ZIBELINE INTERNATIONAL™ PUBLISHING

Matrix Science Mathematic (MSMK)

DOI: http://doi.org/10.26480/msmk.02.2025.66.72

ISSN: 2521-0831 (Print)
ISSN: 2521-084X(Online)
CODEN: MSMAD

CrossMark

RESEARCH ARTICLE

SOFT UNION-THETA PRODUCT OF GROUPS

Zeynep Ay[a], Aslıhan Sezgin[b,*]

[a] *Department of Mathematics, Graduate School of Natural and Applied Sciences, Amasya University, Amasya, Türkiye*
[b] *Department of Mathematics and Science Education, Faculty of Education, Amasya University, Amasya, Türkiye*
Corresponding Author Email: aslihan.sezgin@amasya.edu.tr

ARTICLE DETAILS

Article History:

Received 7 May 2025
Revised 14 June 2025
Accepted 20 June 2025
Available online 10 July 2025

ABSTRACT

Soft set theory provides a mathematically robust and algebraically versatile framework for modeling systems characterized by epistemic indeterminacy, vagueness, and parameter-dependent variability—features that pervade foundational inquiries in decision theory, engineering, economics, and the information sciences. At the heart of this formalism lies an extensive suite of algebraic operations and binary product constructs that collectively confer a rich internal structure upon the universe of soft sets, capable of faithfully representing intricate parametric interrelations. Within this conceptual setting, we introduce and rigorously investigate a novel soft product, referred to as the soft union–theta product, defined over soft sets whose parameter spaces are endowed with an intrinsic group-theoretic structure. The operation is meticulously axiomatized to ensure compatibility with generalized soft subsethood and equality relations, thereby preserving the formal algebraic integrity of the resulting system. A comprehensive algebraic analysis is undertaken to characterize the operation's fundamental properties—including closure, associativity, commutativity, idempotency, and interactions with identity and absorbing elements—as well as its behavior in relation to the null and absolute soft sets. In parallel, the proposed product is analytically juxtaposed with existing soft binary operations within the stratified hierarchy of soft subset classifications, offering deeper insights into their relative expressive capacities and mutual structural coherence. Our results affirm that the soft union–theta product respects the algebraic constraints imposed by group-parameterized domains while generating a coherent and structurally consistent algebraic system over the space of soft sets. Two core algebraic contributions emerge from this study: (i) the integration of this product fortifies the internal operational harmony of soft set theory by embedding it into an axiomatic framework that preserves and extends fundamental algebraic behaviors; and (ii) the operation lays the groundwork for a generalized soft group theory, wherein soft sets over group-based parameter domains replicate the axiomatic signatures of classical group structures under suitably defined soft operations. By addressing the critical need for algebraic operations grounded in semantically meaningful and structurally sound axioms, this work significantly advances the algebraic unification and generalization of soft set theory. Beyond its theoretical depth, the proposed operation enables the construction of abstract algebra-driven soft computational models, with direct implications for multi-criteria decision-making, algebraic classification mechanisms, and uncertainty-sensitive data analysis over group-structured semantic spaces. Consequently, the algebraic apparatus formulated herein not only extends the theoretical boundaries of soft algebra but also solidifies its foundational relevance in both abstract mathematical logic and applied analytical domains.

KEYWORDS

Soft sets; Soft subsets; Soft equalities; Soft union-theta product.

1. Introduction

A wide spectrum of mathematically sophisticated frameworks has been developed to model and analyze phenomena governed by uncertainty, vagueness, and indeterminacy—conditions that routinely manifest in domains such as engineering, economics, the social sciences, and medical diagnostics. Notwithstanding their theoretical depth, classical paradigms such as fuzzy set theory and probabilistic models continue to exhibit intrinsic epistemological and algebraic constraints. For instance, fuzzy set theory, as introduced, hinges on the subjectivity inherent in the assignment of membership functions, whereas probabilistic approaches presuppose the availability of repeatable events and known distributional structures—assumptions that are frequently violated in epistemically ambiguous or non-replicable real-world environments by (Zadeh, 1965).

In a landmark contribution, proposed soft set theory as a formally minimalistic yet structurally adaptable framework that circumvents the limitations of conventional models by encoding uncertainty through parameter dependence rather than probabilistic likelihoods or fuzzy memberships (Molodtsov, 1999). Since its inception, the algebraic underpinnings of soft set theory have undergone substantial refinement. Foundational operations such as union, intersection, and AND/OR products were first introduced, while reinterpreted these constructions within an information-theoretic paradigm, thereby facilitating their applications to multivalued and relational systems (Maji et al., 2003; Pei and Miao, 2005). It further enriched the theoretical apparatus by defining restricted and extended variants of classical operations, which improved the expressive granularity and operational scope of soft systems (Ali et al., 2009). A sequence of subsequent investigations—including those—systematically addressed conceptual ambiguities and introduced new binary products and generalized equalities, significantly advancing the algebraic landscape of soft set theory by (Yang, 2008; Feng et al., 2010;

Quick Response Code	Access this article online	
	Website: www.matrixsmathematic.com	**DOI:** 10.26480/msmk.02.2025.66.72

Jiang et al., 2010; Ali et al., 2011; Neog and Sut, 2011; Fu, 2011; Ge and Yang, 2011; Singh and Onyeozili, 2012a–d; Zhu and Wen, 2013; Onyeozili and Gwary, 2014; Sen, 2014). In recent years, the discipline has witnessed a marked expansion of its algebraic framework through the introduction of rigorously defined novel operations. Distinguished among these are the works, whose contributions have collectively established a robust, extensible, and internally coherent algebraic infrastructure for soft set theory (Eren and Çalışıcı, 2019; Stojanović, 2021; Sezgin et al., 2023a, 2023b; Sezgin and Dağtoros, 2023; Sezgin and Demirci, 2023; Sezgin and Çalışıcı, 2024; Sezgin and Yavuz, 2023a, 2023b; 2024; Sezgin and Çağman, 2024, 2025; Sezgin and Sarıalioğlu, 2024a, 2024b; Sezgin and Şenyiğit, 2025).

A pivotal area within this evolving body of work concerns the formalization and generalization of soft equality and soft inclusion. The original conception of soft subsets was generalized by (Maji et al., 2003; Pei and Miao, 2005; Feng et al., 2010). They advanced the theoretical framework through the introduction of soft congruences, embedding equivalence classes into the soft set universe (Qin and Hong, 2010). They further extended the algebraic semantics by developing the notion of J-soft equality, together with associated distributive principles (Jun and Yang, 2011). Subsequently proposed the concepts of L-soft subsets and L-equality, uncovering foundational deviations from classical algebraic norms—most notably, the failure of distributive identities in generalized soft settings (Liu et al., 2012). Formalized a typology of soft subsets under L-equality and established the validity of associativity, commutativity, and distributivity within certain quotient structures, which were shown to form commutative semigroups (Feng and Yongming, 2013). Broader generalizations—including g-soft, gf-soft, and T-soft equalities—have also been proposed, facilitating a lattice-theoretic and congruence-based reinterpretation of soft algebraic systems (Abbas et al., 2014, 2017; Alshami, 2019; Alshami and El-Shafei, 2020).

