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SOFT INTERSECTION-SYMMETRIC DIFFERENCE PRODUCT OF GROUPS

Aslihan Sezgin^{a*}, İbrahim Durak^b^aDepartment of Mathematics and Science Education, Faculty of Education, Amasya University, Amasya, Türkiye^bDepartment of Mathematics, Graduate School of Natural and Applied Sciences, Amasya University, Amasya, Türkiye*Corresponding Author Email: aslihan.sezgin@amasya.edu.tr

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ABSTRACT

Soft set theory constitutes a mathematically robust and structurally versatile formalism for modeling real-world systems characterized by epistemic uncertainty, vagueness, and parameter-contingent variability—ubiquitous features across decision theory, engineering, economics, and information sciences. At the core of this framework lies a spectrum of algebraic operations and binary product constructions that endow the soft set universe with a rich internal structure, capable of encapsulating intricate interdependencies among parameters. In this context, we introduce and investigate a novel product of soft sets, termed the soft intersection–symmetric difference product, formulated specifically for soft sets whose parameter domains are structured as groups. This product is rigorously defined and analyzed within an axiomatic framework that ensures compatibility with generalized soft subsethood and equality relations. The structural analysis of the soft intersection–symmetric difference product includes the examination of essential algebraic properties—such as closure, associativity, commutativity, and idempotency. In addition, the interplay between this product and pre-existing soft products, is explored regarding the subsets. Theoretical investigations reveal that the operation not only respects the algebraic architecture of the underlying group-parameterized domain but also induces a cohesive and well-behaved algebraic system on the collection of soft sets. This analytical framework yields two central algebraic insights: (i) the internal algebraic cohesion of soft set theory is significantly enhanced by embedding the newly defined product into a logically sound and operation-preserving environment; and (ii) the product itself possesses the formal potential to serve as a foundational construct for a generalized soft group theory, wherein soft sets over group-parameter spaces mimic the axiomatic behavior of classical group-theoretic constructs through suitably defined soft operations. Given that the maturation of soft algebraic systems is contingent upon the rigorous formulation of operations satisfying structurally meaningful axioms, the contributions of this study represent a notable advancement in the algebraic consolidation of soft set theory. Beyond theoretical enrichment, the proposed operation offers tangible utility in the construction of abstract algebra-based soft computational models, with applications spanning multi-criteria decision-making, algebraically-driven classification mechanisms, and uncertainty-aware data analysis governed by group-parametrized semantic domains. Thus, the framework established herein not only extends the theoretical boundaries of soft algebra but also fortifies its role as a foundational tool in both pure and applied mathematical discourse

KEYWORDS

Soft sets; Soft subsets; Soft equalities; Soft intersection-symmetric difference product.

1. INTRODUCTION

A vast array of advanced mathematical frameworks has been devised to represent and analyze systems permeated by uncertainty, vagueness, and indeterminacy—phenomena that frequently emerge in engineering, economics, social sciences, and medical diagnostics. Despite the conceptual sophistication of such paradigms, including fuzzy set theory and probabilistic models, critical epistemological and algebraic limitations persist. Fuzzy set theory, as pioneered hinges on subjectively chosen membership functions, while probabilistic approaches presuppose the availability of repeatable events and known distributional profiles—assumptions which are often untenable in real-world contexts governed by epistemic ambiguity (Zadeh, 1965). In a groundbreaking contribution, circumvented these structural constraints by formulating

soft set theory as a mathematically elastic yet axiomatically minimalistic framework that models uncertainty relative to parameter sets, rather than probabilistic or membership-theoretic axioms (Molodtsov, 1999).

The initial formalism of soft sets has undergone systematic algebraic refinement since 2003. Foundational operations such as union, intersection, and AND/OR-products were introduced, while recontextualized these operations through information-theoretic lenses, rendering them compatible with relational and multivalued contexts (Maji et al., 2003; Pei et al., 2005). This study, extended the operational schema by defining restricted and extended variants, enhancing the algebraic granularity of soft systems (Ali et al., 2009). Subsequent research—including the works of further deepened the algebraic infrastructure of soft set theory, resolving semantic ambiguities and introducing a spectrum of new product operations and generalized equalities (Yang, 2008; Feng et al., 2010; Jiang et al., 2010; Ali et al., 2011;

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Jiang et al., 2010; Ali et al., 2011; Neog and Sut, 2011; Fu, 2011; Ge and Yang, 2011; Singh and Onyeozili, 2012a–d; Zhu and Wen, 2013; Onyeozili and Gwary, 2014; Sen, 2014). Recent developments have significantly enriched the algebraic foundations of soft set theory through the introduction of a wide array of novel operations, each rigorously analyzed within formal algebraic frameworks. Noteworthy contributions in this regard include the works of whose investigations have collectively established a robust and extensible algebraic landscape for the continued advancement of soft set theory (Eren and Çalışıcı, 2019; Stojanović, 2021; Sezgin et al., 2023a, 2023b; Sezgin and Dağtoros, 2023; Sezgin and Demirci, 2023; Sezgin and Çalışıcı, 2024; Sezgin and Yavuz, 2023a, 2023b; 2024; Sezgin and Çağman, 2024, 2025; Sezgin and Sarıaloğlu, 2024a, 2024b; Sezgin and Şenyiğit, 2025).

