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REVIEW ARTICLE

ANALYSIS OF MATHEMATICAL MODELING THE DEPLETION OF FORESTRY RESOURCE: EFFECTS OF POPULATION AND INDUSTRIALIZATION

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ABSTRACT

This paper is attempted to study the system of nonlinear differential equations in assessing the depletion of forest resource with population density and industrialization. It is evidenced that the forest resources are depleted with increase of population and industrialization. The asymptotic method of differential equations and numerical simulation are used to analyze this model. These analytical results are confirmed by using numerical simulation. Further, the graph of proposed model is compared with the real life data of the forestry resources, population density and industrialization in Tamil Nadu.

KEYWORDS

Forestry Biomass, Population, Asymptotic method; Simulation; Industrialization.

1. INTRODUCTION

The forestry resources are chiefly depleted due to over population and the highest industrialization rate. The human population uses forestry uses for its intrinsic growth in the form of fuel, fodder for cattle required for milk production, medicine, etc. directly by cutting trees, plants, herbs, grasses, etc, without clearing the forest land. Clearing of forests in the name of industrialization and development includes construction of buildings, complexes, roads, resorts, industries and other commercial activities. The rate of population is negatively correlated with natural stock and forest resources [1].

The depletion of forest biomass by human population and industrialization has been investigated both theoretically as well as experimentally by many [2-7]. In particular, a group researchers proposed a mathematical model for the depletion of forest resources [8,9]. The forest resources were doomed to extinct under growing population living in the forest or by industrialization which was wholly depended on forest resources. Agarwal and Pathak have analyzed a mathematical model for conservation of forestry biomass and wildlife population [10].

Recently, in a study had analyzed the model for the effect of population and industrialization on forestry resources by using stability analysis [11]. In view of the above literature, it is observed that none of them considered the effect of depletion of forest resources due to population growth human population and industrialization for sustainable management of forest resources.

To the best of our knowledge, no rigorous analytical solutions for the non-steady-state concentrations for depletion of forestry resource for experimental values of the parameters have been published. In this paper, we have derived analytical expression of the concentrations of population density, biomass density and industrialization using the Homotopy perturbation method.

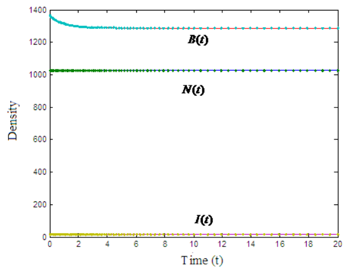

Figure 1: Variation of density N, B and I with time t for values of other parameters indicated in (Shyam Sundar et al., (2017)).

2. MATHEMATICAL MODEL

Let B be the cumulative biomass density of forest resources, N is the population density and I is the density of industrialization due to population biomass density is growing at intrinsic growth rate r, s *and* k with carrying capacity L. It is assumed further that the rural population density is also growing logistically with growth rate r and carrying capacity K *and* M. s_1, ϕ is the depletion rate coefficient of forestry resource, δ is the growth rate coefficient of the industrialization due to population, u_0 is the natural depletion rate coefficient of industrialization and u_1 is the depletion coefficient due to crowding effect. Keeping in view of these considerations, the non-linear model is proposed as follows [1]:

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) + v_1 s_1 BN \tag{1}$$

$$\frac{dB}{dt} = sB\left(1-\frac{B}{L}\right) - s_1 BN - \phi BI \tag{2}$$

$$\frac{dI}{dt} = uI\left(1-\frac{I}{M}\right) + \delta NI - u_0 I + u_1 I^2 \tag{3}$$

The initial conditions become

$$N(0) = N^*; \quad B(0) = B^*; \quad I(0) = I^* \tag{4}$$

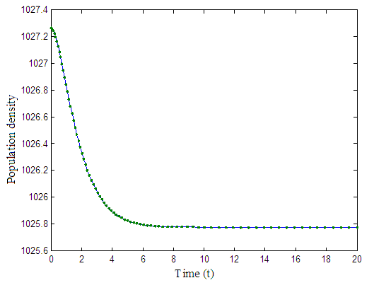

Figure 2: Variation of population density N with time t for values of other parameters indicated in (Shyam Sundar et al., (2017)).