The definitional foundation of soft set theory was significantly restructured, who eliminated internal inconsistencies and provided an operationally coherent axiomatic basis, enabling a more rigorous algebraic treatment (Çağman and Enginoğlu, 2010). Concurrently, efforts to develop binary soft products over algebraic structures have flourished. The soft intersection–union product has been extended to rings, semigroups, and groups, producing soft algebraic entities such as soft union rings, semigroups, and groups (Sezer, 2012; Sezgin, 2016; Muştuoğlu et al., 2016). Its dual, the soft union–intersection product, has likewise been formulated within group-theoretic, semigroup-theoretic, and ring-theoretic contexts, with their structural behaviors critically determined by the presence or absence of identity and inverse elements in the parameter domain (Kaygısız, 2012; Sezer et al., 2015; Sezgin et al., 2017).

Building upon this corpus, the present study introduces a new soft product—the soft union–theta product—defined over soft sets whose parameter sets are equipped with group-theoretic structure. This operation is rigorously formalized and subjected to comprehensive algebraic scrutiny. We examine its fundamental properties, including closure, associativity, commutativity, idempotency, and distributivity, and its interactions with identity and absorbing elements. The compatibility of the proposed operation with generalized soft inclusion and equality is established, ensuring its integration into the broader algebraic architecture of soft set theory. Moreover, a comparative analysis is conducted with preexisting soft products to evaluate its relative expressive power and algebraic coherence within soft subset hierarchies. The product's behavior with respect to null and absolute soft sets is also formally characterized. Our theoretical findings confirm that the soft union–theta product satisfies desirable axiomatic criteria and facilitates a coherent algebraic mechanism for aggregating soft information across group-structured parameter domains. In doing so, it extends classical group-theoretic concepts into the soft set framework and lays the conceptual groundwork for the development of a generalized soft group theory defined via rigorously constructed binary operations. The remainder of this manuscript is structured as follows: Section 2 presents the foundational preliminaries, including definitions and basic algebraic structures relevant to soft sets. Section 3 introduces the soft union–theta product and systematically develops its algebraic theory. Section 4 synthesizes the primary theoretical results and outlines directions for future inquiry, particularly in relation to the expansion of soft algebra and its applications in abstract algebraic systems and uncertainty modeling.

2. Preliminaries

This section undertakes a rigorous and systematic re-examination of the foundational definitions and algebraic axioms that underpin the theoretical architecture developed in the subsequent discourse. While soft set theory was originally formulated as a parameter-dependent formalism for modeling uncertainty, its structural and operational framework underwent a significant axiomatic refinement in the seminal reformulation by (Çağman and Enginoğlu, 2010; Molodtsov, 1999). This revision not only rectified formal inconsistencies in the original model but also substantially enhanced the theory's internal logical coherence and broadened its applicability across a diverse spectrum of algebraic, computational, and decision-theoretic contexts. The present investigation adopts this refined formalism as its foundational axiomatic substrate. Accordingly, all algebraic constructs, operational definitions, and theoretical generalizations introduced in this study are rigorously formulated within this enhanced framework, thereby ensuring maximal internal consistency, structural soundness, and adherence to contemporary standards governing the algebraic theory of soft systems.

Definition 2.1. (Çağman and Enginoğlu, 2010) Let E be a parameter set, U be a universal set, $P(U)$ be the power set of U, and $\mathcal{H} \subseteq E$. Then, the soft set f_Y over U is a function such that $f_{\mathcal{H}}: E \to P(U)$, where for all $w \notin \mathcal{H}$, $f_{\mathcal{H}}(w) = \emptyset$. That is,

$$f_{\mathcal{H}} = \{(w, f_{\mathcal{H}}(w)) : w \in E\}$$

From now on, the soft set over U is abbreviated by $\mathcal{SS}$.

Definition 2.2. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. If $f_{\mathcal{H}}(w) = \emptyset$ for all $w \in E$, then $f_{\mathcal{H}}$ is called a null SS and indicated by $\emptyset_E$, and if $f_{\mathcal{H}}(w) = U$, for all $w \in E$, then $f_{\mathcal{H}}$ is called an absolute SS and indicated by U_E.

Definition 2.3. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. If $f_{\mathcal{H}}(w) \subseteq g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is a soft subset of $g_{\aleph}$ and indicated by $f_{\mathcal{H}} \widetilde{\subseteq} g_{\aleph}$. If $f_{\mathcal{H}}(w) = g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is called soft equal to $g_{\aleph}$, and denoted by $f_{\mathcal{H}} = g_{\aleph}$.

Definition 2.4. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. Then, the complement of $f_{\mathcal{H}}$, denoted by $f_{\mathcal{H}}{}^{c}$, is defined by the soft set $f_{\mathcal{H}}{}^{c}: E \to P(U)$ such that $f_{\mathcal{H}}{}^{c}(e) = U \setminus f_{\mathcal{H}}(e) = (f_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 2.5. (Sezgin et al., 2025b) Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft S-subset of $g_{\aleph}$, denoted by $f_K \widetilde{\subseteq}_S g_{\aleph}$, if for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two SSs f_K and $g_{\aleph}$ are said to be soft S-equal, denoted by $f_K =_S g_{\aleph}$, if $f_K \widetilde{\subseteq}_S g_{\aleph}$ and $g_{\aleph} \widetilde{\subseteq}_S f_K$.

It is obvious that if $f_K =_S g_{\aleph}$, then f_K and $g_{\aleph}$ are the same constant functions, that is, for all $w \in E$, $f_K(w) = g_{\aleph}(w) = \mathcal{M}$, where $\mathcal{M}$ is a fixed set.

Definition 2.6. (Sezgin et al., 2025b) Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft A-subset of $g_{\aleph}$, denoted by $f_K \widetilde{\subseteq}_A g_{\aleph}$, if, for each $a, b \in E$, $f_K(a) \subseteq g_{\aleph}(b)$.

Definition 2.7. (Sezgin et al., 2025b) Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft S-complement of $g_{\aleph}$, denoted by $f_K =_S (g_{\aleph})^c$, if, for all w∈E, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} = \mathcal{D}'$. Here, $\mathcal{D}' = U \setminus \mathcal{D}$.