The algebraic formalization of soft equality and soft inclusion has evolved into a cornerstone of modern soft algebra. The classical notion of soft subsets proposed was generalized and developed soft congruence relations that embedded equivalence structures into the soft set universe (Maji et al., 2003; Pei and Miao, 2005; Feng et al., 2010; Qin and Hong, 2010). This study, advanced the algebraic semantics by introducing J-soft equalities and new distributive frameworks, and revealed profound algebraic divergence in their definition of L-soft subsets and L-equalities, wherein traditional distributive identities fail to hold universally (Çağman and Enginoğlu, 2010; Liu et al., 2012). These foundational results were extended by who rigorously categorized soft subset types and established associativity, commutativity, and distributivity criteria under L-equality, proving that certain classes of quotient soft algebras admit commutative semigroup structures (Feng et al., 2013). In addition, the interplay between this product and pre-existing soft products, is explored to regarding the subsets. Further generalizations—such as g-soft, gf-soft, and T-soft equalities—have been developed within lattice-enriched frameworks by marking a paradigm shift toward lattice-theoretic and congruence-based perspectives (Abbas et al., 2014, 2017; Alshami, 2019; Alshami et al., 2020).

In a pivotal intervention, reconstructed the definitional foundations of soft set operations to rectify inconsistencies in the original formulation, thereby enabling a robust algebraic treatment (Çağman and Enginoğlu, 2010). Parallel developments have focused on soft products over algebraic domains. The soft intersection–union product has been extended to rings, semigroups and groups, yielding structurally consistent notions of soft union rings, semigroups, and groups (Sezer, 2012; Sezgin, 2016; Muştuoğlu et al., 2016). Conversely, the soft union–intersection product has been investigated in group-theoretic, semigroup-theoretic, and ring-theoretic contexts, with algebraic properties contingent on the behavior of identity and inverse elements in the parameter set (Kaygısız, 2012; Sezer et al., 2015; Sezgin et al., 2017).

In response to these advances, the present study proposes a novel product of soft sets—the soft intersection–symmetric difference product—formulated over soft sets whose parameter domains are endowed with group structure. This operation is subjected to a comprehensive algebraic examination, emphasizing its compatibility with generalized soft inclusion and equality relations. The structural analysis of the soft intersection–symmetric difference product includes the examination of essential algebraic properties—such as closure, associativity, distributivity (both left and right), and compatibility with identity and absorbing elements. Moreover, the proposed operation is subjected to a comprehensive comparative assessment against previously formulated soft products within the structured hierarchy of soft subset classifications, providing enhanced theoretical clarity regarding their respective expressive strengths and algebraic coherence. In parallel, a rigorous examination of the product’s interaction with both the null and absolute soft sets is undertaken to further articulate its foundational structural properties. Our results demonstrate that the proposed product adheres to desirable axiomatic criteria, while introducing a structurally coherent mechanism for combining soft information across parametric group domains. The product facilitates a natural extension of classical algebraic concepts to the soft domain, allowing the construction of soft analogues of group-theoretic entities, and laying the groundwork for a new mathematical branch—soft group theory—defined through rigorously axiomatized binary operations. The remainder of this manuscript is structured as follows: Section 2 revisits foundational definitions and formal preliminaries. Section 3 introduces the soft intersection–symmetric difference product and develops its algebraic theory in detail. Section 4 synthesizes the primary results and delineates directions for future research aimed at expanding the algebraic universe of soft sets and exploring their applications in abstract algebra and uncertainty modeling.

2. PRELIMINARIES

This section presents a rigorous and methodical re-evaluation of the foundational definitions and algebraic underpinnings that serve as the formal substrate for the theoretical constructs elaborated in the subsequent discourse. While the original conception of soft set theory was introduced as a parameterized generalization for modeling uncertainty, its formal definitional schema and operational calculus were substantially restructured in the influential reformulation (Molodtsov, 1999; Çağman and Enginoğlu, 2010). Their axiomatic revision endowed the theory with heightened structural coherence and broadened its applicability across diverse algebraic and decision-theoretic settings. The present investigation adopts this refined formalism as the axiomatic foundation upon which all further constructions are based. Accordingly, every algebraic development, operational specification, and theoretical generalization in the forthcoming sections is rigorously articulated within this enhanced framework, ensuring both internal consistency and formal adherence to contemporary standards in soft algebraic systems.

