Matrix Science Mathematic (MSMK)

**DOI** : http://doi.org/10.26480/msmk.02.2018.28.31

***ISSN: 2521-0831 (Print)***
***ISSN: 2521-084X (Online)***
***Coden : MSMADH***

# SOFT B W-HAUSDORFF SPACE IN SOFT BI TOPOLOGICAL SPACES

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**ARTICLE DETAILS**

***Article History:***

Received 12 November 2017
Accepted 12 December 2017
Available online 1 January 2018

**ABSTRACT**

In this article the concept of Soft β W-$T_2$ structure in soft bi topological spaces is introduced in different ways. Fleix Hausdorff was a German Mathematician who is supposed to be the forefather of up-to-the-minute Topology. There are many topological structures in soft topology but Hausdorff topological structure is interesting and more practical, that is why it catches our attention to the best.

**KEYWORDS**

Soft set, soft β open set, soft β closed set, soft β W-Hausdorff space.

## 1. INTRODUCTION

Soft set theory is one of the young topics achieving importance in finding balanced and reasonable way out in day to day life activities problems which involve uncertainty and ambiguity. In 1999, a group researcher introduces new concept of soft set theory, which is absolutely a new method for modelling vagueness and uncertainty [1]. A researcher studied soft bi topological structure and also showed the prettiness of soft separation axioms in soft bi topological spaces with respect to pair wise approach with full depth [2]. In 2011, other researcher has defined soft topological spaces and studied separation axioms [3]. In 1963, Kelly, first commenced the notion of bi topological space [4]. He defined a bi topological space (X,$\tau_1$,$\tau_2$) to be a set X full of two topologies $\tau_1$ and $\tau_2$on X and initiated the systematic study of bitopological space. Also, he left no stone un-turn to studied separation properties of bi topological space. In section II of this paper, preliminary definitions concerning soft sets, soft topological spaces and soft bi topological spaces are given. In section 3 of this article, the notion of Soft β W-Hausdorff in soft bi topological spaces is familiarized in different ways and its beauty is discussed with full make up.

## 2. PRELIMINARY

Throughout this paper, $\widehat{X}$ denotes the master set and $\breve{E}$ denotes the set of parameters for the master set$\widehat{X}$ .

**Definition 1** [5].

Let $\widehat{X}$ be the master and $\breve{E}$ be a set of parameters. Let $P(\widehat{X})$ denotes the power set of$\widehat{X}$ and $\breve{A}$ be a nonempty subset of $\breve{E}$ . A pair $(F, \breve{A})$ denoted by $F_{\breve{A}}$ is named a soft set over$\widehat{X}$ , where $F$ is a mapping given by $F: \breve{A} \rightarrow P(\widehat{X})$.In other words, a soft set over $\widehat{X}$ is a parameterized family of subsets of the master$X$. For a specific $e \in \breve{A}$ , $F(e)$ may be considered the set of e-approximate elements of the soft set $(F, \breve{A})$ and if $e \notin \breve{A}$, then $F(e) = \phi$

$i.e. F_A = \{F(e): e \in \breve{A} \subseteq \breve{E} \ ; F: \breve{A} \rightarrow P(\widehat{X})\}$.

The family of all these soft sets over $X$ with respect to the restriction set $\breve{E}$ is signified by$SS(X)_{\breve{E}}$ .

**Definition 2** [5]. Let$F_{\breve{A}}, G_{\breve{B}} \in SS(X)_{\breve{E}}$. Then $F_{\breve{A}}$ is soft sub set of $G_{\breve{B}}$ , denoted by $F_{\breve{A}} \widetilde{\subseteq} G_{\breve{B}}$ , if

(1)$\breve{A} \widetilde{\subseteq} \breve{B}.$, and
(2)$F(e) \widetilde{\subseteq} G(e), \forall e \in \breve{A}$.

In this case, $F_{\breve{A}}$ is supposed to be a soft subset of $G_{\breve{B}}$ and $G_{\breve{B}}$ is said to be a soft super set of $F_{\breve{A}}, G_{\breve{B}.} \widetilde{\supseteq} F_{\breve{A}}$

**Definition 3** [6]. Two soft subsets $F_A$ and $G_B$ over a common universe $X$ are said to be soft equal if $F_A$ is a soft subset of $G_B$ and $G_B$ is a soft subset of$F_A$.