From now on, let G be a group, and $S_G(U)$ denotes the collection of all $\mathcal{SS}$s over U, whose parameter sets are G; that is, each element of $S_G(U)$ is an $\mathcal{SS}$ parameterized by G.

Definition 2.8. (Muştuoğlu et al., 2016) Let f_G and g_G be two $\mathcal{SS}$s. Then, the soft intersection-union product $f_G \otimes_{i/u} g_G$ is defined by

$$(f_G \otimes_{i/u} g_G)(x) = \bigcap_{x=yz} (f_G(y) \cup g_G(z)), \qquad y, z \in G$$

for all $x \in G$.

For additional information on SSs, we refer to (Aktas and Çağman, 2007; Alcantud et al., 2024; Ali et al., 2015; Ali et al., 2022; Atagün et al., 2019; Atagün and Sezgin, 2015; Atagün and Sezgin, 2017; Atagün and Sezgin, 2018; Atagün and Sezgin, 2022; Feng et al., 2008; Gulistan and Shahzad, 2014; Gulistan et al., 2018; Jana et al., 2019; Karaaslan, 2019; Khan et al., 2017; Mahmood et al., 2015; Mahmood et al., 2018; Manikantan et al., 2023; Memiş, 2022; Özlü and Sezgin, 2020; Riaz et al., 2023; Sezer and Atagün, 2016; Sezer et al., 2017; Sezer et al., 2013; Sezer et al., 2014; Sezgin and İlgin, 2024; Sezgin et al., 2022; Sezgin and Onur, 2024; Sezgin et al., 2024; Sezgin and Orbay, 2022; Sezgin et al., 2019; Sun et al., 2008; Tunçay and Sezgin, 2016; Ullah et al., 2018; Sezgin et al., 2024a, 2024b.).

3. Soft Union-Theta Product of Groups

In this section, we formally introduce a novel binary operation on soft sets, designated as the soft union–theta product, defined over parameter domains equipped with group-theoretic structure. A rigorous algebraic investigation is conducted to systematically delineate the foundational structural properties of this operation, including its compatibility with generalized soft equalities and its behavior under various soft inclusion hierarchies. Special attention is devoted to analyzing the interplay between the proposed product and established soft subset taxonomies, thereby situating it within the broader algebraic landscape of soft set operations. To concretize the theoretical exposition, a carefully selected suite of illustrative examples is provided, elucidating the operational dynamics and subtle algebraic features of the construction. In addition, the product's interaction with preexisting soft binary operations is examined within the context of soft subset classifications, offering refined insight into its algebraic coherence and integrability. Collectively, these results underscore the structural consistency of the soft union–theta product and demonstrate its potential as a foundational component in the ongoing algebraic enrichment of soft set theory.

Definition 3.1. Let f_G and g_G be two $\mathcal{SS}$s. Then, the soft union-theta product $f_G \otimes_{u/t} g_G$ is defined by

$$(f_G \otimes_{u/t} g_G)(x) = \bigcup_{x=yz} (f_G(y)\theta g_G(z)) = \bigcup_{x=yz} ((f_G(y))' \cap (g_G(z))'),$$

$y, z \in G$

for all $x \in G$.

Note here that since G is a group, there always exist $y, z \in G$ such that $= yz$, for all $x \in G$. Let the order of the group G be n, that is, $|G| = n$. Then, it is obvious that there exist n different combinations of writing styles for each $x \in G$ such that $x = yz$, where $y, z \in G$. Besides, for more on the theta (θ) operation of sets, we refer to Sezgin et al. (2023c).

Note 3.2. The soft union-theta product is well-defined in $S_G(U)$. In fact, let f_G, g_G, σ_G, $k_G \in S_G(U)$ such that $(f_G, g_G) = (\sigma_G, k_G)$. Then, $f_G = \sigma_G$ and $g_G = k_G$, implying that $f_G(x) = \sigma_G(x)$ and $g_G(x) = k_G(x)$ for all $x \in G$. Thereby, for all $x \in G$,

$$(f_G \otimes_{u/t} g_G)(x) = \bigcup_{x=yz} (f_G{}^c(y) \cap g_G{}^c(z))$$

$$= \bigcup_{x=yz} (\sigma_G{}^c(y) \cap k_G{}^c(z))$$

$$= (\sigma_G \otimes_{u/t} k_G)(x)$$

Hence, $f_G \otimes_{u/t} g_G = \sigma_G \otimes_{u/t} k_G$.

Example 3.3. Consider the group $G = \{Ձ, Ϭ\}$ with the following operation:

$\cdot$	Ձ	Ϭ
Ձ	Ձ	Ϭ
Ϭ	Ϭ	Ձ

Let f_G and g_G be two $\mathcal{SS}$s over $U = D_2 = \{< x, y >: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$f_G = \{(Ձ, \{e, x, y\}), (Ϭ, \{e, yx\})\}$ and $g_G = \{(Ձ, \{x, yx\}), (Ϭ, \{e, y\})\}$

Since $Ձ = ՁՁ = ϬϬ$, $(f_G \otimes_{u/t} g_G)(Ձ) = (f_G{}^c(Ձ) \cap g_G{}^c(Ձ)) \cup (f_G{}^c(Ϭ) \cap g_G{}^c(Ϭ)) = \{x\}$ and since $Ϭ = ՁϬ = ϬՁ$, $(f_G \otimes_{u/t} g_G)(Ϭ) = (f_G{}^c(Ձ) \cap g_G{}^c(Ϭ)) \cup (f_G{}^c(Ϭ) \cap g_G{}^c(Ձ)) = \{y, yx\}$ is obtained. Hence,

$f_G \otimes_{u/t} g_G = \{(Ձ, \{x\}), (Ϭ, \{y, yx\})\}$

Proposition 3.4. The set $S_G(U)$ is closed under the soft union-theta product. That is, if f_G and g_G are two $\mathcal{SS}$s, then so is $f_G \otimes_{u/t} g_G$.

PROOF. It is obvious that the soft union-theta product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft union-theta product.

Proposition 3.5. The soft union-theta product is not associative in $S_G(U)$

PROOF. Consider the group G and the $\mathcal{SS}$s f_G and g_G over $U = \{e, x, y, yx\}$ in Example 3.3. Let $h_G = \{(Ձ, \{e, y\}), (Ϭ, \{x, yx\})\}$ be an $\mathcal{SS}$ over U. Since $f_G \otimes_{u/t} g_G = \{(Ձ, \{x\}), (Ϭ, \{y, yx\})\}$, then

$(f_G \otimes_{u/t} g_G) \otimes_{u/t} h_G = \{(Ձ, \{e, yx\}), (Ϭ, \{e, x, y\})\}$

Moreover, since $g_G \otimes_{u/t} h_G = \{(Ձ, \emptyset), (Ϭ, U)\}$, then

$f_G \otimes_{u/t} (g_G \otimes_{u/t} h_G) = \{(Ձ, \{yx\}), (Ϭ, \{x, y\})\}$

Thereby, $(f_G \otimes_{u/t} g_G) \otimes_{u/t} h_G \neq f_G \otimes_{u/t} (g_G \otimes_{u/t} h_G)$.