Definition 2.1. (Çağman and Enginoğlu, 2010) Let E be a parameter set, U be a universal set, $P(U)$ be the power set of U , and $\mathcal{H} \subseteq E$. Then, the soft set $f_{\mathcal{H}}$ over U is a function such that $f_{\mathcal{H}}: E \rightarrow P(U)$, where for all $w \notin \mathcal{H}$, $f_{\mathcal{H}}(w) = \emptyset$. That is,

$$f_{\mathcal{H}} = \{(w, f_{\mathcal{H}}(w)): w \in E\}$$

From now on, the soft set over U is abbreviated by \mathcal{SS} .

Definition 2.2. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an \mathcal{SS} . If $f_{\mathcal{H}}(w) = \emptyset$ for all $w \in E$, then $f_{\mathcal{H}}$ is called a null \mathcal{SS} and indicated by \emptyset_E , and if $f_{\mathcal{H}}(w) = U$, for all $w \in E$, then $f_{\mathcal{H}}$ is called an absolute \mathcal{SS} and indicated by U_E .

Definition 2.3. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be a soft set over U . Then, the complement of $f_{\mathcal{H}}$ denoted by $f_{\mathcal{H}}^c$, is defined by the soft set $f_{\mathcal{H}}^c: E \rightarrow P(U)$ such that $f_{\mathcal{H}}^c(e) = U \setminus f_{\mathcal{H}}(e) = (f_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 2.4. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ and $g_{\mathcal{N}}$ be two \mathcal{SS} s. Then, the difference of $f_{\mathcal{H}}$ and $g_{\mathcal{N}}$ is the \mathcal{SS} $f_{\mathcal{H}} \setminus g_{\mathcal{N}}$, where $(f_{\mathcal{H}} \setminus g_{\mathcal{N}})(w) = f_{\mathcal{H}}(w) \setminus g_{\mathcal{N}}(w)$, for all $w \in E$.

Definition 2.5. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ and $g_{\mathcal{N}}$ be two \mathcal{SS} s. If $f_{\mathcal{H}}(w) \subseteq g_{\mathcal{N}}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is a soft subset of $g_{\mathcal{N}}$ and indicated by $f_{\mathcal{H}} \subseteq g_{\mathcal{N}}$. If $f_{\mathcal{H}}(w) = g_{\mathcal{N}}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is called soft equal to $g_{\mathcal{N}}$, and denoted by $f_{\mathcal{H}} = g_{\mathcal{N}}$.

Definition 2.6. (Sezgin et al., 2025b) Let $f_{\mathcal{K}}$ and $g_{\mathcal{N}}$ be two \mathcal{SS} s. Then, $f_{\mathcal{K}}$ is called a soft S-subset of $g_{\mathcal{N}}$, denoted by $f_{\mathcal{K}} \subseteq_S g_{\mathcal{N}}$ if for all $w \in E$, $f_{\mathcal{K}}(w) = \mathcal{M}$ and $g_{\mathcal{N}}(w) = \mathcal{D}$, where \mathcal{M} and \mathcal{D} are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two \mathcal{SS} s $f_{\mathcal{K}}$ and $g_{\mathcal{N}}$ are said to be soft S-equal, denoted by $f_{\mathcal{K}} =_S g_{\mathcal{N}}$, if $f_{\mathcal{K}} \subseteq_S g_{\mathcal{N}}$ and $g_{\mathcal{N}} \subseteq_S f_{\mathcal{K}}$.

It is obvious that if $f_{\mathcal{K}} =_S g_{\mathcal{N}}$, then $f_{\mathcal{K}}$ and $g_{\mathcal{N}}$ are the same constant functions, that is, for all $w \in E$, $f_{\mathcal{K}}(w) = g_{\mathcal{N}}(w) = \mathcal{M}$, where \mathcal{M} is a fixed set.

Definition 2.7. (Sezgin et al., 2025b) Let $f_{\mathcal{K}}$ and $g_{\mathcal{N}}$ be two \mathcal{SS} s. Then, $f_{\mathcal{K}}$ is called a soft A-subset of $g_{\mathcal{N}}$, denoted by $f_{\mathcal{K}} \subseteq_A g_{\mathcal{N}}$ if, for each $a, b \in E$, $f_{\mathcal{K}}(a) \subseteq g_{\mathcal{N}}(b)$.