**Definition 4** [7]. The complement of a soft set $(F, A)$ denoted by $(F, A)'$ is defined by $(F, A)' = (F', A)$ , $F' : A \longrightarrow P(X)$ is a mapping given by $F'(e) = X - F(e)$; $\forall e \in A$ and $F'$ is called the soft complement function of $F$.Clearly $(F')'$ is the same as $F$ and $((F, A)')' = (F, A)$.

**Definition 5** [6]. A soft set $(F, A)$ over $X$ is said to be a Null soft set denoted by $\widetilde{\phi}$or $\phi_A$ if for all$e \in A, F(e) = \phi$ $(vacuous\ set)$.

**Definition 6** [6]. A soft set $(F, A)$ over $X$ is said to be an absolute soft set denoted by $\tilde{A}$ or $X_A$ if for all $e \in A$, $F(e) = X$. obviously we have $X'_A = \phi_A\ and\ \phi'_A = X_A$.

**Definition 7** [6]. The union of two soft sets $(F, A)$ and $(G, B)$ over the common universe $X$ is the soft set $(H, C)$, where $C = A \cup B$ & $\forall e \in C$,

$$H(e) = \begin{cases} F(e), e \in A - B \\ G(e), e \in B - A \\ F(e) \,\breve{\cap}\, G(e), e \in A \,\breve{\cap}\, B \end{cases}$$

**Definition 8** [6]. The intersection of two soft sets $(F, A)$ and $(G, B)$ over the common universe $X$ is the soft set$(H, C)$, where $C = A \,\breve{\cap}\, B$ and for all $e \in C, H(e) = F(e) \,\breve{\cap}\, G(e)$.

**Definition 9** [6]. Let $\tilde{\tau}$ be the collection of soft sets over $\widetilde{X}$, then $\tilde{\tau}$ is said to be a soft topology on $\widetilde{X}$, if

(1) $\phi, \widetilde{X} \in \tilde{\tau}$
(2) Union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$
(3) Intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

**Definition 10** [8]. Let $(X, \tau, E)$ be a soft topological space and $(F, E) \subseteq SS(X)_E$ then $(F, E)$ is called β open soft set $((F, E) \subseteq Cl(int(Cl(F, E))))$.

The set of all β *open soft* set is denoted by $S\beta O(X, \tau, E)$ or $S\beta O(X)$ and the set of all β *closed soft* set is denoted by $\beta CS(X, \tau, E)$ or $\beta CS(X)$.

**Definition 11** [5]. Let $(\widetilde{X}, \tilde{\tau}, E)$ be a soft topological space, $(F, E) \in SS(X)_E$ and $\tilde{Y}$ be a non-vacuous subset of $\widetilde{X}$. Then the soft subset of $(F, E)$ over $\tilde{Y}$ signified by $(F_{\tilde{Y}}, E)$ is defined as follows:

$F_Y(e) = \tilde{Y} \tilde{\cap} F(e), \forall\, e \in E$
In other words, $(F_{\tilde{Y}}, E) = \tilde{Y}_E \tilde{\cap} (F, E)$.

**Definition 12** [9]. Let $(X, \tilde{\tau}, E)$ be a soft topological space and $\tilde{Y}$ be a non-vacuous subset of $X$. Then $\tilde{\tau}_{\tilde{Y}} = \{(F_{\tilde{Y}}, E) : (F, E) \in \tilde{\tau}\}$ is said to be the relative soft topology on $\tilde{Y}$ and $(Y, \tilde{\tau}_Y, E)$ is called a soft subspace of $(\widetilde{X}, \tilde{\tau}, E)$.

**Definition 13** [5]. Let $F_A \in SS(X)_E$ & $G_B \in SS(Y)_K$. The Cartesian product $F_A \circledast G_B$ is defined by $(F_A \circledast G_B)(e, k) = F_A(e) \times G_B(k)$, $\forall\, (e, k) \in A \times B$. According to this definition $F_A \circledast G_B$ is a soft set over $X \times Y$ and its parameter set is $E \times K$.

**Definition 14** [7]. Let $(\widetilde{X}, \tilde{\tau}_X, E)$ and $(\tilde{Y}, \tilde{\tau}_{\tilde{Y}}, K)$ be two soft topological spaces. The soft product topology $\tilde{\tau}_X \circledast \tau_Y$ over $X \times Y$ with respect to $E \times K$ is the soft topology having the collection $\{F_E \circledast G_K / F_E \in \tilde{\tau}_{\widetilde{X}}, G_K \in \tilde{\tau}_{\tilde{Y}}\}$ as the basis.