Proposition 3.6. The soft union-theta product is not commutative in $S_G(U)$. However, if G is an abelian group, then the soft union-theta product is commutative in $S_G(U)$.

PROOF. Let f_G and g_G be two SSs and G be an abelian group. Then, for all $x \in G$,

$$(f_G \otimes_{u/t} g_G)(x) = \bigcup_{x=yz} (f_G{}^c(y) \cap g_G{}^c(z))$$

$$= \bigcup_{x=zy} (g_G{}^c(z) \cap f_G{}^c(y))$$

$$= (g_G \otimes_{u/t} f_G)(x)$$

Example 3.7. Consider the $\mathcal{SS}$s f_G and g_G over $U = \{e, x, y, yx\}$ in Example 3.3. Then,

$f_G \otimes_{u/t} g_G = \{(Ձ, \{x\}), (Ϭ, \{y, yx\})\}$, and $g_G \otimes_{u/t} f_G = \{(Ձ, \{x\}), (Ϭ, \{y, yx\})\}$

implying that $f_G \otimes_{u/t} g_G = g_G \otimes_{u/t} f_G$.

Proposition 3.8. The soft union-theta product is not idempotent in $S_G(U)$.

PROOF. Consider the f_G $\mathcal{SS}$ in Example 3.3. Then,

$$f_G \otimes_{u/t} f_G = \{(Ձ, \{x, y, yx\}), (Ϭ, \emptyset)\}$$

implying that $f_G \otimes_{u/t} f_G \neq f_G$. □

Proposition 3.9. Let f_G be a constant $\mathcal{SS}$. Then, $f_G \otimes_{u/t} f_G = f_G{}^c$.

PROOF. Let f_G be a constant $\mathcal{SS}$ such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(f_G \otimes_{u/t} f_G)(x) = \bigcup_{x=yz} (f_G{}^c(y) \cap f_G{}^c(z))$$

$$= f_G{}^c(x)$$

Thereby, $f_G \otimes_{u/t} f_G = f_G{}^c$.□

Remark 3.10. Let $S_G{}^*(U)$ be the collection of all constant $\mathcal{SS}$s. Then, the soft union-theta product is not idempotent in $S_G{}^*(U)$ either.

Proposition 3.11. Let f_G be an $\mathcal{SS}$. Then, $U_G \otimes_{u/t} f_G = f_G \otimes_{u/t} U_G = \emptyset_G$.

PROOF. Let f_G be an $\mathcal{SS}$. Then, for all $x \in G$,

$$(U_G \otimes_{u/t} f_G)(x) = \bigcup_{x=yz} (U_G{}^c(y) \cap f_G{}^c(z))$$

$$= \bigcup_{x=yz} (\emptyset \cap f_G{}^c(z))$$

$$= \emptyset_G(x)$$

Similarly,

$$(f_G \otimes_{u/t} U_G)(x) = \bigcup_{x=yz} (f_G{}^c(y) \cap U_G{}^c(z))$$

$$= \bigcup_{x=yz} (f_G{}^c(y) \cap \emptyset)$$

$$= \emptyset_G(x)$$

Thereby, $U_G \otimes_{u/t} f_G = f_G \otimes_{u/t} U_G = \emptyset_G$.□

Proposition 3.12. Let f_G be a constant $\mathcal{SS}$. Then, $\emptyset_G \otimes_{u/t} f_G = f_G \otimes_{u/t} \emptyset_G = f_G{}^c$.

PROOF. Let f_G be a constant $\mathcal{SS}$ such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(\emptyset_G \otimes_{u/t} f_G)(x) = \bigcup_{x=yz} (\emptyset_G{}^c(y) \cap f_G{}^c(z))$$

$$= \bigcup_{x=yz} (U \cap f_G{}^c(z))$$

$$= f_G{}^c(x)$$

Similarly, for all $x \in G$,

$$(f_G \otimes_{u/t} \emptyset_G)(x) = \bigcup_{x=yz} (f_G{}^c(y) \cap \emptyset_G{}^c(z))$$

$$= \bigcup_{x=yz} (f_G{}^c(y) \cap U)$$

$$= f_G{}^c(x)$$

Thereby, $\emptyset_G \otimes_{u/t} f_G = f_G \otimes_{u/t} \emptyset_G = f_G{}^c$.□

Proposition 3.13. Let f_G be a constant $\mathcal{SS}$. Then, $f_G{}^c \otimes_{u/t} f_G = f_G \otimes_{u/t} f_G{}^c = \emptyset_G$.

PROOF. Let f_G be a constant SS such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(f_G{}^c \otimes_{u/t} f_G)(x) &= \bigcup_{x=yz} \left(({f_G}^c)^c(y) \cap {f_G}^c(z)\right)\\ &= \bigcup_{x=yz} \left(f_G(y) \cap {f_G}^c(z)\right)\\ &= \emptyset_G(x)\end{aligned}$$

Similarly, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{u/t} {f_G}^c)(x) &= \bigcup_{x=yz} \left({f_G}^c(y) \cap ({f_G}^c)^c(z)\right)\\ &= \bigcup_{x=yz} \left({f_G}^c(y) \cap f_G(z)\right)\\ &= \emptyset_G(x)\end{aligned}$$

Thereby, ${f_G}^c \otimes_{u/t} f_G = f_G \otimes_{u/t} {f_G}^c = \emptyset_G$.□

Proposition 3.14. Let f_G and g_G be two SSs. If one of the following assertions is satisfied, then $f_G \otimes_{u/t} g_G = \emptyset_G$:

i. $f_G =_S (g_G)^c$

ii. $f_G = U_G$ or $g_G = U_G$

iii. $(f_G)^c \tilde{\subseteq}_A g_G$

iv. $(f_G)^c \tilde{\subseteq}_S g_G$

PROOF. Let f_G and g_G be two SSs.

i. Let $f_G =_S (g_G)^c$. Thus, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$ where A and B are two fixed sets and $A = B'$. Then, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{u/t} g_G)(x) &= \bigcup_{x=yz} \left({f_G}^c(y) \cap {g_G}^c(z)\right)\\ &= \bigcup_{x=yz} \left(({g_G}^c)^c(y) \cap {g_G}^c(z)\right)\\ &= \bigcup_{x=yz} \left(g_G(y) \cap {g_G}^c(z)\right)\\ &= \emptyset_G(x)\end{aligned}$$