Definition 2.8. (Sezgin et al., 2025b) Let $f_{\mathcal{K}}$ and $g_{\mathcal{N}}$ be two \mathcal{SS} s. Then, $f_{\mathcal{K}}$ is called a soft S-complement of $g_{\mathcal{N}}$, denoted by $f_{\mathcal{K}} =_S (g_{\mathcal{N}})^c$, if, for all $w \in E$, $f_{\mathcal{K}}(w) = \mathcal{M}$ and $g_{\mathcal{N}}(w) = \mathcal{D}$, where \mathcal{M} and \mathcal{D} are two fixed sets and $\mathcal{M} = \mathcal{D}'$.

From now on, let G be a group, and $S_G(U)$ denotes the collection of all \mathcal{SS} s over U , whose parameter sets are G ; that is, each element of $S_G(U)$ is an \mathcal{SS} parameterized by G .

Definition 2.9. (Sezgin and Ay, 2025) Let f_G and g_G be two \mathcal{SS} s. Then, the intersection-difference product $f_G \otimes_{i/d} g_G$ is defined by

$$(f_G \otimes_{i/d} g_G)(x) = \bigcap_{x=yz} (f_G(y) \setminus g_G(z)), \quad y, z \in G$$

for all $x \in G$.

For additional information on \mathcal{SS} s, we refer to (Aktas and Çağman, 2007; Alcantud et al., 2024; Ali et al., 2015; Ali et al., 2022; Atagün et al., 2019; Atagün and Sezer, 2015; Atagün and Sezgin, 2017; Atagün and Sezgin, 2018; Atagün and Sezgin, 2022; Feng et al., 2008; Gulistan and Shahzad, 2014; Gulistan et al., 2018; Jana et al., 2019; Karaaslan, 2019; Khan et al., 2017; Mahmood et al., 2015; Mahmood et al., 2018; Manikantan et al., 2023; Memiş, 2022; Özlü and Sezgin, 2020; Riaz et al., 2023; Sezer and

Atağün, 2016; Sezer et al., 2017; Sezer et al., 2013; Sezer et al., 2014; Sezgin and İlgin, 2024; Sezgin et al., 2022; Sezgin and Onur, 2024; Sezgin et al., 2024; Sezgin and Orbay, 2022; Sezgin et al., 2019; Sun et al., 2008; Tunçay and Sezgin, 2016; Ullah et al., 2018; Sezgin et al., 2024a, 2024b).

3. SOFT INTERSECTION-SYMMETRIC DIFFERENCE PRODUCT OF GROUPS

In this section, we introduce a novel product off soft sets, termed the soft intersection-symmetric difference product, defined over parameter sets endowed with group structures. We undertake a thorough algebraic analysis aimed at rigorously characterizing the operation's fundamental structural attributes. Special attention is devoted to elucidating the interplay between this product and various generalized soft equality relations, alongside the hierarchical classification of soft subsets under diverse inclusion frameworks. To bridge abstract theory with concrete insight, the exposition incorporates a curated collection of illustrative examples that demonstrate the operational dynamics and algebraic subtleties inherent to the product. Additionally, we explore the relation between the proposed product and some other certain soft products with respect to the soft subsets, thereby clarifying its algebraic compatibility within the existing operational landscape. This examination effectively highlights the proposed soft product's structural coherence and potential for integration into more comprehensive soft algebraic systems.

Definition 3.1:

Let f_G and g_G be two \mathcal{SS} s. Then, the soft intersection-symmetric difference product $f_G \otimes_{i/s} g_G$ is defined by

$$(f_G \otimes_{i/s} g_G)(x) = \bigcap_{x=yz} (f_G(y) \Delta g_G(z)), \quad y, z \in G$$

for all $x \in G$.

Note here that since G is a group, there always exist $y, z \in G$ such that $x = yz$, for all $x \in G$. Let the order of the group G be n , that is, $|G| = n$. Then, it is obvious that there exist n different combinations of writing styles for each $x \in G$ such that $x = yz$, where $y, z \in G$.

Note 3.2: The soft intersection-symmetric difference product is well-defined in $S_G(U)$. In fact, let $f_G, g_G, m_G, k_G \in S_G(U)$ such that $(f_G, g_G) = (m_G, k_G)$. Then, $f_G = m_G$ and $g_G = k_G$, implying that $f_G(x) = m_G(x)$ and $g_G(x) = k_G(x)$, for all $x \in G$. Thereby, for all $x \in G$,

$$\begin{aligned} (f_G \otimes_{i/s} g_G)(x) &= \bigcap_{x=yz} (f_G(y) \Delta g_G(z)) \\ &= \bigcap_{x=yz} (m_G(y) \Delta k_G(z)) \\ &= (m_G \otimes_{i/s} k_G)(x) \end{aligned}$$

Hence, $f_G \otimes_{i/s} g_G = m_G \otimes_{i/s} k_G$.