**Definition 15** [2]. Let $\left(\widetilde{X}, \tilde{\tau}_{1\widetilde{X}}, E\right)$ and $\left(\widetilde{X}, \tilde{\tau}_{2\widetilde{X}}, E\right)$ be two not the same soft topological spaces on $\widetilde{X}$. Then $(X, \tilde{\tau}_{1\widetilde{X}}, , \tilde{\tau}_{2\widetilde{X}}, E)$ is called a Soft bi topological space if the two soft topologies $\tilde{\tau}_{1\widetilde{X}}$ and $\tilde{\tau}_{2\widetilde{X}}$ individually gratify the axioms of soft topology [10-13]. The participants of $\tilde{\tau}_{1\widetilde{X}}$ are called $\tilde{\tau}_{1\widetilde{X}}$ soft open sets and the complements of $\tilde{\tau}_{1\widetilde{X}}$ soft open sets are named $\tilde{\tau}_{1\widetilde{X}}$ soft closed sets. Similarly, The participants of $\tilde{\tau}_{2\widetilde{X}}$ are called $\tilde{\tau}_{2\widetilde{X}}$ soft open sets and the complements of $\tilde{\tau}_{2\widetilde{X}}$ soft open sets are named $\tilde{\tau}_{2\widetilde{X}}$ soft closed sets.

**Definition 16** [2]. Let $(X, \tilde{\tau}_{1\widetilde{X}}, \tilde{\tau}_{2\widetilde{X}}, E)$ be a soft bi topological space over $X$ and $Y$ be a non-empty subset of $X$. Then $\tau_{1\tilde{Y}} = \{(F_Y, E) : (F, E) \in \tilde{\tau}_{1\widetilde{X}}\}$ and $\tilde{\tau}_{2\tilde{Y}} = \{(G_{\tilde{Y}}, E) : (G, E) \in \tilde{\tau}_{2\widetilde{X}}\}$ are said to be the relative topologies on $Y$ and $\{Y, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ is named

## 3. MAIN RESULTS

### 3.1 New Results in Soft Bi Topological Spaces

**Definition 17**. A soft topological space $(\widetilde{X}, \tilde{\tau}, E)$ is said to be soft β-W-Hausdorff space of type 1 signified by $\beta(SW-H)_1$ if for every $e_1, e_2 \in E, e_1 \neq e_2$ there exists soft β open sets $(F, \check{A})$, $(G, \check{B})$ such that $F_{\check{A}}(e_1) = X, G_{\check{B}}(e_2) = X$ *and* $(F, \check{A}) \tilde{\cap} (G, \check{B}) = \tilde{\phi}$.

**Definition 18**. Let $(\widetilde{X}, \tilde{\tau}, E)$ be a soft topological space and $H \subseteq E$. Then $(X, \tilde{\tau}_H, H)$ is called soft p-subspace of $(X, \tilde{\tau}, E)$ relative to the parameter set $H$ where $\tilde{\tau}_H = \{(F_{\check{A}}) / H : H \subseteq \check{A} \subseteq E, \beta$ *open set* $F_{\check{A}} \in \tilde{\tau}\}$ and $(F_{\check{A}}) / H$ is the restriction map on $H$.

**Theorem 1**. ***Proof***

(1) Soft subspace of a β $(SW-H)_1$ space is soft β$(SW-H)_1$.
(2) Soft p-subspace of a β $(SW-H)_1$ space is soft β$(SW-H)_1$.
(3) Product of two soft β $(SW-H)_1$ space is soft β$(SW-H)_1$.

**(1) Proof**. Let $(\widetilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ be a β $(SW-H)_1$ space. Let $Y$ be a non-vacuous sub set of $\widetilde{X}$. Let $(\tilde{Y}, \tilde{\tau}_Y, E)$ be a soft sub space of $(\widetilde{X}, \tau, E)$ where $\tilde{\tau}_Y = \{(\tilde{F}_Y, E) : (F, E) \in \tilde{\tau}\}$ is the relative soft topology on $\tilde{Y}$. Consider $e_1, e_2 \in E, e_1 \neq e_2$ there exists β open sets $(F, A)$, $(G, B)$ such that $F_{\check{A}}(e_1) = X, G_{\check{B}}(e_2) = X$ *and* $(F, \check{A}) \tilde{\cap} (G, \check{B}) = \tilde{\phi}$. Therefore $((F_{\check{A}})_Y, E), ((G_{\check{B}})_Y, E) \in \tilde{\tau}_Y$. Also $(F_{\check{A}})_Y(e_1) = Y \cap F_{\check{A}}(e_1) = Y \tilde{\cap} X = Y$. $(F_{\check{B}})_Y(e_2) = Y \cap G_{\check{B}}(e_2) = Y \tilde{\cap} X = Y$. $((F_A)Y \tilde{\cap} G_B)\tilde{Y})(e) = ((F_A \tilde{\cap} G_{\check{B}})\tilde{Y})(e)$
$= \tilde{Y} \tilde{\cap} (F_A \tilde{\cap} G_B)(e)$
$= \tilde{Y} \cap \tilde{\phi}(e)$
$= \tilde{Y} \cap \phi$
$= \phi$
$(F_A)_Y \cap (G_{\check{B}})_Y = \phi$. Hence $(\tilde{Y}, \tilde{\tau}_Y, E)$ is $(SW-H)_1$.