Thereby, $f_G \otimes_{u/t} g_G = \emptyset_G$.□

ii. Without loss of generality, let $f_G = U_G$. Thus, for all $x \in G$, $f_G(x) = U_G(x) = U$. Then, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{u/t} g_G)(x) &= \bigcup_{x=yz} \left({f_G}^c(y) \cap {g_G}^c(z)\right)\\ &= \bigcup_{x=yz} \left({U_G}^c(y) \cap {g_G}^c(z)\right)\\ &= \bigcup_{x=yz} \left(\emptyset \cap {g_G}^c(z)\right)\\ &= \emptyset_G(x)\end{aligned}$$

Thereby, $f_G \otimes_{u/t} g_G = \emptyset_G$.

iii. Let $(f_G)^c \tilde{\subseteq}_A g_G$. Then, $(f_G)^c(y) \tilde{\subseteq}_A g_G(z)$, for each $y, z \in G$. Thus, for all $x \in G$,

$$(f_G \otimes_{u/t} g_G)(x) = \bigcup_{x=yz} \left({f_G}^c(y) \cap {g_G}^c(z)\right) = \emptyset_G(x)$$

Thereby, $f_G \otimes_{u/t} g_G = \emptyset_G$. Here note that, in classical set theory, if $A' \subseteq B$, then $A' \cap B' = \emptyset$, where A and B are fixed sets**.**

iv. The proof of similar to Proposition 3.14 (iii). □

Proposition 3.15. Let f_G and g_G be two SSs. If $f_G = g_G = \emptyset_G$, then $f_G \otimes_{u/t} g_G = U_G$.

PROOF. Let $f_G = g_G = \emptyset_G$. Then, for all $x \in G$, $f_G(x) = g_G(x) = \emptyset_G(x) = \emptyset$. Thus, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{u/t} g_G)(x) &= \bigcup_{x=yz} \left({f_G}^c(y) \cap {g_G}^c(z)\right)\\ &= \bigcup_{x=yz} \left({\emptyset_G}^c(y) \cap {\emptyset_G}^c(z)\right)\\ &= \bigcup_{x=yz} (U \cap U)\\ &= U_G(x)\end{aligned}$$

Thereby, $f_G \otimes_{u/t} g_G = U_G$.□

Proposition 3.16. Let f_G and g_G be two SSs. If $f_G \tilde{\subseteq}_S g_G$, then $f_G \otimes_{u/t} g_G = {g_G}^c$.

PROOF. Let f_G and g_G be two SSs and $f_G \tilde{\subseteq}_S g_G$. Hence, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$, where A and B are two fixed sets and $A \subseteq B$. Moreover, since ${g_G}^c(x) \subseteq {f_G}^c(x)$, for all $x \in G$,

$$(f_G \otimes_{u/t} g_G)(x) = \bigcup_{x=yz} \left({f_G}^c(y) \cap {g_G}^c(z)\right) = {g_G}^c(x)$$

Thereby, $f_G \otimes_{u/t} g_G = {g_G}^c$. □

Proposition 3.17. Let f_G and g_G be two SSs. If $g_G \tilde{\subseteq}_S f_G$, then $f_G \otimes_{u/t} g_G = {f_G}^c$.

PROOF. Let f_G and g_G be two SSs and $g_G \tilde{\subseteq}_S f_G$. Hence, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$, where A and B are two fixed sets and $B \subseteq A$. Moreover, since ${f_G}^c(x) \subseteq {g_G}^c(x)$, for all $x \in G$,

$$(f_G \otimes_{u/t} g_G)(x) = \bigcup_{x=yz} \left({f_G}^c(y) \cap {g_G}^c(z)\right) = {f_G}^c(x)$$

Thereby, $f_G \otimes_{u/t} g_G = {f_G}^c$. □

Proposition 3.18. Let f_G and g_G be two SSs. Then, $(f_G \otimes_{u/t} g_G)^c = f_G \otimes_{i/u} g_G$.

PROOF. Let f_G and g_G be two SSs. Then, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{u/t} g_G)^c(x) &= \left(\bigcup_{x=yz} \left({f_G}^c(y) \cap {g_G}^c(z)\right)\right)'\\ &= \bigcap_{x=yz} \left({f_G}^c(y) \cup {g_G}^c(z)\right)'\\ &= \bigcap_{x=yz} \left(f_G(y) \cup g_G(z)\right)\\ &= f_G \otimes_{i/u} g_G(x)\end{aligned}$$

Thereby, $(f_G \otimes_{u/t} g_G)^c = f_G \otimes_{i/u} g_G$.

Proposition 3.19. Let f_G, g_G, and h_G be three SSs. If $f_G \tilde{\subseteq} g_G$, then $g_G \otimes_{u/t} h_G \tilde{\subseteq} f_G \otimes_{u/t} h_G$ and $h_G \otimes_{u/t} g_G \tilde{\subseteq} h_G \otimes_{u/t} f_G$.

PROOF. Let f_G, g_G, and h_G be three SSs such that $f_G \tilde{\subseteq} g_G$. Then, for all $x \in G$, $f_G(x) \subseteq g_G(x)$. Moreover, since ${g_G}^c(x) \subseteq {f_G}^c(x)$ for all $x \in G$,

$$\begin{aligned}(g_G \otimes_{u/t} h_G)(x) &= \bigcup_{x=yz} \left({g_G}^c(y) \cap {h_G}^c(z)\right)\\ &\subseteq \bigcup_{x=yz} \left({f_G}^c(y) \cap {h_G}^c(z)\right)\\ &= (f_G \otimes_{u/t} h_G)(x)\end{aligned}$$

is obtained, implying that $g_G \otimes_{u/t} h_G \tilde{\subseteq} f_G \otimes_{u/t} h_G$. Moreover, for all $x \in G$,

$$\begin{aligned}(h_G \otimes_{u/t} g_G)(x) &= \bigcup_{x=yz} \left({h_G}^c(y) \cap {g_G}^c(z)\right)\\ &\subseteq \bigcup_{x=yz} \left({h_G}^c(y) \cap {f_G}^c(z)\right)\\ &= (h_G \otimes_{u/t} f_G)(x)\end{aligned}$$

implying that $h_G \otimes_{u/t} g_G \tilde{\subseteq} h_G \otimes_{u/t} f_G$.

Proposition 3.20. Let f_G, g_G, σ_G, and k_G be four SSs. If $k_G \tilde{\subseteq} \sigma_G$, and $f_G \tilde{\subseteq} g_G$, then $\sigma_G \otimes_{u/t} g_G \tilde{\subseteq} k_G \otimes_{u/t} f_G$ and $g_G \otimes_{u/t} \sigma_G \tilde{\subseteq} f_G \otimes_{u/t} k_G$.