Example 3.3: Consider the group $G = \{\rho, \tau\}$ with the following operation:

| | | |
|---|---|---|
| · | ρ | τ |
| ρ | ρ | τ |
| τ | τ | ρ |

Let f_G and g_G be two \mathcal{SS} s over $U = D_2 = \{<x, y>: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$$f_G = \{(\rho, \{e, x\}), (\tau, \{x, yx\})\} \text{ and } g_G = \{(\rho, \{x, y\}), (\tau, \{e, x, yx\})\}$$

Since $\rho = \rho\rho = \tau\tau$, $(f_G \otimes_{i/s} g_G)(\rho) = (f_G(\rho) \Delta g_G(\rho)) \cap (f_G(\tau) \Delta g_G(\tau)) = \{e\}$, and since $\tau = \rho\tau = \tau\rho$, $(f_G \otimes_{i/s} g_G)(\tau) = (f_G(\rho) \Delta g_G(\tau)) \cap (f_G(\tau) \Delta g_G(\rho)) = \{yx\}$ is obtained. Hence,

$$f_G \otimes_{i/s} g_G = \{(\rho, \{e\}), (\tau, \{yx\})\}$$

Proposition 3.4:

The set $S_G(U)$ is closed under the soft intersection-symmetric difference product. That is, if f_G and g_G are two \mathcal{SS} s, then so is $f_G \otimes_{i/s} g_G$.

PROOF. It is obvious that the soft intersection-symmetric difference product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft intersection-symmetric difference product. \square

Proposition 3.5:

The soft intersection-symmetric difference product is not associative in $S_G(U)$.

PROOF. Consider the group G and the \mathcal{SS} s f_G and g_G in Example 3.3, and let $h_G = \{(\rho, \{yx\}), (\tau, \{e, x, y\})\}$ be an \mathcal{SS} over $U = \{e, x, y, yx\}$.

Since $f_G \otimes_{i/s} g_G = \{(\rho, \{e\}), (\tau, \{yx\})\}$, then

$$(f_G \otimes_{i/s} g_G) \otimes_{i/s} h_G = \{(\rho, \{e, yx\}), (\tau, \emptyset)\}$$

Moreover, since $g_G \otimes_{i/s} h_G = \{(\rho, \{y, yx\}), (\tau, \{e\})\}$, then

$$f_G \otimes_{i/s} (g_G \otimes_{i/s} h_G) = \{(\rho, \{e, x, yx\}), (\tau, \{x\})\}$$

Thereby, $(f_G \otimes_{i/s} g_G) \otimes_{i/s} h_G \neq f_G \otimes_{i/s} (g_G \otimes_{i/s} h_G)$. \square

Proposition 3.6:

The soft intersection-union product is not commutative in $S_G(U)$. However, if G is an abelian group, then the intersection-union product is commutative in $S_G(U)$.

PROOF. Let f_G, g_G be two \mathcal{SS} s and G be an abelian group. Then, for all $x \in G$,

$$\begin{aligned} (f_G \otimes_{i/s} g_G)(x) &= \bigcap_{x=yz} (f_G(y) \Delta g_G(z)) \\ &= \bigcap_{x=zy} (g_G(z) \Delta f_G(y)) \\ &= (g_G \otimes_{i/s} f_G)(x) \end{aligned}$$

implying that $f_G \otimes_{i/s} g_G = g_G \otimes_{i/s} f_G$. \square

Proposition 3.7:

The soft intersection-symmetric difference product is not idempotent in $S_G(U)$.

PROOF. Consider the \mathcal{SS} f_G in Example 3.3. Then,

$$f_G \otimes_{i/s} f_G = \{(\rho, \emptyset), (\tau, \emptyset)\}$$

implying that $f_G \otimes_{i/s} f_G \neq f_G$. \square

Proposition 3.8:

Let f_G be a constant \mathcal{SS} . Then, $f_G \otimes_{i/s} f_G = \emptyset_G$.

PROOF. Let f_G be a constant \mathcal{SS} such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(f_G \otimes_{i/s} f_G)(x) = \bigcap_{x=yz} (f_G(y) \Delta f_G(z)) = \emptyset_G(x)$$

Thereby, $f_G \otimes_{i/s} f_G = \emptyset_G$. \square

Remark 3.9:

Let $S_G^*(U)$ be the collection of all constant \mathcal{SS} s. Then, the soft intersection-symmetric difference product is not idempotent in $S_G^*(U)$ either.

