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REVIEW ARTICLE

ANALYTICAL APPROXIMATE SOLUTION OF NON-LINEAR PROBLEM BY HOMOTOPY PERTURBATION METHOD (HPM)

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ABSTRACT

In this article, we want to find the analytic approximate solution of nonlinear problems by using Homotopy Perturbation Method. Using the Homotopy Perturbation Method once we express the nonlinear problem into infinite number of sub linear problems and then obtain the solution of linear problems.

KEYWORDS

Exact and Approximate Solutions, Non-Linear partial differential equations System of equations, Homotopy Perturbation Method

1. INTRODUCTION

Nonlinear phenomena played a very important role in science especially in the field of applied Mathematics, Physics and Engineering etc., since after the appearance of super computer; it is not difficult to obtain the solution of linear problem. But unfortunately, it is still difficult to solve nonlinear problem analytically. Commonly, the nonlinear problem is determined to be the type of nonlinear equation and then using the analytic method for its solution. The analytic methods are fast developing, but still have some deficiencies. Homotopy Perturbation Method was first presented [1,2]. The method of Homotopy Perturbation Method applied by many authors to find the solution of various nonlinear problem in the field of science and engineering [3-6].

Homotopy Perturbation Method provide an opportunity that is not require a small parameter like perturbation method in the equation, which is a significant advantage of this method and it gives us an analytic approximate solution to a wide range of nonlinear problem in applied sciences [2]. Homotopy Perturbation Method also applicable to different types of equations like Volterra equation, Integro equation, nonlinear oscillator equation, bifurcation equation, nonlinear wave equation etc [7]. In most cases Homotopy Perturbation Method provide very rapid and fast convergence [7]. Thus, a method which can solve different types of nonlinear equations is known as Homotopy Perturbation Method. But still these methods have some have some deficiencies and did not provide a convenience result in most cases. Also, it is proved in most that Homotopy Perturbation Method is a powerful and efficient method for the solution of nonlinear problems.

Like other nonlinear analytic techniques perturbation techniques have their own advantages, restrictions and limitations. First the Perturbation methods are depending on the parameters, which may be small or large, so at least one unknown must be expressed in a series of small parameters but unfortunately not every nonlinear equation has such small parameter, secondly there exist such a parameter, the result by perturbation method, which is given in most cases valid only for the small values of the parameter. So, the limitation of the perturbation methods arises from the small parameter supposition. Then it seems to be necessary to develop a new type of analytic techniques which does not require small parameter to all. Also, analytic approximation of nonlinear problem is break down often, as non-linearity becomes strong and perturbation techniques valid for nonlinear problem only in case of weak nonlinearity [8]. Perturbation techniques are essentially based on the existence of small or large parameters or variables called perturbation quantity. Briefly speaking, perturbation techniques use perturbation quantities to transfer a nonlinear problem into an infinite number of sub linear problems and then approximate it by the sum of solutions of the first several sub-problems. There are many analytical methods to solve nonlinear problems, such as perturbation techniques, which are widely applied [8]. With the help of perturbation methods, a lot of useful and interesting axioms for nonlinear problems have been discovered. The role of perturbation method is that to express the given nonlinear problem into an infinite number of sublinear problem and then approximating the solution using first several sub-linear problems by its sum, whose small parameter or variable is known perturbation quantity [9-12]. However, the perturbation quantity through the nonlinear problem transfers to a system of linear sub problem brings some restrictions. These restrictions are described as, firstly when nonlinearity become stronger, the analytic approximation of nonlinear problem by perturbation methods often breaks down, secondly it is not possible in every nonlinear problem, that it must contain perturbation quantity, which shows that perturbation method is valid only in the case of weak nonlinearity [13-18].