PROOF. Let f_G, g_G, σ_G, and k_G be four SSs such that $k_G \tilde{\subseteq} \sigma_G$, and $f_G \tilde{\subseteq} g_G$. Then, for all $x \in G$, $k_G(x) \subseteq \sigma_G(x)$ and $f_G(x) \subseteq g_G(x)$. Moreover, since ${\sigma_G}^c(x) \subseteq {k_G}^c(x)$ and ${g_G}^c(x) \subseteq {f_G}^c(x)$, for all $x \in G$, then

$$\begin{aligned}(\mathcal{r}_G \otimes_{u/t} \mathcal{g}_G)(x) &= \bigcup_{x=yz} \left(\mathcal{r}_G{}^c(y) \cap \mathcal{g}_G{}^c(z)\right) \\ &\subseteq \bigcup_{x=yz} \left(\mathcal{h}_G{}^c(y) \cap \mathcal{f}_G{}^c(z)\right) \\ &= (\mathcal{h}_G \otimes_{u/t} \mathcal{f}_G)(x)\end{aligned}$$

is obtained, implying that $\mathcal{r}_G \otimes_{u/t} \mathcal{g}_G \widetilde{\subseteq} \mathcal{h}_G \otimes_{u/t} \mathcal{f}_G$. Similarly, for all $x \in G$,

$$\begin{aligned}(\mathcal{g}_G \otimes_{u/t} \mathcal{r}_G)(x) &= \bigcup_{x=yz} \left(\mathcal{g}_G{}^c(y) \cap \mathcal{r}_G{}^c(z)\right) \\ &\subseteq \bigcup_{x=yz} \left(\mathcal{f}_G{}^c(y) \cap \mathcal{h}_G{}^c(z)\right) \\ &= (\mathcal{f}_G \otimes_{u/t} \mathcal{h}_G)(x)\end{aligned}$$

is obtained, implying that $\mathcal{g}_G \otimes_{u/t} \mathcal{r}_G \widetilde{\subseteq} \mathcal{f}_G \otimes_{u/t} \mathcal{h}_G$. □

4. CONCLUSION

This study commences with the formal introduction of a novel soft product on soft sets—termed the soft union–theta product—defined over parameter domains equipped with group-theoretic structure. Anchored in this foundational formulation, we undertake a comprehensive algebraic investigation of the operation, focusing in particular on its structural behavior across various taxonomies of soft subsethood and its coherence with generalized soft equality relations. The proposed product is further subjected to a comparative analysis with previously established soft binary operations, systematically positioned within the hierarchical lattice of soft subset classifications. This yields sharpened theoretical insights into the relative representational expressiveness and algebraic compatibility of competing soft operations. Concurrently, an in-depth structural analysis is conducted to examine the interaction of the soft union–theta product with both the null and absolute soft sets, as well as with other soft binary products defined over group-structured parameter domains, thereby further elucidating its foundational role within the broader algebraic topology of soft systems. The systematic study of such operations within an axiomatized algebraic framework aligns with core pursuits in abstract algebra, where structural features—including closure, associativity, commutativity, idempotency, and the presence or absence of identity, inverse, and absorbing elements—serve as critical invariants for classifying induced systems within the established algebraic hierarchy. The algebraic regularities and structural phenomena uncovered through this analysis not only affirm the internal consistency of the proposed construction but also underscore its capacity to generalize classical algebraic forms, thereby extending the expressive reach of soft algebraic theory. From a foundational perspective, the formal apparatus developed herein addresses salient gaps in the literature and establishes a rigorous platform for the advancement of a generalized soft group theory—an emerging paradigm in which soft sets over group-parameterized domains simulate classical group-theoretic behavior through carefully defined soft operations. Prospective research may build upon this framework by synthesizing additional algebraic operations in soft contexts and refining generalized notions of soft equality, thereby broadening both the theoretical scope and the methodological applicability of soft set theory in algebraic modeling, computational abstractions, and uncertainty-aware decision frameworks.

REFERENCES

Abbas, M., Ali, B. and Romaguera, S., 2014. On generalized soft equality and soft lattice structure. Filomat, 28(6), Pp. 1191-1203.

Abbas, M., Ali, M. I. and Romaguera, S., 2017. Generalized operations in soft set theory via relaxed conditions on parameters. Filomat, 31(19), Pp. 5955-5964.

Aktas, H. and Çağman, N., 2007. Soft sets and soft groups. Information Science, 177(13), Pp. 2726-2735.

Alcantud, J.C.R. and Khameneh, A.Z., Santos-García, G. and Akram, M. A systematic literature review of soft set theory. Neural Computing and Applications, 36, Pp. 8951–8975.

Ali, B., Saleem, N., Sundus, N., Khaleeq, S., Saeed, M. and George, R., 2022. A contribution to the theory of soft sets via generalized relaxed operations. Mathematics, 10(15), Pp. 26-36.

Ali, M. I., Feng, F., Liu, X., Min, W. K. and Shabir, M., 2009. On some new operations in soft set theory. Computers and Mathematics with Applications, 57(9) Pp. 1547-1553.

Ali, M. I., Shabir, M. and Naz, M., 2011. Algebraic structures of soft sets associated with new operations. Computers and Mathematics with Applications, 61(9), Pp. 2647-2654.

Ali, M. I., Mahmood, M., Rehman, M.U. and Aslam, M. F., 2015. On lattice ordered soft sets, Applied Soft Computing, 36, Pp. 499-505.

Alshami, T. and El-Shafei, M., 2020. T-soft equality relation. Turkish Journal of Mathematics, 44(4), Pp. 1427-1441.

Al-Shami, T. M., 2019. Investigation and corrigendum to some results related to g-soft equality and gf -soft equality relations. Filomat, 33(11), Pp. 3375-3383.

Atagün, A. O. and Sezer, A. S. 2015. Soft sets, soft semimodules and soft substructures of semimodules. Mathematical Sciences Letters, 4(3), Pp. 235-242.

Atagün, A. O. and Sezgin, A., 2018. A new view to near-ring theory: Soft near-rings, South East Asian Journal of Mathematics & Mathematical Sciences, 14(3), Pp. 1-14.

Atagün, A. O. and Sezgin, A., 2022. More on prime, maximal and principal soft ideals of soft rings. New mathematics and natural computation, 18(1), Pp. 195-207.

Atagün, A.O. and Sezgin, A., 2017. Int-soft substructures of groups and semirings with applications, Applied Mathematics & Information Sciences, 11(1), Pp. 105-113.