3. ASYMTOTIC TOOLS OF NON-LINEAR DIFFERENTIAL EQUATION ON FOREST RESOURCE

Nonlinear differential equations are useful to get the best solution by little iteration. However, finding analytical solutions for this class of equations always has been a challenging task. In the recent years, several approximate methods were proposed for the analytical solution of nonlinear differential equations that do not depend on the existence of a small or large parameter in the equation. In addition, in the recent years, some novel methods for approximate solution of nonlinear differential equations emerged, such as Optimal Homotopy Asymptotic Method (OHAM) and Generalized Homotopy Method (GHM) [12,13]. In the past, many scientists attempted to suggest an improvement to HPM. Their studies were mainly focussed on enlarging convergence radius and accelerating convergence of the solution and new suggestions for homotopy construction as well as alternation of linear and nonlinear part of homotopy based on the applied problem. A group researchers discussed convergency of the proposed method [14]. In order to demonstrate efficiency, accuracy, and superiority of the suggested method, the new modification was applied to some experiments in their paper. An effective method for convergence improvement of HPM for the solution of fractional differential equations was suggested [15]. They proposed a method to select the linear part in the HPM to keep the inherent stability of fractional equations. Recently, we have analysed the analysis of modelling for induced resistance to plant disease using biological control agents by using Homotopy perturbation method have analyzed the non linear differential equation describing the depletion of forestry resources using this method [16]. Further, have analyzed the modelling the depletion of atmospheric fuzzy plankton-oxygen dynamics in a lake due to photosynthetic activity of ecosystem under Climate change using this HPM (see "Appendix A") [17]. The system of solution of above equation with corresponding initial conditions can be rewritten as:

$$N(t) = N^* e^{rt} + \left[\frac{N^{*2}}{k} - \frac{v_1 s_1 B^* I^*}{s+u-u_0-r}\right] e^{rt} + \left[\frac{v_1 s_1 B^* I^*}{s+u-u_0-r}\right] e^{(s+u-u_0)t} - \frac{N^{*2}}{k} e^{2rt} \tag{5}$$

$$B(t) = B^* e^{st} + \left[\frac{B^{*2}}{L} + \frac{s_1 B^* N^*}{r} + \frac{\phi B^* I^*}{u-u_0}\right] e^{st} - \frac{B^{*2} e^{2st}}{L} - \frac{s_1 B^* N^* e^{(s+r)t}}{r} - \frac{\phi B^* N^* e^{(s+u-u_0)t}}{u-u_0} \tag{6}$$

$$I(t) = I^* e^{(u-u_0)t} + \left[\frac{uI^{*2}}{M(u-u_0)} - \frac{\delta N^* I^*}{r} + \frac{u_1 I^{*2}}{u-u_0}\right] e^{(u-u_0)t} - \frac{uI^{*2}}{M(u-u_0)} e^{2(u-u_0)t} + \frac{\delta N^* I^* e^{(r+u-u_0)t}}{r} - \frac{u_1 I^{*2} e^{2(u-u_0)t}}{u-u_0} \tag{7}$$

4. NUMERICAL SIMULATION

In this section, the values of parameters which are used in simulations for system of equations (1-3) are following parameters [1]:

$r = 1, K = 1000, v_1 = 0.2, s_1 = 0.0001, s = 1.2, L = 1500, \phi = 0.0005, u = 0.1, M = 10,$
$\delta = 0.0003, u_0 = 0.01, u_1 = 0.02.$

Numerical simulations are observed to know the effect the variation of parameters. It can be easily verified that the condition of nonlinear differential equations (1-3) could be solved using analytical method and compared with help of function pdex in MATLAB. Satisfactory results are observed. Appendix B denotes Matlab program. The estimation of surveillance of Industrial growth rate, population density and biomass density during 1990-2017 is shown in Tables 1-3 and Figures 5-7. This model equation is compared with the real-life data of the forestry resources, population density and industrialization in Tamil Nadu using graphical representation which gives satisfactory agreement is noted.

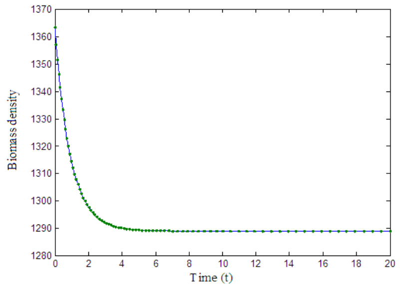

Figure 3: Variations of biomass density B versus time t for values of other parameters indicated in (Shyam Sundar et al., (2017)).