2. NONLINEAR PROBLEMS OF PARTIAL DIFFERENTIAL EQUATIONS

Problem 1. Consider a two-component evolutionary system of a homogeneous Partial Differential Equations of order three, such that

$$\frac{\partial u}{\partial t}-\frac{\partial^3 u}{\partial^3 x}-u\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial x}=0, \tag{1}$$

$$\frac{\partial v}{\partial t}+2\frac{\partial^3 v}{\partial^3 x}+u\frac{\partial v}{\partial x}=0, \tag{2}$$

subject to

$$\begin{pmatrix} u(x,0)=3-6\tanh^2(\frac{x}{2}), \\ v(x,0)=-3t\sqrt{2}\tanh^2(\frac{x}{2}) \end{pmatrix}. \tag{3}$$

By Homotopy Perturbation Method, construction Homotopy for system (1)-(2), we obtain

$$\frac{\partial u}{\partial t}+p\left(\frac{\partial u}{\partial t}-\frac{\partial^3 u}{\partial x^3}-u\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial x}\right)=0 \tag{4}$$

$$\frac{\partial v}{\partial t}+p\left(2\frac{\partial^3 v}{\partial x^3}+u\frac{\partial v}{\partial x}\right)=0. \tag{5}$$

Suppose that the solution of Systems (1)-(2) has the form

$$u=u_0+pu_1+p^2u_2+p^3u_3+\dots \tag{6}$$

$$v=v_0+pv_1+p^2v_2+p^3v_3+\dots \tag{7}$$

Substituting equation (6)-(7) in equation (4)-(5) respectively and comparing the coefficients of identical degrees of p, we obtain

$$\frac{\partial}{\partial t}\left(u_0+pu_1+p^2u_2+p^3u_3+\dots\right)+p\begin{bmatrix} -\frac{\partial^3}{\partial x^3}\left(u_0+pu_1+p^2u_2+p^3u_3+\dots\right) \\ -\left(u_0+pu_1+p^2u_2+p^3u_3+\dots\right) \\ \frac{\partial}{\partial x}\left(u_0+pu_1+p^2u_2+p^3u_3+\dots\right) \\ -\left(v_0+pv_1+p^2v_2+p^3v_3+\dots\right) \\ \frac{\partial}{\partial x}\left(v_0+pv_1+p^2v_2+p^3v_3+..\right) \end{bmatrix}=0,$$

$$\frac{\partial}{\partial t}\left(v_0+pv_1+p^2v_2+p^3v_3+..\right)+p\begin{bmatrix} 2\frac{\partial^3}{\partial x^3}\left(v_0+pv_1+p^2v_2+p^3v_3+..\right) \\ +\left(u_0+pu_1+p^2u_2+p^3u_3+\dots\right) \\ \frac{\partial}{\partial x}\left(v_0+pv_1+p^2v_2+p^3v_3+..\right) \end{bmatrix}=0.$$

Now comparing the coefficients of identical degrees of p, we obtain

$$p^0:\frac{\partial u_0}{\partial t}=0, \tag{8}$$

$$p^0:\frac{\partial v_0}{\partial t}=0, \tag{9}$$

$$p^1:\frac{\partial u_1}{\partial t}-u_0\frac{\partial u_0}{\partial x}-\frac{\partial^3 u_0}{\partial x^3}-v_0\frac{\partial v_0}{\partial x}=0, \tag{10}$$

$$p^1:\frac{\partial v_1}{\partial t}+u_0\frac{\partial v_0}{\partial x}+2\frac{\partial^3 u_0}{\partial x^3}=0, \tag{11}$$

$$p^2:\frac{\partial u_2}{\partial t}-u_1\frac{\partial u_0}{\partial x}-u_0\frac{\partial u_1}{\partial x}-v_1\frac{\partial v_0}{\partial x}-v_0\frac{\partial v_1}{\partial x}-\frac{\partial^3 u_1}{\partial x^3}=0, \tag{12}$$

$$p^2:\frac{\partial v_2}{\partial t}+u_1\frac{\partial v_0}{\partial x}+u_0\frac{\partial v_1}{\partial x}+2\frac{\partial^3 v_1}{\partial x^3}=0. \tag{13}$$