Atagün, A.O., Kamacı, H., Taştekin, İ. and Sezgin, A., 2019. P-properties in near-rings. Journal of Mathematical and Fundamental Sciences, 51(2), Pp. 152-167.

Çağman, N. and Enginoğlu, S., 2010. Soft set theory and uni-int decision making. European Journal of Operational Research, 207(2), Pp. 848-855.

Eren, Ö. F. and Çalışıcı, H., 2019. On some operations of soft sets. The Fourth International Conference on Computational Mathematics and Engineering Sciences.

Feng, F. and Li, Y., 2013. Soft subsets and soft product operations. Information Sciences, 232(20), Pp. 44-57.

Feng, F., Jun, Y. B. and Zhao, X., 2008. Soft semirings. Computers and Mathematics with Applications, 56(10), Pp. 2621-2628.

Feng, F., Li, Y. M., Davvaz, B. and Ali, M. I., 2010. Soft sets combined with fuzzy sets and rough sets: a tentative approach. Soft Computing, 14, Pp. 899-911.

Fu, L., 2011. Notes on soft set operations, ARPN Journal of Systems and Software, 1, Pp. 205-208.

Ge, X. and Yang, S., 2011. Investigations on some operations of soft sets, World Academy of Science, Engineering and Technology, 75, Pp. 1113-1116.

Gulistan, M., Feng, F., Khan, M., and Sezgin, A., 2018. Characterizations of right weakly regular semigroups in terms of generalized cubic soft sets. Mathematics, No: 6, 293.

Gulistan, M., Shahzad, M., 2014. On soft KU-algebras, Journal of Algebra, Number Theory: Advances and Applications, 11(1), Pp. 1-20.

Jana, C., Pal, M., Karaaslan, F. and Sezgin, A., 2019. (α, β)-soft intersectional rings and ideals with their applications. New Mathematics and Natural Computation, 15(2), Pp. 333–350.

Jiang, Y., Tang, Y., Chen, Q., Wang, J. and Tang, S., 2010. Extending soft sets with description logics. Computers and Mathematics with Applications, 59(6), Pp. 2087-2096.

Jun, Y. B. and Yang, X, 2011. A note on the paper combination of interval-valued fuzzy set and soft set. Computers and Mathematics with Applications, 61(5), Pp. 1468-1470.

Karaaslan, F., 2019. Some properties of AG*-groupoids and AG-bands under SI-product Operation. Journal of Intelligent and Fuzzy Systems, 36(1), Pp. 231-239.

Kaygisiz, K., 2012. On soft int-groups. Annals of Fuzzy Mathematics and Informatics, 4(2), 363–375.

Khan, A., Izhar, I., and Sezgin, A., 2017. Characterizations of Abel Grassmann's Groupoids by the properties of their double-framed soft ideals, International Journal of Analysis and Applications, 15(1), Pp. 62-74.

Khan, M., Ilyas, F., Gulistan, M. and Anis, S., 2015. A study of soft AG-groupoids, Annals of Fuzzy Mathematics and Informatics, 9(4), Pp. 621–638.

Liu, X., Feng, F. and Jun, Y. B., 2012. A note on generalized soft equal relations. Computers and Mathematics with Applications, 64(4), Pp.

572-578.

Mahmood, T., Rehman, Z. U., and Sezgin, A., 2018. Lattice ordered soft near rings. Korean Journal of Mathematics, 26(3), Pp. 503-517

Mahmood, T., Waqas, A., and Rana, M. A., 2015. Soft intersectional ideals in ternary semiring. Science International, 27(5), Pp. 3929-3934.

Maji, P. K., Biswas, R. and Roy, A. R., 2003. Soft set theory. Computers and Mathematics with Application, 45(1), Pp. 555-562.

Manikantan, T., Ramasany, P., and Sezgin, A., 2023. Soft quasi-ideals of soft near-rings, Sigma Journal of Engineering and Natural Science, 41(3), Pp. 565-574.

Memiş, S., 2022. Another view on picture fuzzy soft sets and their product operations with soft decision-making. Journal of New Theory, 38, Pp. 1-13.

Molodtsov, D., 1999. Soft set theory. Computers and Mathematics with Applications, 37(1), Pp. 19-31.

Muştuoğlu, E., Sezgin, A., and Türk, Z.K., 2016. Some characterizations on soft uni-groups and normal soft uni-groups. International Journal of Computer Applications, 155(10), Pp. 1-8.

Neog, I.J and Sut, D.K., 2011. A new approach to the theory of softset, International Journal of Computer Applications, 32(2), Pp. 1-6.

Onyeozili, I. A. and Gwary T. M., 2014. A study of the fundamentals of soft set theory, International Journal of Scientific & Technology Research, 3(4), Pp. 132-143.

Özlü, Ş. and Sezgin, A., 2020. Soft covered ideals in semigroups. Acta Universitatis Sapientiae Mathematica, 12(2), Pp. 317-346.

Pei, D. and Miao, D. From soft sets to information systems, In: Proceedings of Granular Computing (Eds: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang) IEEE, 2, Pp. 617-621.

Qin, K. and Hong, Z., 2010. On soft equality. Journal of Computational and Applied Mathematics, 234(5), Pp. 1347-1355.

Riaz, M., Hashmi, M. R., Karaaslan, F., Sezgin, A., Shamiri, M. M. A. A. and Khalaf, M. M., 2023. Emerging trends in social networking systems and generation gap with neutrosophic crisp soft mapping. CMES-computer modeling in engineering and sciences, 136(2), Pp. 1759-1783.

Sen, J., 2014. On algebraic structure of soft sets. Annals of Fuzzy Mathematics and Informatics, 7(6), Pp. 1013-1020.

Sezer, A. S. and Atagün, A. O., 2016. A new kind of vector space: soft vector space, Southeast Asian Bulletin of Mathematics, 40(5), Pp. 753-770.

Sezer, A. S., 2012. A new view to ring theory via soft union rings, ideals and bi-ideals. Knowledge-Based Systems, 36, Pp. 300–314.

Sezer, A. S., Çağman, N. and Atagün, A. O., 2014. Soft intersection interior ideals, quasi-ideals and generalized bi-ideals; A new approach to semigroup theory II. J. Multiple-Valued Logic and Soft Computing, 23(1-2), Pp. 161-207.

Sezer, A. S., Çağman, N., Atagün, A. O., Ali, M. I. and Türkmen, E., 2015. Soft intersection semigroups, ideals and bi-Ideals; A New application on semigroup theory I. Filomat, 29(5), Pp. 917-946.

Sezer, A., Atagün, A. O. and Çağman, N., 2013. A new view to N-group theory: soft N-groups, Fasciculi Mathematici, 51, Pp. 123-140.

