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REVIEW ARTICLE

PRINCIPLES AND CONCEPT OF DEFORMABLE GEOMETRY

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ABSTRACT

Geometrical objects are not always static, they are subject to changes in real life problems. This work underlines the principles of the topic and gives a systematic reasoning for such problems. The initial geometry, the rules/constraints of deformation and the final deformed geometry are the major steps in treating such problems. Real life example problems are given to exploit the ideas.

KEYWORDS

Geometry, deformation, calculus

1. Introduction And Preliminary Concepts

Geometry is one of the fundamental branches of mathematics for which the education starts in the elementary school curriculum with a continuation in the secondary school and undergraduate level. The geometry has strong ties with other branches of mathematics such as algebra also. The convergence of an alternating series can be visualized by triangular areas (Sinha, 2022). Geometric solutions of quadratic algebraic equations are well-known in the history (Allaire and Bradley, 2001). Usually, geometry is treated as a static discipline, that is, the geometric shape is preserved for all time periods (See Kaufman 2022, 2022a for sample problems). Even the static problems of geometry can be proposed to be extremely hard and there are always misconceptions and misunderstandings in treating geometric problems (George, 2019)

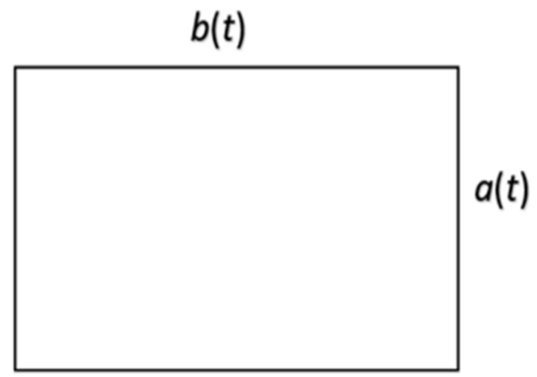
Knowledge of geometry is vital for applied oriented researchers such as engineers, without it, their problems remain unsolved. In real life problems, we usually encounter with geometric shapes subject to changes within time. Such problems can be classified under the topic of “Deformable Geometry” to distinguish it from the static problems. A deformable geometry problem has three crucial steps: 1) The initial geometrical shape, 2) The rules of time variation and/or constraints of deformation, 3) The final shape. To extract the knowledge of the final deformed/changed shape, it is necessary to know the deformation process or constraints under which such deformations take place. The relationships giving the variation of geometric elements such as sides, areas, volumes etc. or changes under the assumptions of constant perimeters/areas/volumes may be considered as the rules/constraints of deformation. This might introduce additional complexity to geometrical problems but we cannot abstain from handling such problems if we are to solve the so-called real-life problems.

Depending on the level of education, such deformable geometry problems can be introduced to students at the levels of secondary school and undergraduate with suitable complexities to their levels. The paper is limited to outline the essential steps and concepts of deformable geometry, and to reach this task, sample problems are treated using the approach.

2. Sample Problems

Several example problems for deformable geometries are posed and solved in this section.

2.1 Example Problem 1



Consider a rectangle with sides a(t) and b(t) changing with time according to the relations

$a(t) = a_0 + a_1 t, \qquad b(t) = b_0 - b_1 t$

with $a_0, a_1, b_0, b_1 > 0$ and $t \le \frac{b_0}{b_1}$ for non-negative side length. The deformation rules are the changes in side lengths in time. The problem is to find the time when the area of the rectangle is maximum and the specific maximum area.