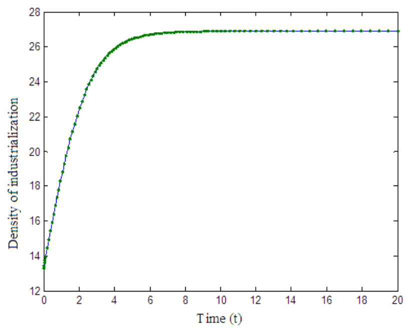

Figure 4: Variations of density of industrialization I versus time t for values of other parameters indicated in (Shyam Sundar et al., (2017)).

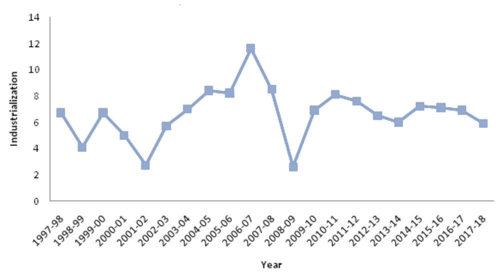

Figure 5: The graph of proposed model is compared with the real life data of the Industrial growth rate in India versus time in years.

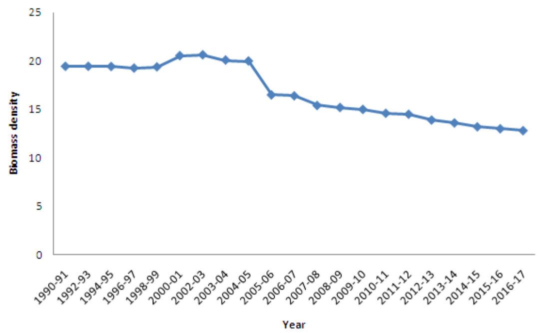

Figure 6: The graph of proposed model is compared with the real life data of the biomass density in India versus time in years.

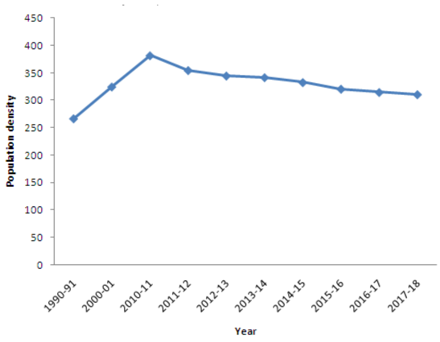

Figure 7: The graph of proposed model is compared with the real life data of the population density in India versus time in years.

Table 1: Industrial growth rate versus time in years for depletion rate coefficient in www.indiastat.com

Year	Analytical value	Numerical value	Error(%)
1997-98	6.7	6.71	0.1493
1998-99	4.1	4.12	0.4878
1999-00	6.7	6.69	0.1493
2000-01	5	5.01	0.2
2001-02	2.7	2.65	1.8519
2002-03	5.7	5.75	0.8772
2003-04	7	7.12	1.7143
2004-05	8.4	8.34	0.7143
2005-06	8.2	8.15	0.6098
2006-07	11.6	11.57	0.2586
2007-08	8.5	8.54	0.4706
2008-09	2.6	2.63	1.1538
2009-10	6.9	6.78	1.7391
2010-11	8.1	7.99	1.3580
2011-12	7.6	7.56	0.5263
2012-13	6.5	6.56	0.9231
2013-14	6	5.99	0.1667
2014-15	7.2	7.1	1.3889
2015-16	7.1	7.14	0.5634
2016-17	6.9	6.97	1.0145
2017-18	5.9	5.87	0.5085

Table 2: Biomass density versus time in years for depletion rate coefficient in www.indiastat.com.

Year	Analytical value	Numerical value	Error(%)
1990-91	19.45	19.44	0.0514
1992-93	19.45	19.44	0.0514
1994-95	19.43	19.4	0.1544
1996-97	19.27	19.25	0.1038
1998-99	19.39	19.3	0.4642
2000-01	20.55	20.5	0.2433
2002-03	20.64	20.61	0.1453
2003-04	20.1	20	0.4975
2004-05	20	20.1	0.5
2005-06	16.5	16.7	1.2121
2006-07	16.4	16.3	0.6098
2007-08	15.4	15.43	-0.1948
2008-09	15.2	15.32	-0.7895
2009-10	15	15.12	-0.8
2010-11	14.6	14.59	0.0685
2011-12	14.5	14.34	1.1034
2012-13	13.9	13.78	0.8633
2013-14	13.6	13.56	0.2941
2014-15	13.2	13.3	-0.7576
2015-16	13	12.97	0.2308
2016-17	12.8	12.65	1.1719

Table 3: Population density versus time in years for depletion rate coefficient in www.indiastat.com.

