.19. Let f_G , g_G , and h_G be three SSs. If $f_G \cong g_G$, then $g_G \otimes_{u/t} h_G \cong f_G \otimes_{u/t} h_G$ and $h_G \otimes_{u/t} g_G \cong h_G \otimes_{u/t} f_G$.

PROOF. Let f_G , g_G , and h_G be three SSs such that $f_G \cong g_G$. Then, for all $x \in G$, $f_G(x) \subseteq g_G(x)$. Moreover, since $g_G{}^c(x) \subseteq f_G{}^c(x)$ for all $x \in G$,

$$\begin{split} \left(g_{G} \otimes_{u/t} \hbar_{G}\right)(x) &= \bigcup_{x=yz} \left(g_{G}^{c}(y) \cap \hbar_{G}^{c}(z)\right) \\ &\subseteq \bigcup_{x=yz} \left(f_{G}^{c}(y) \cap \hbar_{G}^{c}(z)\right) \\ &= \left(f_{G} \otimes_{u/t} \hbar_{G}\right)(x) \end{split}$$

is obtained, implying that $g_G \otimes_{u/t} \hbar_G \cong f_G \otimes_{u/t} \hbar_G$. Moreover, for all $x \in G$,

$$(\hbar_{G} \otimes_{u/t} \mathcal{G}_{G})(x) = \bigcup_{x=yz} (\hbar_{G}^{c}(y) \cap \mathcal{G}_{G}^{c}(z))$$

$$\subseteq \bigcup_{x=yz} (\hbar_{G}^{c}(y) \cap \mathcal{G}_{G}^{c}(z))$$

$$= (\hbar_{G} \otimes_{u/t} \mathcal{G}_{G})(x)$$

implying that $\hbar_G \otimes_{u/t} g_G \cong \hbar_G \otimes_{u/t} f_G$.

Proposition 3.20. Let f_G , g_G , σ_G , and k_G be four SSs. If $k_G \cong \sigma_G$, and $f_G \cong g_G$, then $\sigma_G \otimes_{u/t} g_G \cong k_G \otimes_{u/t} f_G$ and $g_G \otimes_{u/t} \sigma_G \cong f_G \otimes_{u/t} k_G$.

PROOF. Let f_G , g_G , σ_G , and k_G be four SSs such that $k_G \subseteq \sigma_G$, and $f_G \subseteq g_G$. Then, for all $x \in G$, $k_G(x) \subseteq \sigma_G(x)$ and $f_G(x) \subseteq g_G(x)$. Moreover, since $\sigma_G^{\ c}(x) \subseteq k_G^{\ c}(x)$ and $g_G^{\ c}(x) \subseteq f_G^{\ c}(x)$, for all $x \in G$, then

$$\begin{split} \left(\sigma_G \otimes_{u/t} \mathcal{G}_G\right)(x) &= \bigcup_{x=yz} \left(\sigma_G^c(y) \cap \mathcal{G}_G^c(z)\right) \\ &\subseteq \bigcup_{x=yz} \left(\mathcal{R}_G^c(y) \cap \mathcal{R}_G^c(z)\right) \\ &= \left(\mathcal{R}_G \otimes_{u/t} \mathcal{R}_G\right)(x) \end{split}$$

is obtained, implying that $\sigma_G \otimes_{u/t} g_G \cong k_G \otimes_{u/t} f_G$. Similarly, for all $x \in G$,

$$(g_G \otimes_{u/t} \sigma_G)(x) = \bigcup_{x = y_Z} (g_G^c(y) \cap \sigma_G^c(z))$$

$$\subseteq \bigcup_{x = y_Z} (f_G^c(y) \cap k_G^c(z))$$

$$= (f_G \otimes_{u/t} k_G)(x)$$

is obtained, implying that $g_G \bigotimes_{u/t} \sigma_G \cong f_G \bigotimes_{u/t} k_G$. \square

4. CONCLUSION

This study commences with the formal introduction of a novel soft product on soft sets-termed the soft union-theta product-defined over parameter domains equipped with group-theoretic structure. Anchored in this foundational formulation, we undertake a comprehensive algebraic investigation of the operation, focusing in particular on its structural behavior across various taxonomies of soft subsethood and its coherence with generalized soft equality relations. The proposed product is further subjected to a comparative analysis with previously established soft binary operations, systematically positioned within the hierarchical lattice of soft subset classifications. This yields sharpened theoretical insights into the relative representational expressiveness and algebraic compatibility of competing soft operations. Concurrently, an in-depth structural analysis is conducted to examine the interaction of the soft union-theta product with both the null and absolute soft sets, as well as with other soft binary products defined over group-structured parameter domains, thereby further elucidating its foundational role within the broader algebraic topology of soft systems. The systematic study of such operations within an axiomatized algebraic framework aligns with core pursuits in abstract algebra, where structural features—including closure, associativity, commutativity, idempotency, and the presence or absence of identity, inverse, and absorbing elements—serve as critical invariants for classifying induced systems within the established algebraic hierarchy. The algebraic regularities and structural phenomena uncovered through this analysis not only affirm the internal consistency of the proposed construction but also underscore its capacity to generalize classical algebraic forms, thereby extending the expressive reach of soft algebraic theory. From a foundational perspective, the formal apparatus developed herein addresses salient gaps in the literature and establishes a rigorous platform for the advancement of a generalized soft group theory—an emerging paradigm in which soft sets over group-parameterized domains simulate classical group-theoretic behavior through carefully defined soft operations. Prospective research may build upon this framework by synthesizing additional algebraic operations in soft contexts and refining generalized notions of soft equality, thereby broadening both the theoretical scope and the methodological applicability of soft set theory in algebraic modeling, computational abstractions, and uncertainty-aware decision frameworks.

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