The solution of Equations (8)-(13) can be obtained by using Equation (3)

$$u_0=u(x,0)=3-\tanh^2\left(\frac{x}{2}\right),$$

$$v_0=v(x,0)=3t\sqrt{2}\tanh^2\left(\frac{x}{2}\right).$$

Now the solution of Equations (10) and (11) is

$$p^1:\frac{\partial u_1}{\partial t}-u_0\frac{\partial u_0}{\partial x}-\frac{\partial^3 u_0}{\partial x^3}-v_0\frac{\partial v_0}{\partial x}=0,$$

$$\frac{\partial u_1}{\partial t}=u_0\frac{\partial u_0}{\partial x}+\frac{\partial^3 u_0}{\partial x^3}+v_0\frac{\partial v_0}{\partial x}.$$

Now taking the integral we get:

$$\int\frac{\partial u}{\partial t}=\int\left(u_0\frac{\partial u_0}{\partial x}+\frac{\partial^2 u_0}{\partial x^2}+v_0\frac{\partial v_0}{\partial x}\right)dt,$$

$$u_1=\int_0^1\left(u_0\frac{\partial u_0}{\partial x}+\frac{\partial^2 u_0}{\partial x^2}+v_0\frac{\partial v_0}{\partial x}\right)dt. \tag{14}$$

Now putting the values of u_0 and v_0 in Equation (14) we get:

$$u_1=-48scsh^3\left(x\left(\sinh^4(\frac{x}{2})\right)\right).$$

Similarly, for y_1, we

$$v_1=\int_0^t(-u_0\frac{\partial v_0}{\partial x}-2\frac{\partial^3 v_0}{\partial x^3}dt). \tag{15}$$

Now putting the values of v_0 and u_0 in Equation (15), we get:

$$v_1=t\begin{pmatrix} 9t\sqrt{2}\sec h^2(\frac{x}{2})\tanh(\frac{x}{2})-12t\sqrt{2}\sec h^4(\frac{x}{2})\tanh(\frac{x}{2}) \\ -12t\sqrt{2}\sec h^2(\frac{x}{2})\tanh^3(\frac{x}{2}). \end{pmatrix}$$

Considering the first eight terms of Equations (6)-(7), then the approximate solution of the System (1)-(2) by setting $p=1$ is

$$\begin{aligned} u_{app}(x,t)&=\sum_{i=0}^{7}u_i \\ &=3-6\tanh^2\left(\frac{x}{2}\right)-48tscsh^3(x)\left(\sinh^4\left(\frac{x}{2}\right)+\dots\right), \end{aligned}$$

$$\begin{aligned} v_{app}&=\sum_{i=0}^{7}v_i \\ &=3t\sqrt{2}\tanh^2\left(\frac{x}{2}\right)+t\left(9t\sqrt{2}\sec h^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)\right) \\ &\quad -12t\sqrt{2}\sec h^4\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)-12t\sqrt{2}\sec h^2\left(\frac{x}{2}\right)\tanh^3\left(\frac{x}{2}\right)+\dots, \end{aligned}$$

Which are the required solutions of the (1) and (2) by Homotopy Perturbation Method.

Problem 2. Consider a tow component evolutionary system of homogeneous PDE equation of order 3,

$$\frac{\partial u}{\partial t}-\frac{\partial^3 u}{\partial x^3}-2v\frac{\partial u}{\partial x}-u\frac{\partial v}{\partial x}=0, \qquad (16)$$

$$\frac{\partial v}{\partial t}-u\frac{\partial u}{\partial x}=0. \qquad (17)$$

subject to

$$u(x,o)=-\tanh\left(\frac{x}{\sqrt{3}}\right),$$
$$v(x,0)=\frac{-1}{6}-\frac{1}{2}\tanh^2\left(\frac{x}{\sqrt{3}}\right). \qquad (18)$$