**(2) Proof:** Let (X,$\tau$, E) be a β $(SW-H)_1$ space. Let $H \subseteq E$. Let $(\widetilde{X}, \tilde{\tau}_H, H)$ be a soft p-subspace of $(\widetilde{X}, \tau, E)$ relative to the parameter set $H$. $\tilde{\tau}_H = \{(F_{\check{A}}) / H : H \subseteq A \subseteq E, F_A \in \tau$. Suppose $\tilde{h}_1, \tilde{h}_2 \in H, h_1 \neq h_2$. Then $\tilde{h}_1, \tilde{h}_2 \in E$. Therefore, there happens soft β open sets $(F, A), (G, B)$ such that $F_{\check{A}}(h_1) = X, G_{\check{B}}(h_2) = \widetilde{X}$ & $(F, \check{A}) \cap (G, \check{B}) = \tilde{\phi}$. Therefore $(F_{\check{A}})/H\ (G_{\check{B}})/H \in \tilde{\tau}_H$. Also $((F_{\check{A}}) / H)(h_1) = F_{\check{A}}(h_1) = \widetilde{X}$

$((G_B) / H)(h_2) = G_B(h_2) = \widetilde{X}$ &

$((F_{\check{A}}) / H) \cap ((G_{\check{B}}) / H) = (F_{\check{A}} \cap \beta G_{\check{B}}) / H$

$= \tilde{\phi} / H$
$= \tilde{\phi}$

Hence $(\widetilde{X}, \tilde{\tau}_H, H)$ is β$(SW-H)_1$.

**(3) Proof:** Let $(\widetilde{X}, \tilde{\tau}_x, E)$ and $\{\tilde{Y}, \tilde{\tau}_Y, K\}$ be two $\beta(SBW-H)_1$ spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2 \in E \times K$ either $e_1 \neq e_2$ or $k_1 \neq k_2$. Suppose $e_1 \neq e_2$. Since $(\widetilde{X}, \tilde{\tau}_x, E)$ is $\beta(SW-H)_1$, there exist soft $\beta$ open sets $(F, \check{E})$, $(G, \check{B})$ such that $F_{\check{A}}(e_1) = \widetilde{X}, G_B(e_2) = \widetilde{X}$ &$(F, \check{A}) \cap (G, \check{B}) = \tilde{\phi}$.

Therefore $F_A \circledast Y_k \in \tilde{\tau}_{1X} \circledast \tilde{\tau}_{1Y}, G_{\check{B}} \circledast Y_K \in \tilde{\tau}_X \circledast \tilde{\tau}_Y$
$(F_{\check{A}} \circledast \tilde{Y}_K)\ (e_1, k_1) = F_{\check{A}}(e_1) \times \tilde{Y}_K(k_1) = \widetilde{X} \times \tilde{Y}$
$(G_{\check{B}} \circledast Y_K)\ (e_2, k_2) = G_{\check{B}}(e_2) \times Y_K(k_2) = \widetilde{X} \times \tilde{Y}$

If for any $(e, k) \in (E \times K), (F_A \circledast \tilde{Y}_K)\ (e, k) \neq \phi \Rightarrow F_A(e) \times \tilde{Y}_K(k) \neq \phi$
$\Rightarrow F_{\check{A}}(e) \times \tilde{Y} \neq \phi \Rightarrow \beta F_{\check{A}}(e) \neq \phi \Rightarrow G_{\check{B}}(e) = \phi$

Since ( $F_A \cap G_{\check{B}} = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$ )
$\Rightarrow G_{\check{B}}(e) \times \tilde{Y}_K(k) = \phi \Rightarrow (G_B \otimes \tilde{Y}_K)(e, k) = \phi$
$\Rightarrow (F_{\check{A}} \circledast \tilde{Y}_K) \cap (G_{\check{B}} \circledast \tilde{Y}_K) = \tilde{\phi}$