Sezer, A., Atagün, A. O. and Çağman, N., 2017. N-group SI-action and its applications to N-group theory, Fasciculi Mathematici, 52, Pp. 139-153.

Sezgin, A., 2016. A new approach to semigroup theory I: Soft union semigroups, ideals and bi-ideals. Algebra Letters, 2016, 3, Pp. 1-46.

Sezgin, A. and Çağman, N., 2024. A new soft set operation: Complementary soft binary piecewise difference operation. Osmaniye Korkut Ata University Journal of the Institute of Science and Technology, 7(1), Pp. 1-37.

Sezgin, A. and Çağman, N., 2025. An extensive study on restricted and extended symmetric difference operations of soft sets, Utilitas Mathematica. in press.

Sezgin, A. and Çalışıcı, H., 2024. A comprehensive study on soft binary piecewise difference operation, Eskişehir Teknik Üniversitesi Bilim ve Teknoloji Dergisi B- Teorik Bilimler, 12(1), Pp. 1-23.

Sezgin, A. and Dagtoros, K., 2023. Complementary soft binary piecewise symmetric difference operation: A novel soft set operation. Scientific Journal of Mehmet Akif Ersoy University, 6(2), Pp. 31-45.

Sezgin, A. and Demirci, A. M., 2023. A new soft set operation: Complementary soft binary piecewise star operation. Ikonion Journal of Mathematics, 5(2), Pp. 24-52.

Sezgin, A. and İlgin, A., 2024. Soft intersection almost subsemigroups of semigroups. International Journal of Mathematics and Physics, 15(1), Pp. 13-20.

Sezgin, A. and Onur, B., 2024. Soft intersection almost bi-ideals of semigroups. Systemic Analytics, 2(1), Pp. 95-105.

Sezgin, A. and Orbay, M., 2022. Analysis of semigroups with soft intersection ideals, Acta Universitatis Sapientiae, Mathematica, 14(2), Pp. 166-210.

Sezgin, A. and Sarıalioğlu, M., 2024a. A new soft set operation: Complementary soft binary piecewise theta operation. Journal of Kadirli Faculty of Applied Sciences, 4(2), Pp. 325-357.

Sezgin, A. and Sarıalioğlu, M., 2024b. Complementary extended gamma operation: A new soft set operation, Natural and Applied Sciences Journal, 7(1), Pp. 15-44.

Sezgin, A. and Şenyiğit, E., 2025. A new product for soft sets with its decision-making: soft star-product. Big Data and Computing Visions, 5(1), Pp. 52-73.

Sezgin, A. and Yavuz, E., 2023a. A new soft set operation: Soft binary piecewise symmetric difference operation. Necmettin Erbakan University Journal of Science and Engineering, 5(2), Pp. 150-168.

Sezgin, A. and Yavuz, E., 2023b. A new soft set operation: Complementary soft binary piecewise lambda operation. Sinop University Journal of Natural Sciences, 8(2), Pp. 101-133.

Sezgin, A. and Yavuz, E., 2024. Soft binary piecewise plus operation: A new type of operation for soft sets, Uncertainty Discourse and Applications, 1(1), Pp. 79-100.

Sezgin, A, Yavuz, E, and Özlü, Ş. 2024b. Insight into soft binary piecewise lambda operation: a new operation for soft sets. Journal of Umm Al-qura University for Applied Sciences, https://doi.org/10.1007/s43994-024-00187-1, in press.

Sezgin, A., Atagün, A. O and Çağman N., 2025a. A complete study on and-product of soft sets. Sigma Journal of Engineering and Natural Sciences, 43(1), Pp. 1–14.

Sezgin, A., Atagün, A. O., Çağman, N. and Demir, H., 2022. On near-rings with soft union ideals and applications. New Mathematics and Natural Computation, 18(2), Pp. 495-511.

Sezgin, A., Aybek, F. and Atagün, A. O., 2023a. A new soft set operation: Complementary soft binary piecewise intersection operation. Black Sea Journal of Engineering and Science, 6(4), Pp. 330-346.

Sezgin, A., Aybek, F. and Güngör, N. B., 2023b. A new soft set operation: Complementary soft binary piecewise union operation. Acta Informatica Malaysia, 7(1), Pp. 38-53.

Sezgin, A., Çağman, N. and Atagün, A. O., 2017. A completely new view to soft intersection rings via soft uni-int product, Applied Soft Computing, 54, Pp. 366-392.

Sezgin, A., Çağman, N., Atagün, A. O. and Aybek, F., 2023c. Complemental binary operations of sets and their application to group theory. Matrix Science Mathematic, 7(2), Pp. 99-106.

Sezgin, A., Durak, İ. and Ay, Z., 2025b. Some new classifications of soft subsets and soft equalities with soft symmetric difference-difference product of groups. Amesia, 6(1), Pp. 16-32.

Sezgin, A., Onur, B. and İlgin, A., 2024a. Soft intersection almost tri-ideals of semigroups. SciNexuses, 1, Pp. 126-138.

Sezgin, A., Shahzad, A. and Mehmood, A., 2019. A new operation on soft sets: Extended difference of soft sets. Journal of New Theory, 27, Pp. 33-42.

Singh, D. and Onyeozili, I. A., 2012a. Notes on soft matrices operations. ARPN Journal of Science and Technology, 2(9), Pp. 861-869.

Singh, D. and Onyeozili, I. A., 2012c. Some results on distributive and absorption properties on soft operations. IOSR Journal of Mathematics (IOSR-JM), 4(2), Pp. 18-30.

Singh, D. and Onyeozili, I. A., 2012d. Some conceptual misunderstanding of the fundamentals of soft set theory. ARPN Journal of Systems and Software, 2(9), Pp. 251-254.

Singh, D. and Onyeozili, I. A., 2012b. On some new properties on soft set operations. International Journal of Computer Applications, 59(4), Pp. 39-44.

Stojanovic, N. S., 2021. A new operation on soft sets: Extended symmetric difference of soft sets. Military Technical Courier, 69(4), Pp. 779-791.

Tunçay, M. and Sezgin, A., 2016. Soft union ring and its applications to ring theory, International Journal of Computer Applications, 151(9), Pp. 7-13.

Ullah, A., Karaaslan, F. and Ahmad, I., 2018. Soft uni-abel-grassmann's groups. European Journal of Pure and Applied Mathematics, 11(2), Pp. 517-536.

Yang, C. F., 2008. A note on: Soft set theory. Computers and Mathematics with Applications, 56(7), Pp. 1899-1900.

Zadeh, L. A., 1965. Fuzzy sets. Information Control, 8(3), Pp. 338-353.

Zhu, P. and Wen, Q., 2013. Operations on soft sets revisited, Journal of Applied Mathematics, 2013, Article ID 105752, 7 pages.