Constructing Homotopy for Equations (16)-(17), we get

$$\frac{\partial u}{\partial t}+p\left(-\frac{\partial^3 u}{\partial x^3}-2v\frac{\partial u}{\partial x}-u\frac{\partial v}{\partial x}\right)=0, \qquad (19)$$

$$\frac{\partial v}{\partial t}+p\left(-u\frac{\partial u}{\partial x}\right)=0. \qquad (20)$$

Suppose that the solution of Systems (16)-(17) has the form

$$u=u_0+pu_1+p^2u_2+p^3u_3+\ldots \qquad (21)$$

$$v=v_0+pv_1+p^2v_2+p^3v_3+\ldots \qquad (22)$$

Substituting Equations (21)-(22) in Equations (19)-(20), respectively, we get

$$\frac{\partial}{\partial t}\left(u_0+pu_1+p^2u_2+\ldots\right)+p\begin{bmatrix}-\frac{\partial^3}{\partial x^3}\left(u_0+pu_1+p^2u_2+\ldots\right)\\ -2\left(v_0+pv_1+p^2v_2+\ldots\right)\\ \frac{\partial}{\partial x}\left(u_0+pu_1+p^2u_2+\ldots\right)\\ -\left(u_0+pu_1+p^2u_2+\ldots\right)\\ \frac{\partial}{\partial x}\left(v_0+pv_1+p^2v_2+\ldots\right)\end{bmatrix}=0 \qquad (23)$$

Putting the values of *u* and *v* in Equation (20)

$$\frac{\partial}{\partial t}\left(v_0+pv_1+p^2v_2+p^3v_3+..\right)+p\begin{bmatrix}-(u_0+pu_1+p^2u_2+p^3u_3+\ldots)\\ \frac{\partial}{\partial x}\left(-(u_0+pu_1+p^2u_2+p^3u_3+\ldots)\right)\end{bmatrix}=0. \qquad (24)$$

Now comparing the coefficients of identical degrees of p, in Equations (23) and (24), we obtain

$$p^0:\frac{\partial u_0}{\partial t}=0, \qquad (25)$$

$$p^0:\frac{\partial v_0}{\partial t}=0, \qquad (26)$$

$$p^1:\frac{\partial u_1}{\partial t}-2v_0\frac{\partial u_0}{\partial x}-\frac{\partial^3 u_0}{\partial x^3}-u_0\frac{\partial v_0}{\partial x}=0, \qquad (27)$$

$$p^1:\frac{\partial v_1}{\partial t}+u_0\frac{\partial u_0}{\partial x}=0, \qquad (28)$$

$$p^2:\frac{\partial u_2}{\partial t}-2v_1\frac{\partial u_0}{\partial x}-2v_0\frac{\partial u_1}{\partial x}-u_1\frac{\partial v_0}{\partial x}-u_0\frac{\partial v_1}{\partial x}-\frac{\partial^3 u_1}{\partial x^3}=0, \qquad (29)$$

$$p^2:\frac{\partial v_2}{\partial t}-u_1\frac{\partial u_0}{\partial x}-u_0\frac{\partial u_1}{\partial x}=0. \qquad (30)$$

The solution of Equation (25) and (26) can be obtained by using (18)

$$v_0=u(x,0)=-\tanh\left(\frac{x}{\sqrt{3}}\right),$$

$$v_0(x,0)=\frac{-1}{6}-\frac{1}{2}\tanh^2\left(\frac{x}{\sqrt{3}}\right).$$

Then we derive the solutions of Equations (27) (28) as

$$\frac{\partial u_1}{\partial t}-2v_0\frac{\partial u_0}{\partial x}-u_0\frac{\partial v_0}{\partial x}-\frac{\partial^3 u_0}{\partial x^3}=0,$$

$$\frac{\partial v_1}{\partial t}-u_0\frac{\partial u_0}{\partial x}=0,$$

$$\frac{\partial u_1}{\partial t}=2v_0\frac{\partial u_0}{\partial x}+u_0\frac{\partial v_0}{\partial x}-\frac{\partial^3 u_0}{\partial x^3},$$