Proposition 3.10:

Let f_G be a constant \mathcal{SS} . Then, $f_G \otimes_{i/s} \emptyset_G = \emptyset_G \otimes_{i/s} f_G = f_G$.

PROOF. Let f_G be a constant \mathcal{SS} such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(f_G \otimes_{i/s} \emptyset_G)(x) = \bigcap_{x=yz} (f_G(y) \Delta \emptyset_G(z)) = \bigcap_{x=yz} (f_G(y) \Delta \emptyset) = f_G(x)$$

Thereby, $f_G \otimes_{i/s} \emptyset_G = f_G$. Similarly, for all $x \in G$,

$$\begin{aligned}
 (\emptyset_G \otimes_{i/s} \mathcal{f}_G)(x) &= \bigcap_{x=yz} (\emptyset_G(y) \Delta \mathcal{f}_G(z)) = \bigcap_{x=yz} (\emptyset \Delta \mathcal{f}_G(z)) = \mathcal{f}_G(x) &= \bigcup_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z))' \\
 & &= \bigcup_{x=yz} (\mathcal{f}_G(y) \sqcup \mathcal{g}_G(z)) \\
 & &= (\mathcal{f}_G \otimes_{u/s'} \mathcal{g}_G)^c(x)
 \end{aligned}$$

Thereby, $\emptyset_G \otimes_{i/s} \mathcal{f}_G = \mathcal{f}_G$.

Remark 3.11:

\emptyset_G is the identity element of the soft intersection-symmetric difference product in $S_G^*(U)$. Besides, the inverse of each element is itself in $S_G^*(U)$ with respect to the soft intersection-symmetric difference product by Proposition 3.8.

Proposition 3.12:

Let \mathcal{f}_G be a constant \mathcal{SS} . Then, $\mathcal{f}_G \otimes_{i/s} U_G = U_G \otimes_{i/s} \mathcal{f}_G = \mathcal{f}_G^c$.

PROOF. Let \mathcal{f}_G be a constant \mathcal{SS} such that, for all $x \in G$, $\mathcal{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(\mathcal{f}_G \otimes_{i/s} U_G)(x) = \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta U_G(z)) = \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta U) = \mathcal{f}_G^c(x)$$

Thereby, $\mathcal{f}_G \otimes_{u/s} U_G = \mathcal{f}_G^c$. Similarly, for all $x \in G$,

$$(U_G \otimes_{i/s} \mathcal{f}_G)(x) = \bigcap_{x=yz} (U_G(y) \Delta \mathcal{f}_G(z)) = \bigcap_{x=yz} (U \Delta \mathcal{f}_G(z)) = \mathcal{f}_G^c(x)$$

Thereby, $U_G \otimes_{i/s} \mathcal{f}_G = \mathcal{f}_G^c$.

Proposition 3.13:

Let \mathcal{f}_G be a constant \mathcal{SS} . Then, $\mathcal{f}_G \otimes_{i/s} \mathcal{f}_G^c = \mathcal{f}_G^c \otimes_{i/s} \mathcal{f}_G = U_G$.

PROOF. Let \mathcal{f}_G be a constant \mathcal{SS} such that, for all $x \in G$, $\mathcal{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(\mathcal{f}_G \otimes_{i/s} \mathcal{f}_G^c)(x) = \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{f}_G^c(z)) = U = U_G(x)$$

Thereby, $\mathcal{f}_G \otimes_{i/s} \mathcal{f}_G^c = U_G$. Similarly, for all $x \in G$,

$$(\mathcal{f}_G^c \otimes_{i/s} \mathcal{f}_G)(x) = \bigcap_{x=yz} (\mathcal{f}_G^c(y) \Delta \mathcal{f}_G(z)) = U = U_G(x)$$

Thus, $\mathcal{f}_G^c \otimes_{i/s} \mathcal{f}_G = U_G$. \square

Theorem 3.14:

Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s. Then, $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = U_G$ if only if $\mathcal{f}_G =_s (\mathcal{g}_G)^c$.

PROOF. Let $\mathcal{f}_G =_s (\mathcal{g}_G)^c$. Then, for all $x \in G$, $\mathcal{f}_G(x) = A$ and $\mathcal{g}_G(x) = B$, where A and B are two fixed sets and $A = B'$. Hence, for all $x \in G$,

$$(\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)(x) = \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) = U = U_G(x)$$

Thereby, $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = U_G$.