Solution

The area is:

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$A(t) = (a_0 + a_1 t)(b_0 - b_1 t)$

Differentiating with respect to time, equating to zero

$$\frac{dA}{dt} = a_1 b_0 - a_0 b_1 - 2a_1 b_1 t = 0$$

the time for which the area becomes optimum is

$$t_m = \frac{1}{2}\left(\frac{b_0}{b_1} - \frac{a_0}{a_1}\right)$$

with the requirement of $0 \leq t_m \leq \frac{b_0}{b_1}$ f for physical solutions. This area is the maximum area since the second derivative

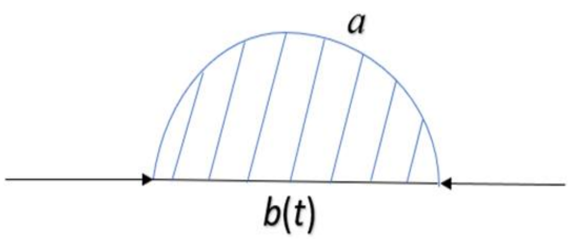
$$\frac{d^2 A}{dt^2} = -2a_1 b_1 < 0$$

turns out to be negative. The maximum area is then

$$A_m = a_0 b_0 + \frac{1}{4}\frac{(a_1 b_0 - a_0 b_1)^2}{a_1 b_1}$$

For a rectangular area with sides $a = 1 + 2t$, $b = 2 - t$, the time where maximum area occurs is $t_m = \frac{3}{4}$ satisfying the condition $0 < \frac{3}{4} < 2$ and the maximum area is $A_m = \frac{25}{8}$.

2.2 Example Problem 2



A strip of length a lays on the ground. When it is squeezed from the ends, it forms a circular arc. The ends approach each other in time with

$$b(t) = a - \alpha\, t$$

where α is the rate of approach. The constraint for the problem is that the length a remains constant during the deformation. The final stage is the half circle. Required quantities are:

- The time to reach the final stage
- The final half circle area between the ground and the strip

Solution

In the final stage, b becomes the diameter of the half circle, i.e., $b = b_f = \frac{2a}{\pi}$ and inserting this quantity into the given relationship and solving for the final time

$$t_f = \frac{a}{\alpha}\left(1 - \frac{2}{\pi}\right)$$

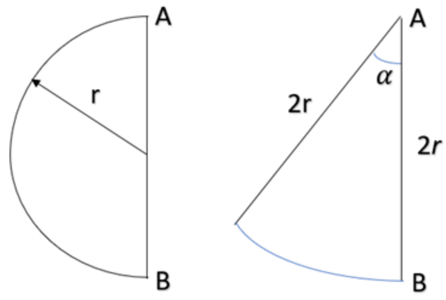
and the corresponding shaded area is

$$A_f = \frac{1}{2}\pi\left(\frac{b_f}{2}\right)^2 = \frac{a^2}{2\pi}$$

For numerical quantities of $a = 40cm, \alpha = 2\frac{cm}{s}$, the results are $t_f = 7.27s$, and $A_f = 254.65cm^2$.

Using calculus, it can be shown that the half circle area is indeed the maximum area during the deformation process. If we squeeze further, the stage of full circle is achieved with an area of $\frac{a^2}{4\pi}$ which is half the area of the half circle.

2.3 Example Problem 3



The half circle with radius r deforms into a piece of circular arc region with radius $2r$ under the effect of an external force. The required quantity is the angle α *if* under deformation

- periphery remains constant
- area remains constant

Solution

- For the first case, since the periphery remains constant during deformation

$$\pi r = 2r + 2r\alpha \qquad \rightarrow \qquad \alpha = \frac{\pi - 2}{2} = 0.57\, rad = 32.7^o$$

The final area is $A_f = (\pi - 2)r^2$ and the initial area is $A_0 = \frac{1}{2}\pi r^2$ with the ratio being

$$\frac{A_f}{A_0} = \frac{2(\pi - 2)}{\pi} = 0.73$$

resulting in a 27% reduction in the area.