Assume $k_1 \neq k_2$. *Since* $\{Y, \tilde{\tau}_{1Y}, \tilde{\tau}_{2Y}, K\}$ is $(SW-H)_1$, there exists soft β open sets $(F, \check{E})$, $(G, \check{B})$ $F_{\check{A}} \in \tilde{\tau}_Y, G_B \in \tilde{\tau}_Y$ such that $F_{\check{A}}(k_1) =, \tilde{Y}\ G_{\check{B}}(k_2) = \tilde{Y}$ & $F_{\check{A}} \cap G_{\check{B}} = \tilde{\phi}$.

Therefore $X_E \circledast F_A \in \tilde{\tau}_Y, \widetilde{X}_E \circledast G_{\check{B}} \in \tilde{\tau}_Y$
$(\widetilde{X}_E \circledast \beta F_{\check{A}})\ (e_1, k_1) = \widetilde{X}_E(e_1) \times \beta F_{\check{A}}(k_1)$
$= \widetilde{X} \times \tilde{Y}$
$(X_E \circledast \beta G_{\check{B}})(e_2, k_2) = \widetilde{X}_E(e_2) \times F_{\check{A}}(k_1)$
$= \widetilde{X} \times \tilde{Y}$

If for any $(e, k) \in E \times K, (\widetilde{X}_E \circledast \beta F_{\check{A}})(e, k) \neq \phi \Rightarrow \widetilde{X}_E(e) \times F_{\check{A}}(k) \neq \phi$
$\Rightarrow \widetilde{X} \times F_{\check{A}}(k) \neq \phi \Rightarrow F_{\check{A}}(k) \neq \phi \Rightarrow F_A(k) \neq \phi$. So $G_B(k) = \phi$
$(Since F_A \cap G_B = \tilde{\phi} \Rightarrow F_{\check{A}}(k) \cap G_B(k) = \phi) \Rightarrow X_E(e) \times G_B(k) = \phi \Rightarrow (X_E \circledast G_B)(e, k) = \phi$
$\Rightarrow (\widetilde{X}_E \circledast F_{\check{A}}) \cap (X_E \circledast G_B) = \tilde{\phi}$

Hence, $\left(\widetilde{X} \times \tilde{Y}, \tilde{\tau}_X \circledast \tilde{\tau}_Y, E \times K\right)$ *is* $\beta(SBW-H)_1$

**Definition 19**. Let $\widetilde{X}$ be a non-empty set and $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be two different topologies on $\widetilde{X}$. Then $(\widetilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ is called a bi topological space.

**Definition 20.** A soft bi topological space $(\widetilde{X}, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ is said to be Soft β W-Hausdorff space of type 1 or soft $\beta W - T_2$ space of type 1 denoted by $(SBW-H)_1$ if it is soft β $(SW-H)_1$ With respect to $\tau_{1\widetilde{X}}$ or β $(SW-H)_1$ with respect to $\tilde{\tau}_{2\widetilde{X}}$

**Definition 21**. A soft bi topological space $(\widetilde{X}, \tau_{1\widetilde{X}}, \tau_{2\widetilde{X}}, E)$ is supposed to be Soft β W- Hausdorff space of type 2 signified by $(SBW-H)_2$ if for every $e_1, e_2 \in E, e_1 \neq e_2$ there occurs soft β $(F, \check{A}) \in \tilde{\tau}_{1\widetilde{X}}$ soft $\beta\ (F, \check{A}) \in \tilde{\tau}_{2\widetilde{X}}$ such that $F_{\check{A}}(e_1) = X, G_{\check{B}}(e_2) = \widetilde{X}$ and $(F, \check{A}) \cap (G, \check{B}) = \tilde{\phi}$.

***Theorem 2.*** Soft subspace of a $\beta(SBW-H)_1$ space is $\beta(SBW-H)_1$.

**Proof.** Let $(\widehat{X}, \tilde{\tau}_{1\widehat{X}}, \tau_{2\widehat{X}}, E)$ be a $\beta(SBW-H)_1$ space. Then it is β $(SW-H)_1$ with respect to $\tilde{\tau}_{1X}$ *or* $\beta(SW-H)_1$ with respect to $\tilde{\tau}_{2\widehat{X}}$. Let $\tilde{Y}$ be a non-vacuous subset of $\widehat{X}$, .