Conversely, suppose that $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = U_G$. That is, $(\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)(x) = U_G(x)$, for each $x \in G$. Then, for all $x \in G$,

$$U_G(x) = (\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)(x) = \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) = U$$

This implies that $\mathcal{f}_G(y) \Delta \mathcal{g}_G(z) = U$, for all $y, z \in G$. Thus, $\mathcal{f}_G =_s (\mathcal{g}_G)^c$. \square

Proposition 3.15:

Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s. Then, $(\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)^c = \mathcal{f}_G \otimes_{u/s'} \mathcal{g}_G$.

PROOF. Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s. Then, for all $x \in G$,

$$(\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)^c(x) = \left(\bigcap_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) \right)'$$

Thus, $(\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G) = \mathcal{f}_G \otimes_{u/s'} \mathcal{g}_G$. For more on the symmetric difference complement (\sqcup) operation, we refer to (Ay and Sezgin, 2025).

Proposition 3.16:

Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s. If $\mathcal{g}_G \subseteq_A \mathcal{f}_G$, then $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = \mathcal{f}_G \otimes_{i/d} \mathcal{g}_G$.

PROOF. Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s such that $\mathcal{g}_G \subseteq_A \mathcal{f}_G$. Then, for each $x, y \in G$, $\mathcal{g}_G(x) \subseteq \mathcal{f}_G(y)$. Thus, for all $x \in G$,

$$\begin{aligned}
 (\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)(x) &= \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) = \bigcap_{x=yz} (\mathcal{f}_G(y) \setminus \mathcal{g}_G(z)) \\
 &= (\mathcal{f}_G \otimes_{i/d} \mathcal{g}_G)(x)
 \end{aligned}$$

Thus, $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = \mathcal{f}_G \otimes_{i/d} \mathcal{g}_G$.

Remark 3.17:

Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s. If $\mathcal{g}_G \subseteq_s \mathcal{f}_G$, then $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = \mathcal{f}_G \setminus \mathcal{g}_G$.

Proposition 3.18:

Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s. Then, $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G \subseteq \mathcal{f}_G \otimes_{u/s} \mathcal{g}_G$.

PROOF. Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s. Then, for all $x \in G$,

$$\begin{aligned}
 (\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)(x) &= \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) \\
 &\subseteq \bigcup_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) \\
 &= (\mathcal{f}_G \otimes_{u/s} \mathcal{g}_G)(x)
 \end{aligned}$$

Thus, $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G \subseteq \mathcal{f}_G \otimes_{u/s} \mathcal{g}_G$. \square

Proposition 3.19:

Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s. If one of the following assertions is satisfied, then $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = \mathcal{f}_G \otimes_{u/s} \mathcal{g}_G$:

- i. $\mathcal{f}_G \subseteq_s \mathcal{g}_G$
- ii. $\mathcal{g}_G \subseteq_s \mathcal{f}_G$
- iii. $\mathcal{f}_G =_s (\mathcal{g}_G)^c$

PROOF. Let \mathcal{f}_G and \mathcal{g}_G be two \mathcal{SS} s.

i. Let $\mathcal{f}_G \subseteq_s \mathcal{g}_G$. Hence, for all $x \in G$, $\mathcal{f}_G(x) = A$ and $\mathcal{g}_G(x) = B$, where A and B are two fixed sets and $A \subseteq B$. Then, for all $x \in G$,

$$\begin{aligned}
 (\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)(x) &= \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) \\
 &= \bigcup_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) \\
 &= (\mathcal{f}_G \otimes_{u/s} \mathcal{g}_G)(x)
 \end{aligned}$$

Thereby, $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = \mathcal{f}_G \otimes_{u/s} \mathcal{g}_G$.

ii. Let $\mathcal{g}_G \subseteq_s \mathcal{f}_G$. Hence, for all $x \in G$, $\mathcal{f}_G(x) = A$ and $\mathcal{g}_G(x) = B$, where A and B are two fixed sets and $B \subseteq A$. Then, for all $x \in G$,

$$\begin{aligned}
 (\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G)(x) &= \bigcap_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) \\
 &= \bigcup_{x=yz} (\mathcal{f}_G(y) \Delta \mathcal{g}_G(z)) \\
 &= (\mathcal{f}_G \otimes_{u/s} \mathcal{g}_G)(x)
 \end{aligned}$$

Thereby, $\mathcal{f}_G \otimes_{i/s} \mathcal{g}_G = \mathcal{f}_G \otimes_{u/s} \mathcal{g}_G$.

iii. Let $f_G =_S (g_G)^c$. Then, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$, where A and B are two fixed sets and $A = B'$. Hence, for all $x \in G$,

$$\begin{aligned} (f_G \otimes_{i/s} g_G)(x) &= \bigcap_{x=yz} (f_G(y) \Delta g_G(z)) \\ &= U \\ &= \bigcup_{x=yz} (f_G(y) \Delta g_G(z)) \\ &= (f_G \otimes_{u/s} g_G)(x) \end{aligned}$$

Thereby, $f_G \otimes_{i/s} g_G = f_G \otimes_{u/s} g_G$. \square

Proposition 3.20:

Let f_G and g_G be two \mathcal{SS} s. Then, $f_G \otimes_{i/s} g_G \subseteq f_G \otimes_{i/u} g_G$.