- For the second case, since the area remains constant during deformation

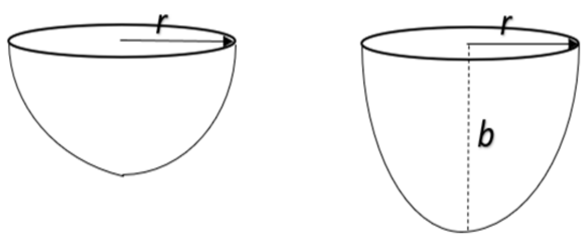
$$\frac{1}{2}\pi r^2 = \frac{1}{2}(2r)^2\alpha \qquad \rightarrow \qquad \alpha = \frac{\pi}{4} = 0.79\, rad = 45^o$$

The final perimeter is $p_f = 2r\alpha + 2(2r) = \left(4 + \frac{\pi}{2}\right)r$ and the initial perimeter is $p_0 = (2 + \pi)r$ with the ratio being

$$\frac{p_f}{p_0} = \frac{4 + \frac{\pi}{2}}{2 + \pi} = 1.08$$

resulting in a 8% increase in the total perimeter.

2.4 Example Problem 4



A half sphere with elastic surface deforms under the action of gravitational forces into a paraboloid. The required quantity is to determine b with the deformation process taking place under the constant volume assumption.

Solution

Since the volume is constant during the deformation,

$$V_{hs} = V_p \qquad \rightarrow \qquad \frac{2}{3}\pi r^3 = \frac{1}{2}\pi r^2 b$$

where it is well known that a paraboloid is half the volume of the cylinder that contains the paraboloid. Hence

$$b = \frac{4}{3}r$$

The woman breast can be considered as half sphere under no external forces and when the upper body of the woman is parallel to the ground, the breast deforms into the shape of a paraboloid. The tissue is elastic and it can be shown from calculus that the surface area increases when the volume remains almost constant under this deformation. Example problem 3, case ii (Constant area assumption) can also be interpreted as a two-dimensional view of the breast before and after deformation when the body is in upright position. Geometry is an idealization and approximation of the real-life problems, hence for more precise solutions, a more involved analysis is inevitable using physical principles.

2.5 Example Problem 5

The circle with radius r_0 deforms and partially becomes a circle of radius r_1 with a small portion of it being triangular due to flattening. The central angle corresponding to the flattened part is α as shown. The problem may be considered as an idealization of the tire problem of an automobile when it touches the ground. The required quantity is to determine r_1 in terms of the given quantities r_0 and α if

- deformation constraint is the constant area assumption
- deformation constraint is the constant periphery assumption

Solution

- Equating the final area to the initial area

$$\frac{2\pi-\alpha}{2\pi}\pi r_1^2 + \frac{1}{2}r_1^2 sin\alpha = \pi r_0^2$$

where the first term is the circular area and the second term is the triangular area of the deformed shape. The final radius is

$$r_1 = \sqrt{\frac{\pi}{\pi-\frac{1}{2}\alpha+\frac{1}{2}sin\alpha}}\, r_0$$

The change in perimeter is

$$\Delta p = p_1 - p_0 = (2\pi-\alpha)r_1 + 2r_1 sin\frac{\alpha}{2} - 2\pi r_0$$

or

$$\Delta p = \left\{\left(2\pi - \alpha + 2sin\frac{\alpha}{2}\right)\sqrt{\frac{\pi}{\pi-\frac{1}{2}\alpha+\frac{1}{2}sin\alpha}} - 2\pi\right\} r_0$$

It can be shown that the quantity is positive. For numerical values of $r_0 = 1$ and $\alpha = \frac{\pi}{18}$, $r_1 = 1.00007$ and $\Delta p = 0.00022$. Under the constant area constraint of deformation, the perimeter increases for this specific problem.

ii) Equating the final periphery to the initial one

$$(2\pi-\alpha)r_1 + 2r_1 sin\frac{\alpha}{2} = 2\pi r_0$$

The final radius is

$$r_1 = \frac{2\pi r_0}{2\pi - \alpha + 2sin\frac{\alpha}{2}}$$

The change in area is

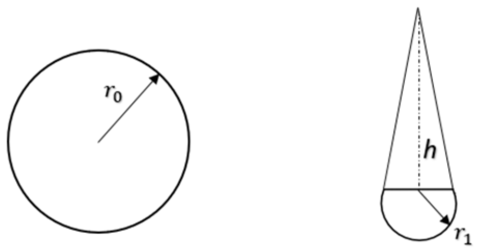
$$\Delta A = \frac{2\pi-\alpha}{2\pi}\pi r_1^2 + \frac{1}{2}r_1^2 sin\alpha - \pi r_0^2$$

or

$$\Delta A = \left\{\left(\pi - \frac{1}{2}\alpha + \frac{1}{2}sin\alpha\right)\frac{4\pi^2}{\left(2\pi-\alpha+2sin\frac{\alpha}{2}\right)^2} - \pi\right\} r_0^2$$