Let $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{1\tilde{Y}}, E\}$ be a soft subspace of $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$. From **Theorem 1** a soft subspace of β $(SW-H)_1$space is $(SW-H)_1$. Therefore, $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ is $\beta(SW-H)1$ with respect to $\tilde{\tau}_{1Y}$ or $\beta(SW-H)_1$ with respect to $\tilde{\tau}_{2Y}$. Hence $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ is $\beta(SBW-H)_1$.

***Theorem 3.*** Soft subspace of a $\beta(SBW-H)_2$ space is $\beta(SBW-H)_2$.

**Proof**. Let $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ be a $\beta(SBW-H)_2$ space. Let $Y$ be a non-vacuous subset of $\widehat{X}$. Let $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ be a soft subspace of $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ where

$\tilde{\tau}_{1\tilde{Y}} = \{(F_{\tilde{Y}}, E) : \beta(F, E) \in \tilde{\tau}_{1\widehat{X}}\}$ &
$\tilde{\tau}_{2\tilde{Y}} = \{(G_{\tilde{Y}}, E) : \beta(G, E) \in \tilde{\tau}_{2\widehat{X}}\}$ are supposed to be the relative topologies on $Y$.

Consider $e_1, e_2 \in E$ *with* $e_1 \neq e_2$ there occur $\beta(F, E) \in \tilde{\tau}_{1\widehat{X}}, \beta(G, E) \in \tilde{\tau}_{2\widehat{X}}$ such that $\beta F_A(e_1) = X, \beta G_B(e_2) = X$ &$\beta(F, E) \tilde{\cap} \beta(G, E) = \tilde{\phi}$. Hence $((F_{\check{A}})\tilde{Y}, E) \in \tilde{\tau}_{1Y}, ((G_B)\tilde{Y}, E) \in \tilde{\tau}_{2\tilde{Y}}$

Also β $(F_{\check{A}})\tilde{Y}(e_1) = \tilde{Y} \tilde{\cap} \beta F_{\check{A}}(e_1)$
$= \tilde{Y} \tilde{\cap} X = Y$
$\beta(G_B)\tilde{Y}(e_2) = \tilde{Y} \tilde{\cap} \beta G_{\check{B}}(e_2)$
$= \tilde{Y} \tilde{\cap} X$
$= \tilde{Y}$
$((\beta F_A)Y \tilde{\cap} \beta G_B)\tilde{Y})(e) = ((\beta F_A \tilde{\cap} \beta G_{\check{B}})\tilde{Y})(e)$
$= \tilde{Y} \tilde{\cap} (\beta F_A \tilde{\cap} \beta G_{\check{B}})(e)$
$= \tilde{Y} \cap \tilde{\phi}(e)$
$= \tilde{Y} \cap \phi$
$= \phi$
$(\beta F_A)Y \cap (\beta G_B)Y = \phi$
Hence $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$is $\beta(SBW-H)_2$.

**Definition 22.** Let $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ be a soft bi-topological space over $\widehat{X}$ & $H \subseteq E$. Then $\{X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H\}$ is called Softβ p-subspace of $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ relative to the parameter set $H$ where $\tilde{\tau}_{1H} = \{(\beta F_A) / H : H \subseteq A \subseteq E, \beta F_A \in \tilde{\tau}_{1\widehat{X}}\}$, $\tilde{\tau}_2 H = \{(\beta G_B) / H : H \subseteq B \subseteq E, \beta G_{\check{B}} \in \tilde{\tau}_{2\widehat{X}}\}$ & $(\beta F_A) / H$, $(\beta G_B) / H$ are the restriction maps on$H$.

***Theorem 4.*** Soft p-subspace of a $\beta(SBW-H)_1$ space isβ$(SBW-H)_1$.

***Proof.*** Let $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ be a $\beta(SBW-H)_1$ space. Then it is $\beta(SW-H)_1$ with respect to $\tilde{\tau}_{1\widehat{X}}$ *or* $\beta(SW-H)_1$ with respect to $\tilde{\tau}_2$. Let $H \subseteq E$. Let $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ be a soft p-subspace of $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ relative to the parameter set$H$. **From the theorem 1,** the soft p-subspace of $\beta(SW-H)_1$ space$\beta(SW-H)_1$. Therefore, the soft p-subspace of $\beta(SBW-H)_1$is $\beta(SW-H)_1$with respect to $\tilde{\tau}_{1H}$ or with respect to$\tilde{\tau}_{2H}$. Hence $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ is $\beta(SBW-H)_1$.