$$\frac{\partial v_1}{\partial t}=u_0\frac{\partial u_0}{\partial x}.$$

Taking the integral of both sides in the above equations

$$u_1=\int_0^t\left(2v_0\frac{\partial u_0}{\partial x}+u_0\frac{\partial v_0}{\partial x}-\frac{\partial^3 u_0}{\partial x^3}\right)dt \qquad (31)$$

$$v_1=\int_0^t u_0\frac{\partial u_0}{\partial x}dt \qquad (32)$$

Now putting the values of u_0 and v_0 in Equation (31) and (32)

$$u_1=\frac{1}{\sqrt{3}}t\sec h^2\left(\frac{x}{\sqrt{3}}\right).$$

$$v_1=\frac{1}{\sqrt{3}}t\tanh\left(\frac{x}{\sqrt{3}}\right)\sec h^2\left(\frac{x}{\sqrt{3}}\right).$$

Similarly, in this way we find u_2 and v_2. Considering the first nine terms of Equations (21)-(22) then the approximate solution of Equations (16)-(17) by selecting $p=1$ is

$$u_{app}(x,t)=\sum_{i=0}^{8}u_i=-\tanh\left(\frac{x}{\sqrt{3}}\right)-\frac{1}{\sqrt{3}}tscsh^2\left(\frac{x}{\sqrt{3}}\right)+\ldots, \qquad (33)$$

$$v_{app}=\sum_{i=0}^{8}v_i=\frac{-1}{6}-\frac{1}{2}\tanh^2\left(\frac{x}{\sqrt{3}}\right)+\frac{1}{\sqrt{3}}t\tanh\left(\frac{x}{\sqrt{3}}\right)\sec h^2\left(\frac{x}{\sqrt{3}}\right)+\ldots., \qquad (34)$$

Problem 3. Consider the generalized coupled Hirota Satsuma PDE system

$$\frac{\partial u}{\partial t}-\frac{1}{2}\frac{\partial^2 u}{\partial x^2}+3u\frac{\partial u}{\partial x}(vw)=0, \qquad (35)$$

$$\frac{\partial v}{\partial t}+\frac{\partial^3 v}{\partial x^3}-3u\frac{\partial v}{\partial x}=0, \tag{36}$$

$$\frac{\partial w}{\partial t}+\frac{\partial^3 w}{\partial x^3}-3u\frac{\partial w}{\partial x}=0. \tag{37}$$

Subject to

$$u(x,0)=\frac{-1}{3}+\tanh^2(x) \tag{38}$$

$$v(x,0)=\tanh(x), \tag{39}$$

$$w(x,0)=\frac{8}{3}\tanh(x). \tag{40}$$

By Homotopy Perturbation Method, constructing Homotopy for Systems (35)-(37), we get

$$\frac{\partial u}{\partial t}+p\left(-\frac{1}{3}\frac{\partial^3 u}{\partial x^3}+3u\frac{\partial u}{\partial x}-3\frac{\partial}{\partial x}(vw)\right)=0, \tag{41}$$

$$\frac{\partial v}{\partial t}+p\left(\frac{\partial^3 v}{\partial x^3}-3u\frac{\partial v}{\partial x}\right)=0, \tag{42}$$

$$\frac{\partial w}{\partial t}+p\left(\frac{\partial^3 w}{\partial x^3}-3u\frac{\partial w}{\partial x}\right)=0, \tag{43}$$

Suppose that the solution of Equation (35)-(36) and Equation (37) has the form

$$u=u_0+pu_1+p^2u_2+p^3u_3+\dots \tag{44}$$

$$v=v_0+pv_1+p^2v_2+p^3v_3+\dots \tag{45}$$

$$w=w_0+pw_1+p^2w_2+p^3w_3+\dots \tag{46}$$

Substituting Equations (44)-(46) in (41)-(43) respectively

$$\frac{\partial}{\partial t}\left(u_0+pu_1+p^2u_2+\dots\right)+p\begin{bmatrix}-\frac{1}{3}\frac{\partial^3}{\partial x^3}\left(u_0+pu_1+p^2u_2+\dots\right)\\ +3\left(u_0+pu_1+p^2u_2+\dots\right)\\ \frac{\partial}{\partial x}\left(u_0+pu_1+p^2u_2+\dots\right)\\ -3\frac{\partial}{\partial x}\left(v_0+pv_1+p^2v_2+..\right)\\ -3\frac{\partial}{\partial x}\left(w_0+pw_1+p^2w_2+..\right)\end{bmatrix}=0, \tag{47}$$