For numerical values of $r_0 = 1$ and $\alpha = \frac{\pi}{18}$, $r_1 = 1.00004$ and $\Delta A = -0.00022$ which resulted in a reduction in the area.

2.6 Example Problem 6



The figures may represent the cross-sectional areas (side view) of a rain drop before and after deformation. The circular section of the drop changes approximately to a small half circle and a triangle with the shaping of drag forces. The required quantity is to find r_1 in terms of r_0 and h if

- the deformation occurs under constant area assumption
- the deformation occurs under constant periphery assumption

Solution

- Initial step is to equate the areas after and before deformation

$$\frac{\pi r_1^2}{2} + r_1 h = \pi r_0^2$$

and solving the equation for the unknown which is a quadratic equation, the final radius is

$$r_1 = -\frac{h}{\pi} + \sqrt{\left(\frac{h}{\pi}\right)^2 + 2r_0^2} \quad .$$

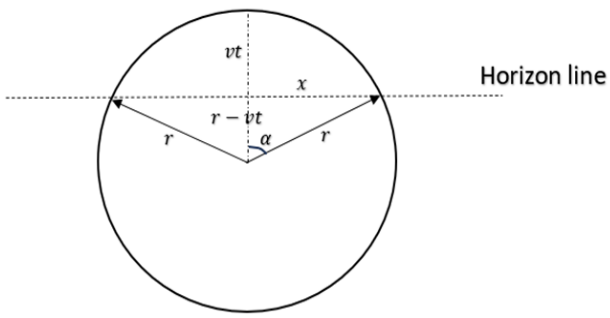
- For equivalent peripheries after and before deformation

$$\pi r_1 + 2\sqrt{r_1^2 + h^2} = 2\pi r_0$$

with the final radius being

$$r_1 = \frac{2\pi^2 r_0}{\pi^2-4} + \frac{2}{\pi^2-4}\sqrt{\pi^2(h^2+4r_0^2) - 4h^2}$$

2.7 Example Problem 7



Assume that the sun rises above the horizon with a constant velocity v. The aim is to calculate the area of the visible top part of the sun as a function of time t $(0 \le t \le \frac{2r}{v})$. The area above the horizon can be found by subtracting the area of the circular arc with a central angle 2α from the triangular area,

$$A(t) = r^2\alpha - x(r - vt)\,.$$

From Pythagorean theorem

$x = \sqrt{2rvt - v^2t^2}$.

The angle in terms of the given quantities is

$\alpha = Arccos\left(1 - \frac{vt}{r}\right)$.

Hence the area variation in time is

$A(t) = r^2 Arccos\left(1 - \frac{vt}{r}\right) - \sqrt{2rvt - v^2t^2}(r - vt)$

The solution can be checked for known cases:

At the initial stage $A(0) = 0$ as there is no area visible and the sun is tangent to the horizon from below. At $t = \frac{r}{v}$, $A\left(\frac{r}{v}\right) = \frac{\pi r^2}{2}$ and the upper half of the sun is visible. Finally, at $t = \frac{2r}{v}$, $A\left(\frac{2r}{v}\right) = \pi r^2$, the sun is completely visible.

3. Concluding Remarks

In real world problems, the geometric shapes are subject to changes which can be called as deformations. The deformable geometry discipline arises from investigating such problems. One needs an initial geometric shape, a time variation rule and/or constraint of deformation to calculate the final geometric shape. Basic examples which do not require a preliminary knowledge of mathematics higher than the junior high school level are treated to exploit the ideas. More complex problems can be posed involving calculus and even variational calculus for undergraduate and beginning graduate level students.

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