***Theorem 5.*** Soft p-subspace of a $\beta(SBW-H)_2$ space is $(SBW-H)_2$ .

***Proof.*** Let $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ be a $\beta(SBW-H)_2$ space. Let $H \subseteq E$. Let $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ be a soft p-subspace of $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ relative to the parameter set H where

$\tilde{\tau}_{1H} = \{(\beta F_{\check{A}}) / H : H \subseteq \check{A} \subseteq E, \beta F_{\check{A}} \in \tilde{\tau}_{1\widehat{X}}\}$,
$\tilde{\tau}_{2H} = \{(\beta G_B) / H : H \subseteq \check{B} \subseteq E, \beta G_{\check{B}} \in \tilde{\tau}_{2\widehat{X}}\}$.

Consider $h_1, h_2 \in H$, $h_1 \neq h_2$.Then$h_1, h_2 \in E$. Therefore, there exist $\beta(A, E) \in \tilde{\tau}_{1\widehat{X}}$ , $\beta(G, \check{B}) \in \tilde{\tau}_{2\widehat{X}}$ such that$\beta F_{\check{A}}(e_1) = X, \beta G_{\check{B}}(e_2) = \widehat{X}$ *and* $\beta(A, \check{E}) \cap \beta(G, \check{B}) = \tilde{\phi}$.

Therefore $(\beta F_{\check{A}}) /) / H \in \tilde{\tau}_{1H}, , (\beta G_B) / H \in \tilde{\tau}_{2H}$
Also $((\beta F_{\check{A}}) / H)(h_1) = \beta F_{\check{A}}(h_1) = \widehat{X}$
$((\beta G_B) / H)(h_2) = \beta G_B(h_2) = \widehat{X}$ and
$((\beta F_{\check{A}}) / H) \cap ((\beta G_{\check{B}}) / H) = (\beta F_{\check{A}} \cap \beta G_{\check{B}}) / H$
$= \tilde{\phi} / H$
$= \tilde{\phi}$

Hence$(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ *is* $\beta(SBW-H)_2$.

***Theorem 6.*** Product of two β $(SBW-H)_1$ spaces is $\beta(SBW-H)_1$.

***Proof.*** Let $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ and $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tau_{2\tilde{Y}}, K\}$ be two $\beta(SBW-H)_1$ spaces. Then $(\widehat{X}, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ is $\beta(SW-H)_1$ with respect to $\tilde{\tau}_{1\widehat{X}}$ or $\beta(SW-H)_1$ with respect to $\tilde{\tau}_{2\widehat{X}}$ and $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, K\}$ is $\beta(SW-H)_1$ with respect to $\tilde{\tau}_{1\tilde{Y}}$ or $\beta(SW-H)_1$ with respect to $\tilde{\tau}_{2\tilde{Y}}$. From theorem 18, the product of two $\beta(SW-H)_1$ spaces is $\beta(SW-H)_1$. Hence the product of two $\beta(SBW-H)_1$spaces is $\beta(SBW-H)_1$.

***Theorem 7.*** Product of two $\beta(SBW-H)_2$ spaces is $\beta(SBW-H)_2$

***Proof.*** Let $(\widehat{X}, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ and$\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tau_{2\tilde{Y}}, K\}$ be two $\beta(SBW-H)_2$ spaces. Consider two distinct points$(e_1, k_1)$, $(e_2, k_2 \in E \times K$ either $e_1 \neq e_2$ or

$k_1 \neq k_2$. Suppose $e_1 \neq e_2$. Since $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ is$\beta(SBW-H)_2$, there exist $\beta(F, \check{E}) \in \tilde{\tau}_{1\widehat{X}}, (G, \check{B}) \in \tilde{\tau}_{2\widehat{X}}$ such that $F_{\check{A}}(e_1) = \widehat{X}, G_B(e_2) = \widehat{X}$ &$(F, \check{A}) \cap (G, \check{B}) = \tilde{\phi}$.