$$\frac{\partial}{\partial t}\left(v_0+pv_1+p^2v_2+\dots\right)+p\begin{bmatrix}\frac{\partial^3}{\partial x^3}\left(v_0+pv_1+p^2v_2+\dots\right)\\ -3(u_0+pu_1+p^2u_2+\dots)\\ \frac{\partial}{\partial x}\left(v_0+pv_1+p^2v_2+\dots\right)\end{bmatrix}=0, \tag{48}$$

$$\frac{\partial}{\partial t}\left(w_0+pw_1+p^2w_2+\dots\right)+p\begin{bmatrix}\frac{\partial^3}{\partial x^3}\left(w_0+pw_1+p^2w_2+\dots\right)\\ -3(u_0+pu_1+p^2u_2+\dots)\\ \frac{\partial}{\partial x}\left(w_0+pw_1+p^2w_2+\dots\right)\end{bmatrix}=0. \tag{49}$$

Now comparing the coefficients of identical degrees of p, in Equations (47)-(49), we obtain:

$$p^0:\frac{\partial u_0}{\partial t}=0, \tag{50}$$

$$p^0:\frac{\partial v_0}{\partial t}=0, \tag{51}$$

$$p^0:\frac{\partial w_0}{\partial t}=0, \tag{52}$$

$$p^1:\frac{\partial u_1}{\partial t}+3u_0\frac{\partial u_0}{\partial x}-3w_0\frac{\partial v_0}{\partial x}-3v_0\frac{\partial u_0}{\partial x}-\frac{1}{2}\frac{\partial^3 u_0}{\partial x^3}=0, \tag{53}$$

$$p^1:\frac{\partial v_1}{\partial t}-3u_0\frac{\partial v_0}{\partial x}+\frac{\partial^3 v_0}{\partial x^3}=0, \tag{54}$$

$$p^1:\frac{\partial w_1}{\partial t}-3u_0\frac{\partial w_0}{\partial x}+\frac{\partial^3 w_0}{\partial x^3}=0, \tag{55}$$

$$\begin{aligned}p^2:\frac{\partial u_2}{\partial t}&+3u_1\frac{\partial u_0}{\partial x}+3u_0\frac{\partial u_1}{\partial x}-3w_1\frac{\partial v_0}{\partial x}-3w_0\frac{\partial v_1}{\partial x}\\ &-3v_1\frac{\partial w_0}{\partial x}-3v_0\frac{\partial w_1}{\partial x}-\frac{1}{2}\frac{\partial^3 u_0}{\partial x^3}=0,\end{aligned} \tag{56}$$

$$p^2:\frac{\partial v_2}{\partial t}-3u_1\frac{\partial v_0}{\partial x}-3u_0\frac{\partial v_1}{\partial x}+\frac{\partial^3 v_0}{\partial x^3}=0, \tag{57}$$

$$p^2:\frac{\partial w_2}{\partial t}-3u_1\frac{\partial w_0}{\partial x}-3u_0\frac{\partial w_1}{\partial x}+\frac{\partial^3 w_0}{\partial x^3}=0, \tag{58}$$