Year	Analytical value	Numerical value	Error(%)
1900-01	77	76	1.2987
1910-11	82	81.4	0.7317
1920-21	81	80.8	0.2469
1930-31	90	89.7	0.3333
1940-41	103	102.4	0.5825
1950-51	117	116.8	0.1709
1960-61	142	143.2	0.8451
1970-71	177	177.6	0.3389
1980-81	216	216.4	0.1852
1990-91	267	268	0.3745
2000-01	325	325.4	0.1231
2010-11	382	382.4	0.1047
2011-12	355	356	0.2817
2012-13	345	345	0
2013-14	342	343.2	0.3509
2014-15	333	334.2	0.3604
2015-16	321	320.7	0.0935
2016-17	315	316.3	0.4127
2017-18	311	312.2	0.3859

5. RESULT AND DISCUSSION

The variation of population, biomass and industrialization densities with time for value of depletion rate coefficient is plotted in figure 1. The graphs show that depletion rate coefficient of the resource biomass reduction corresponding with due to increased population increases or depletion rate coefficient of forestry resource biomass due to industrialization increases, then density of forestry resource biomass decreases [18-22]. The population density as a function of time for value of depletion rate co-efficient of forestry resource due to industrialization is plotted in figure 2. From this figure, we note that the depletion rate coefficient decreases with increase in population. In figure 3, the depletion of forestry resource biomass density due to industrialization decreases as the depletion rate coefficient due to industrialization increases. The variation of industrialization density with time for value of growth rate coefficient of industrialization due to population is depicted in figure 4, from which it is found that the density of industrialization increases with increase in the value of growth rate coefficient

6. CONCLUSIONS

In this paper, we have proposed a nonlinear mathematical model. The nonlinear model is analyzed using the asymptotic tools of differential equations. The effect of population and industrialization is studied analytically and numerically. Numerical simulation has been performed to justify the analytical findings and graphs are plotted to study the variations of important variables of the system with time for different parameters. The conditions for the system of analytical solution have been determined and these conditions are further justified numerically by considering a given set of parameter values. The model analysis has shown that as the rate of awareness programs increases the value of the cumulative biomass density increases. This implies that as human population become aware about the sustainable management of resources, the original level of the biomass density can be maintained.

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APPENDIX A

Solution of the equations (10) to (12) using Homotopy perturbation method.

In this Appendix, we indicate how Eqs. (1) to (3) in this paper is derived. To find the solution of Eqs. (5) to (7) we first construct a Homotopy as follows:

$$(1-p)\left[\frac{dN}{dt}-rN\right]+p\left[\frac{dN}{dt}-rN+\frac{rN^2}{k}-v_1s_1BN\right]=0 \quad \text{(A.1)}$$

$$(1-p)\left[\frac{dB}{dt}-sB\right]+p\left[\frac{dB}{dt}-sB+\frac{sB^2}{L}+s_1BN+\phi BI\right]=0 \quad \text{(A.2)}$$

$$(1-p)\left[\frac{dI}{dt}-(u-u_0)I\right]+p\left[\frac{dI}{dt}-(u-u_0)I+\frac{uI^2}{M}-\delta NI+u_1I^2\right]=0 \quad \text{(A.3)}$$

and the initial approximations are as follows:

$$N(0)=N^*;B(0)=B^*;I(0)=I^* \quad \text{(A.4)}$$

(A.5)

$$t=0;\,N_i=0;\,B_i=0;\;I_i=0$$

And

$$\begin{cases} N=N_0+pN_1+p^2N_2+p^3N_3+\dots\dots\dots \\ B=B_0+pB_1+p^2B_2+p^3B_3+\dots\dots. \\ I=I_0+pI_1+p^2I_2+p^3I_3+\dots\dots. \end{cases} \quad \text{(A.6)}$$

Substituting Eq. (A.6) into Eqs. (A.1) and (A.2) and (A.3) and arranging the coefficients of powers p, we can obtain the following differential equations

$$p^0:\quad \frac{dN_0}{dt}-rN_0=0 \quad \text{(A.7)}$$

$$p^1:\quad \frac{dN_1}{dt}-rN_1+\frac{rN_0^2}{k}-v_1s_1B_0N_0 \quad \text{(A.8)}$$