Therefore $F_A \otimes G_k \in \tilde{\tau}_{1X} \otimes \tilde{\tau}_{1Y}, G_{\check{B}} \otimes Y_K \in \tilde{\tau}_{2\widehat{X}} \otimes \tilde{\tau}_{2Y}$
$(F_{\check{A}} \otimes \tilde{Y}_K)(e_1, k_1) = F_{\check{A}}(e_1) \times \tilde{Y}_K(k_1) = \widehat{X} \times \tilde{Y}$
$(G_{\check{B}} \otimes Y_K)(e_2, k_2) = G_{\check{B}}(e_2) \times Y_K(k_2) = \widehat{X} \times \tilde{Y}$

If for any $(e, k) \in (E \times K), (\beta F_A \otimes \tilde{Y}_K)(e, k) \neq \phi \Rightarrow \beta F_A(e) \times \tilde{Y}_K(k) \neq \phi$
$\Rightarrow F_{\check{A}}(e) \times \tilde{Y} \neq \phi \Rightarrow F_{\check{A}}(e) \neq \phi \Rightarrow G_{\check{B}}(e) = \phi$

Since $(F_A \cap G_{\check{B}} = \phi \Rightarrow F_A(e) \cap G_{\check{B}}(e) = \phi)$
$\Rightarrow G_{\check{B}}(e) \times \tilde{Y}_K(k) = \phi \Rightarrow (G_B \otimes \tilde{Y}_K)(e, k) = \phi$
$\Rightarrow (F_{\check{A}} \otimes \tilde{Y}_K) \cap (G_{\check{B}} \otimes \tilde{Y}_K) = \tilde{\phi}$

Assume $k_1 \neq k_2$.*Since* $\{Y, \tilde{\tau}_{1Y}, \tilde{\tau}_{2Y}, K\}$ is $(SBW-H)_2$ , there exist soft β open sets $(F, \check{E}) \in \tilde{\tau}_{1\tilde{Y}}, (G, \check{B}) \in \tilde{\tau}_{2\tilde{Y}}$ such that $F_{\check{A}}(k_1) =, \tilde{Y}\ G_{\check{B}}(k_2) = \tilde{Y}$ & $F_{\check{A}} \cap G_{\check{B}} = \tilde{\phi}$.

Therefore $X_E \otimes F_A \in \tilde{\tau}_{1X} \otimes \tilde{\tau}_{1\tilde{Y}}, \widehat{X}_E \otimes \beta G_{\check{B}} \in \tilde{\tau}_{2X} \otimes \tilde{\tau}_{2\tilde{Y}}$
$(\widehat{X}_E \otimes F_{\check{A}})(e_1, k_1) = \widehat{X}_E(e_1) \times F_{\check{A}}(k_1)$
$= \widehat{X} \times \tilde{Y}$
$(X_E \otimes G_{\check{B}})(e_2, k_2) = \widehat{X}_E(e_2) \times G_{\check{B}}(k_2)$
$= \widehat{X} \times \tilde{Y}$

If for any $(e, k) \in E \times K, (\widehat{X}_E \otimes F_{\check{A}})(e, k) \neq \phi \Rightarrow \widehat{X}_E(e) \times F_{\check{A}}(k) \neq \phi$
$\Rightarrow \widehat{X} \times \beta F_{\check{A}}(k) \neq \phi \Rightarrow F_{\check{A}}(k) \neq \phi \Rightarrow G_B(k) = \phi$

$(Since F_A \cap \beta G_B = \tilde{\phi} \Rightarrow \beta F_{\check{A}}(k) \cap G_B(k) = \phi) \Rightarrow X_E(e) \times G_B(k) = \phi \Rightarrow$ $(X_E \otimes G_B)(e, k) = \phi$
$\Rightarrow (\widehat{X}_E \otimes F_{\check{A}}) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence, $\left(\widehat{X} \times Y, \tilde{\tau}_{1\widehat{X}} \otimes \tilde{\tau}_{1Y}, \tilde{\tau}_{2\widehat{X}} \otimes \tilde{\tau}_{2\tilde{Y}}, E \times K\right)$ *is* $\beta(SBW-H)_2$

## 4. CONCLUSION

In this paper the concept of β W- Hausdorff structure in soft bi topological spaces is introduced and some basic properties regarding this concept are demonstrated. Topology is the most significant branch of mathematics which deals with mathematical structures. Recently, many investigators have deliberated the soft set theory which is originated by Molodtsov and carefully applied to many complications which comprise uncertainties in our social life. Shabir and Naz in familiarized and intensely studied the notion of soft topological spaces. They also deliberate topological structures and demonstrated their several belongings with respect to ordinary points.

In the present work, in this paper the concept of βW- Hausdorff space in soft bi topological spaces is introduced and some basic properties concerning this idea are verified. This soft structure would be useful for the growth of the theory of soft topology to bury complex problems, comprising doubts in economics, engineering, medical etc. We hope that these results in this paper will help the researchers for reinforcement the toolbox of soft topology. In the next study, we extend the concept of α-open, Pre-open and $b^{**}$open soft sets in soft bi topological spaces with respect to ordinary as well as soft points

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