PROOF. Let f_G and g_G be two \mathcal{SS} s. Then, for all $x \in G$,

$$\begin{aligned} (f_G \otimes_{i/s} g_G)(x) &= \bigcap_{x=yz} (f_G(y) \Delta g_G(z)) \\ &\subseteq \bigcap_{x=yz} (f_G(y) \cup g_G(z)) \\ &= (f_G \otimes_{i/u} g_G)(x) \end{aligned}$$

Thus, $f_G \otimes_{i/s} g_G \subseteq f_G \otimes_{i/u} g_G$. \square

Proposition 3.21:

Let f_G and g_G be two \mathcal{SS} s. If one of the following assertions is satisfied, then $f_G \otimes_{i/s} g_G = f_G \otimes_{i/u} g_G$:

i. $f_G = \emptyset_G$ or $g_G = \emptyset_G$

ii. $f_G =_S (g_G)^c$

PROOF. Let f_G and g_G be two \mathcal{SS} s.

i. Without loss of generality, let $f_G = \emptyset_G$. Then, for all $x \in G$, $f_G(x) = \emptyset$. Thus, for all $x \in G$,

$$\begin{aligned} (f_G \otimes_{i/s} g_G)(x) &= \bigcap_{x=yz} (f_G(y) \Delta g_G(z)) \\ &= \bigcap_{x=yz} (f_G(y) \cup g_G(z)) \\ &= (f_G \otimes_{i/u} g_G)(x) \end{aligned}$$

Thereby, $f_G \otimes_{i/s} g_G = f_G \otimes_{i/u} g_G$.

ii. Let $f_G =_S (g_G)^c$. Then, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$, where A and B are two fixed sets and $A = B'$. Hence, for all $x \in G$,

$$\begin{aligned} (f_G \otimes_{i/s} g_G)(x) &= \bigcap_{x=yz} (f_G(y) \Delta g_G(z)) \\ &= U \\ &= \bigcap_{x=yz} (f_G(y) \cup g_G(z)) \\ &= (f_G \otimes_{i/u} g_G)(x) \end{aligned}$$

Thereby, $f_G \otimes_{i/s} g_G = f_G \otimes_{i/u} g_G$. \square

4. CONCLUSION

This study initiates with the formal definition of a novel product of soft sets, referred to as the soft intersection-symmetric difference product, constructed over a parameter domain equipped with a group-theoretic structure. Anchored in this foundational formulation, we conduct a rigorous algebraic investigation of the operation, with particular focus on its behavior under various taxonomies of soft subsethood and its compatibility with generalized soft equality relations. Furthermore, the proposed operation undergoes an in-depth comparative analysis with earlier soft product constructions within the hierarchical taxonomy of soft subset classifications, thereby yielding refined theoretical insights into their respective representational capacities and algebraic congruence. Concurrently, a meticulous structural investigation into the product's behavior with respect to both the null and absolute soft sets further

elucidates its foundational role within the broader algebraic framework. The systematic development and examination of such operations within an axiomatized algebraic framework constitute a cornerstone of abstract algebra, wherein structural validation through properties such as closure, associativity, commutativity, idempotency, and the existence (or nonexistence) of identity, inverse, and absorbing elements is essential for the formal classification of the induced algebraic system within the established algebraic hierarchy. The algebraic regularities and structural phenomena uncovered through this analysis not only consolidate the internal logical coherence of the framework but also affirm the product's capacity to generalize classical algebraic constructs, thereby extending the expressive reach of soft algebraic systems. From a theoretical standpoint, the framework articulated herein addresses critical gaps in the existing body of literature and provides a robust foundation for the formal emergence of a new line of inquiry: soft group theory, grounded in the structural behavior of the proposed operation. Future explorations may aim to synthesize further algebraic operations within soft environments and refine generalized soft equality notions, ultimately broadening the theoretical landscape and enhancing the methodological applicability of soft set theory in algebraic modeling, computational structures, and decision-theoretic analysis under uncertainty.

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