And

$$p^0:\quad \frac{dB_0}{dt}-sB_0=0 \quad \text{(A.9)}$$

$$p^1:\quad \frac{dB_1}{dt}-sB_1+\frac{sB_0{}^2}{L}+s_1B_0N_0+\phi B_0I_0=0 \quad \text{(A.10)}$$

$$p^0:\quad \frac{dI_0}{dt}-(u-u_0)I_0=0 \quad \text{(A.11)}$$

$$p^1: \quad \frac{dI_1}{dt} - (u-u_0)I_1 + \frac{uI_0^{\ 2}}{M} - \delta N_0 I_0 + u_1 I_0^{\ 2} = 0 \tag{A.12}$$

we can find the following results

$$N_0(t) = N^* e^{rt} \tag{A.13}$$

$$N_1(t) = \left[\frac{N^{*2}}{k} - \frac{v_1 s_1 B^* I^*}{s+u-u_0-r}\right]e^{rt} + \left[\frac{v_1 s_1 B^* I^*}{s+u-u_0-r}\right]e^{(s+u-u_0)t} - \frac{N^{*2}}{k}e^{2rt} \tag{A.14}$$

$$B_0(t) = B^* e^{st} \tag{A.15}$$

$$B_1(t) = \left[\frac{B^{*2}}{L} + \frac{s_1 B^* N^*}{r} + \frac{\phi B^* I^*}{u-u_0}\right]e^{st} - \frac{B^{*2} e^{2st}}{L} - \frac{s_1 B^* N^* e^{(s+r)t}}{r} - \frac{\phi B^* N^* e^{(s+u-u_0)t}}{u-u_0} \tag{A.16}$$

And

$$I_0(t) = I^* e^{(u-u_0)t} \tag{A.17}$$

$$I_1(t) = \left[\frac{uI^{*2}}{M(u-u_0)} - \frac{\delta N^* I^*}{r} + \frac{u_1 I^{*2}}{u-u_0}\right]e^{(u-u_0)t} - \frac{uI^{*2}}{M(u-u_0)}e^{2(u-u_0)t} + \frac{\delta N^* I^* e^{(r+u-u_0)t}}{r} - \frac{u_1 I^{*2} e^{2(u-u_0)t}}{u-u_0} \tag{A.18}$$

According to the HPM, we can conclude that

$$N(\rho) = \lim_{p \to 1} N(\rho) = N_0 + N_1 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{A.19}$$

$$B(\rho) = \lim_{p \to 1} B(\rho) = B_0 + B_1 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{A.20}$$

$$I(\rho) = \lim_{p \to 1} I(\rho) = I_0 + I_1 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{A.21}$$

After putting Eqs.(A.13) and (A.14) into Eq. (A.19), Eqs.(A.15) and (A.16) into Eq. (A.20) and Eqs. (A.17) and (A.18) into Eq.(A.21), the final results can be described in Eqs. (5) to (7) in the text. The remaining components of $N_n(x), B_n(x)$ and $I_n(x)$ be completely determined such that each term is determined by the previous term.

APPENDIX B

MATLAB Programme to find the numerical solution of the non-linear differential equations (1) to (3)

```
function main
options= odeset('RelTol',1e-6,'Stats','on');
%initial conditions
N1=1027.265927;
B1=1363.296347;
I1=13.2726592;
Xo = [N1, B1, I1];
tspan = [0,20];
xspan = [0,15];
tic
[t,X] = ode45 (@TestFunction,tspan, Xo,options);
toc
figure
plot(t,X(:,3))
ylabel('x')
xlabel('t')
return
function [dx_dt]= TestFunction(t,x)
r=1;
k=1000;
v1=0.2;
s1=0.0001;
s=1.2;
L=1500;
o=0.005;
u=0.1;
m=10;
d=0.0005;
u0=0.01;
u1=0.02;
dx_dt(1) =r*x(1)*(1-(x(1)/k))+v1*s1*x(2)*x(1);
dx_dt(2) =s*x(2)*(1-(x(2)/L))-s1*x(2)*x(1)-o*x(2)*x(3);
dx_dt(3) =u*x(3)*(1-(x(3)/m))+d*x(1)*x(3)-u0*x(3)-u1*x(3)^2;
dx_dt = dx_dt';
return
```