The solution of Equations (50)-(52) can be obtained by using the following initial conditions

$$u(x,0)=\frac{-1}{3}+\tanh^2(x)$$

$$v(x,0)=\tanh(x),$$

$$w(x,0)=\frac{8}{3}\tanh(x).$$

Then we derive the solution of Equations (53)-(55):

$$\frac{\partial u_1}{\partial t}+3u_0\frac{\partial u_0}{\partial x}-3w_0\frac{\partial v_0}{\partial x}-3v_0\frac{\partial w_0}{\partial x}-\frac{1}{2}\frac{\partial^3 u_0}{\partial x^3}=0,$$

$$\frac{\partial v_1}{\partial t}-3u_0\frac{\partial v_0}{\partial x}-\frac{\partial^3 v_0}{\partial x^3}=0,$$

$$\frac{\partial w_1}{\partial t}-3u_0\frac{\partial w_0}{\partial x}-\frac{\partial^3 w_0}{\partial x^3}=0.$$

$$\frac{\partial u_1}{\partial t}=-3u_0\frac{\partial u_0}{\partial x}-3w_0\frac{\partial v_0}{\partial x}-3v_0\frac{\partial w_0}{\partial x}-\frac{1}{2}\frac{\partial^3 u_0}{\partial x^3}, \tag{58}$$

$$\frac{\partial v_1}{\partial t}=3u_0\frac{\partial v_0}{\partial x}-\frac{\partial^3 v_0}{\partial x^3}, \tag{59}$$

$$\frac{\partial w_1}{\partial t} = 3u_0 \frac{\partial w_0}{\partial x} - \frac{\partial^3 w_0}{\partial x^3}. \qquad (60)$$

Now taking the integrals of both sides of Equations (58)-(60)

$$u_1 = \int_0^1 \left(-3u_0 \frac{\partial u_0}{\partial x} - 3w_0 \frac{\partial v_0}{\partial x} - 3v_0 \frac{\partial w_0}{\partial x} - \frac{1}{2} \frac{\partial^3 u_0}{\partial x^3} \right) dt \qquad (61)$$

$$v_1 = \int_0^1 \left(3u_0 \frac{\partial v_0}{\partial x} - \frac{\partial^3 v_0}{\partial x^3} \right) dt \qquad (62)$$

$$w_1 = \int_0^1 \left(3u_0 \frac{\partial w_0}{\partial x} - \frac{\partial^3 w_0}{\partial x^3} \right) dt. \qquad (63)$$

Now putting the values of u_0, v_0 and w_0 in Equations (61)-(63)

$$u_1 = 4t \sec h^2(x) \tanh(x).$$

$$v_1 = t \sec h^2(x).$$

$$w_1 = \frac{8}{3} t \sec h^2(x).$$

Similarly, in this way we find u_2, v_2 and w_2. Considering the first seven terms of Equations (44)-(46), then the approximate solutions of (35)-(37) by setting $p = 1$ are

$$u_{app}(x,t) = \sum_{i=0}^{6} u_i = \frac{-1}{3} + 2\tanh^2(x) + 4t \sec h^2(x) \tanh(x) + \ldots, \qquad (64)$$

$$v_{app}(x,t) = \sum_{i=0}^{6} v_i = \tanh(x) + t \sec h^2(x) + \ldots, \qquad (65)$$

$$w_{aap}(x,t) = \sum_{i=0}^{6} w_i = \frac{8}{3} \tanh(x) + \frac{8}{3} t \sec h^2(x) + \ldots \qquad (66)$$

3. CONCLUDING REMARKS

The role of perturbation method is that to express the given nonlinear problem into an infinite number of sub-linear problem and then approximating the solution using first several sub-linear problems by its sum, whose small parameter or variable is known perturbation quantity. However, the perturbation quantity through the nonlinear problem transfers to a system of linear sub problem brings some restrictions. These restrictions are described as, firstly when nonlinearity become stronger, the analytic approximation of nonlinear problem by perturbation methods often breaks down, secondly it is not possible in every nonlinear problem, that it must contain perturbation quantity, which shows that perturbation method is valid only in the case of weak nonlinearity. So, to avoid from this difficulty that is from the restriction of weak non-linearity, we use Homotopy Perturbation Method, which is one of the powerful and efficient technique and can solve the linear and nonlinear problems.

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